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Performance investigation of LAMBDA and bootstrapping methods for PPP narrow-lane ambiguity resolution

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ABSTRACT

Precise point positioning with ambiguity resolution (PPP-AR) is a powerful tool for geodetic and time-constrained applications that require high precision. The performance of PPP-AR highly depends on the reliability of the correct integer carrier-phase ambiguity estimation. In this study, the performance of narrow-lane ambiguity resolution of PPP using the Least-squares AMBiguity Decorrelation (LAMBDA) and bootstrapping methods is extensively investigated using real data from 55 IGS stations over one-month in 2020. Static PPP with 24-, 12-, 8-, 4-, 2-, 1- and ½-h sessions using two different cutoff angles (7° and 30°) was conducted with three PPP modes: i.e. ambiguity-float and two kinds of ambiguity-fixed PPP using the LAMBDA and bootstrapping methods for narrow-lane AR, respectively. The results show that the LAMBDA method can produce more reliable results for 2 hour and shorter observation sessions compared with the bootstrapping method using a 7° cutoff angle. For a 30° cutoff angle, the LAMBDA method outperforms the bootstrapping method for observation sessions of 4 h and less. For long observation times, the bootstrapping method produced much more accurate coordinates compared with the LAMBDA method without considering the wrong fixes cases. The results also show that occurrences of fixing the wrong integer ambiguities using the bootstrapping method are higher than that of the LAMBDA method.

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Ambiguity resolution; narrow-lane; Precise point positioning (PPP); LAMBDA; bootstrapping

1. Introduction

Precise point positioning (PPP) has been increasingly used in many geoscience applications, thanks to its powerful and cost-effective technique (Li et al. 2015; Labib et al. 2019). High positioning accuracy (cm to mm) can be maintained anywhere in the world using a single Global Navigation Satellite System (GNSS) receiver without using reference stations (Zumberge et al. 1997). The main drawback of PPP related to convergence time can be mitigated by implementing ambiguity resolution (AR) (Zhao et al. 2020). The AR realization requires estimating the uncalibrated phase delay (UPD) biases arising in both receivers and satellites. In addition to that, satellite orbit and clock products need to be consistent with the produced UPD biases (Banville et al. 2020). Without AR, lower precision – especially in the east component – and much longer convergence time are generally observed in PPP compared with the relative positioning technique. The majority of online PPP GNSS services, such as MagicGNSS, CSRS-PPP (launch of PPP-AR service planned for the end of 2020) (Alkan et al. 2015), GAPS, and some PPP software such as Bernese, RTKLIB, and GpsTools perform only ambiguity-float PPP (Mohammed et al. 2016).

After removing the hardware delay biases arising from the receiver and satellite, the integer property of

single-station ambiguities can be retrieved in PPP. Several methods have been used for the estimation of integer ambiguities. Commonly used methods are Least-squares AMBiguity Decorrelation Adjustment (LAMBDA) (Teunissen 1995) and bootstrapping (Teunissen 1998).

Several studies have been conducted on the performance of LAMBDA and bootstrapping methods for AR in relative positioning. Teunissen (2007) investigates the effect of ambiguity precision on the performance of bootstrapping. The results show that when the precision of the ambiguities computed from the float solution is higher, the probability of fixing the correct integer becomes larger. Teunissen, Joosten, and Tiberius (2003) investigate LAMBDA, TCAR (Three-Carrier Ambiguity Resolution), and CIR (Cascading Integer Resolution) techniques for relative positioning. According to the results, the highest probability of success rate can be obtained using LAMBDA compared with TCAR and CIR. Teunissen, De Jonge, and Tiberius (1997) analyzes the LAMBDA method performance on relative positioning. The results indicate that the LAMBDA method is generally successfully applicable to any GPS model or application. In theory, the integer least-squares principle maximizes the probability of correct integer estimation. The bootstrapping method can also achieve a high correct estimation

probability and has an advantage over the computation burden. As to the narrow-lane PPP-AR, Loyer et al. (2012) and Katsigianni et al. (2019) employ the bootstrapping method, while many other researchers employ the LAMBDA method (Geng et al. 2009; Li et al. 2016; Xiao et al. 2019). Wen et al. (2020) perform PPP-AR using bootstrapping and LAMBDA methods for wide-lane and narrow-lane ambiguities, respectively, with dual-frequency android GNSS data. The results show that up to 80% of total epochs can be fixed and cm-level three-dimensional accuracy is obtained using the both methods. Zhang, Chen, and Yuan (2019) investigate AR performance of PPP-RTK based on undifferenced and uncombined observations. It is found that double-differenced ambiguities within the Crustal Movement Observation Network of China (CMONOC) network can be resolved efficiently using the LAMBDA method. Zhang, Teunissen, and Odijk (2011) also investigate PPP-RTK approach using two GPS CORS networks with inter-station distances ranging from 60 to 100 km. The LAMBDA method was used for AR within these two CORS networks. The results indicate that ambiguity resolution with a success rate of about 99.7% was obtained.

Access to software to conduct PPP-AR, along with satellite and clock products that are consistent with the UPD bias products, is limited because several analysis centers (ACs) produce these products for internal purposes (Banville et al. 2020; Capilla, Berné-Valero, and Hermosilla-Rodrigo 2017). Wuhan AC have been producing phase/clock bias products using the integer clock model (Laurichesse et al. 2009) which is consistent with CODE (Centre for Orbit Determination in Europe) final orbit. In this study, these phase/clock bias products were used for PPP-AR. The products have been publicly available since 2006. For more detailed information about producing phase/clock biases, the reader is referred to Geng et al. (2019a). The open-source GNSS PPP software named PRIDE PPP-AR has also been publicly released by Wuhan University (Geng et al. 2019b).

In this study, the performance of bootstrapping and LAMBDA methods on PPP narrow-lane (NL) AR was investigated using PRIDE PPP-AR software, while considering different observation length and satellite geometry scenes. In the following sections, the PPP functional model and bootstrapping and LAMBDA methods are briefly introduced. Details of the data processing are given in Section 4. The results are summarized in Section 5, and the conclusions are presented in the last section.

2. Ionosphere-free PPP functional model

The ionosphere-free (IF) code and carrier-phase observations can be expressed for a particular epoch as (Leick, Rapoport, and Tatarnikov 2015)

$$P_{IF,r}^s = \rho + c(dt_r - dt^s) + d_{trop} + HD_{P,r,IF} - HD_{P,s,IF} + \epsilon_{P,IF}^s = (f_1^2 P_1 - f_2^2 P_2) / (f_1^2 - f_2^2) \quad (1)$$

$$\emptyset_{IF,r}^s = \rho + c(dt_r - dt^s) + d_{trop} + \lambda_{IF}^s N_{r,IF}^s + HD_{\emptyset,r,IF} - HD_{\emptyset,s,IF} + \epsilon_{\emptyset,IF}^s = (f_1^2 \emptyset_1 - f_2^2 \emptyset_2) / (f_1^2 - f_2^2) \quad (2)$$

where superscript s and r denote the satellite and receiver, respectively; $P_{IF,r}^s$ and $\emptyset_{IF,r}^s$ are the code and phase IF combination, respectively; ρ denotes the geometric range between the receiver and satellite; dt_r and dt^s denote the receiver and satellite clock offset, respectively; d_{trop} is the tropospheric delay including wet and dry components; λ_{IF}^s and $N_{r,IF}^s$ are IF wavelength and IF carrier phase ambiguity, respectively; $HD_{P,r,IF}$ and $HD_{\emptyset,r,IF}$ denote the IF receiver-related code and phase hardware delay, respectively; $HD_{P,s,IF}$ and $HD_{\emptyset,s,IF}$ are their counterparts for satellite-related delays; $\epsilon_{P,IF}^s$ and $\epsilon_{\emptyset,IF}^s$ are the unmodeled errors of IF for code and carrier phase measurements; f_1 and f_2 are two frequencies; P_1 , P_2 , \emptyset_1 , and \emptyset_2 are the code and phase observations on the two frequencies. $HD_{P,s,IF}$ is lumped into the dt^s . Because clock parameters are computed using IF code measurements, estimation of dt_r absorbs the $HD_{P,r,IF}$. $HD_{\emptyset,r,IF}$ and $HD_{\emptyset,s,IF}$ are lumped into the ambiguities ($N_{r,IF}^s$). Therefore, the integer property of $N_{r,IF}^s$ is destroyed (Xiao et al. 2019).

The IF wavelength (λ_{IF}) and ambiguity (N_{IF}) can be written as

$$\lambda_{IF} = \frac{c}{f_1 + f_2} \quad (3)$$

$$\widetilde{N}_{IF} = N_1 - \frac{1}{2} \times \left(\frac{\lambda_{WL}}{\lambda_{IF}} - 1 \right) \times (N_2 - N_1) \quad (4)$$

where λ_{WL} is the wide-lane wavelength. N_1 and N_2 are the float ambiguities of first and second frequencies. The satellite-related hardware-phase delays ($HD_{\emptyset,s,IF}$) can be estimated within the network using the double differenced (for ambiguity estimation) and undifferenced observations. There are several techniques to estimate $HD_{\emptyset,s,IF}$ at the server side and to calibrate it at the user side, such as the “integer clock model” (Laurichesse et al. 2009), the “decoupled clock model” (Collins et al. 2010), the “uncalibrated phase delay” (Ge et al. 2008) and the “modified phase clock/bias model” (Geng et al. 2019a). After estimating the $HD_{\emptyset,s,IF}$ within the network of GNSS stations, IF ambiguity is usually decomposed into wide- and narrow-lane ambiguities, due to the fact that \emptyset_1 and \emptyset_2 ambiguities cannot be estimated simultaneously in the IF PPP approach because of the rank deficiency in the normal equation system (Ge et al. 2008). The float wide-lane (WL) ambiguity can be expressed by the

Hatch–Melbourne–Wübbena (HMW) combination (Hatch 1982; Melbourne 1985; Wübbena 1985):

$$HMW = \frac{f_1 \times \vartheta_1 - f_2 \times \vartheta_2}{f_1 - f_2} - \frac{f_1 \times P_1 + f_2 \times P_2}{f_1 + f_2} \quad (5)$$

$$= \lambda_{WL} \times \widetilde{N}_{WL}$$

Wide-lane wavelength can be expressed as

$$\lambda_{WL} = \frac{c}{f_1 - f_2} \quad (6)$$

Because the HMW combination is an independent function of geometry, ionosphere, and troposphere, code noise and multipath are the major factors that affect its accuracy. After resolving the wide-lane ambiguities (\widetilde{N}_{WL}), the float NL ambiguities \widetilde{N}_{NL} can be expressed using IF ambiguities as (Li et al. 2018)

$$\lambda_{NL} \times \widetilde{N}_{NL} = \lambda_{IF} \times \widetilde{N}_{IF} - \frac{c \times f_2}{f_1^2 - f_2^2} \times N_{WL} \quad (7)$$

NL wavelength can be expressed as:

$$\lambda_{NL} = \frac{c}{f_1 + f_2} \quad (8)$$

The IF ambiguity can only be fixed when both WL and NL ambiguities are estimated as integers. If some float NL ambiguities cannot be fixed to the integer, partial ambiguity resolution can be conducted (Li and Zhang 2015). Fixing the NL ambiguity is much more difficult than fixing WL ambiguity due to the short wavelength of NL combination (10.7 cm). As a result, the NL combination is easily influenced by residual atmosphere delay and multipath. Moreover, NL ambiguities are more correlated than WL ambiguities because WL ambiguities are directly derived from the HMW combination; as a result, they are not correlated to each other. The bootstrapping method or the rounding strategy can be successfully applied to estimate the correct integer of WL ambiguities, while the LAMBDA method (search strategy) can be chosen to fix the NL ambiguities because of the high correlation between NL ambiguities. In static PPP, the estimated parameters can be written as

$$X = \left[\bar{x}, dt_r, Z_{wet}, Grad_{ns}, Grad_{ew}, \widetilde{N}_{IF} \right] \quad (9)$$

where \bar{x} is the vector of the receiver position update with respect to the a priori position, Z_{wet} is the tropospheric zenith wet delay, and $Grad_{ns}$ and $Grad_{ew}$ denote the north–south and east–west tropospheric gradients, respectively. The bootstrapping and LAMBDA methods are briefly introduced in the following section.

3. Integer ambiguity search and validation

Estimating the correct integer ambiguity requires searching and validating the best candidates of the integer ambiguities. Ambiguity float solutions and their variance–covariance matrix are used to search

for the correct integer ambiguities. Bootstrapping and LAMBDA methods are among the searching methods for integer candidates around the float (real-valued) solutions. The LAMBDA method is based on integer least squares. The LAMBDA method enables the decorrelation of the ambiguities; thus, the search space scales to a large degree of accuracy (Teunissen 1998). The LAMBDA method consists of two stages. First, decorrelation of the ambiguities is maintained using Z-transformation. Then, the search for integer candidates is conducted. When the number of ambiguities increases, the search space also extends. Therefore, the efficiency of LAMBDA decreases. The bootstrapping method is the generalization of integer rounding, and it is based on sequential conditioning rounding while some of the correlations between ambiguities are taken into consideration (Teunissen 2005). Computation of the bootstrapping method is relatively easier than LAMBDA because fixing ambiguities to the integers can be performed directly without considering all correlations between ambiguities (Teunissen 2007).

Ambiguity validation is based on testing the best available candidates and determining whether any candidates should be accepted (Feng and Jokinen 2017). The ambiguity validation procedure generally involves a statistical test, distribution of the ambiguities, and determination of thresholds. Ratio, F-ratio, and W-ratio are commonly used statistical tests for ambiguity validation. For detailed information on the statistical tests, the reader is referred to Li and Wang (2012).

4. Data processing

Fifty-five geographically well-distributed IGS stations were selected to investigate PPP NL AR performance using the LAMBDA and bootstrapping methods. Figure 1 shows the geographic distribution of the selected stations. One month (January) in 2020 was chosen to perform static PPP processes using 24-, 12-, 8-, 4-, 2-, 1- and ½-h non-overlapped sessions for each IGS station. PPP using three different modes was conducted – namely, float, NL AR with LAMBDA (AR_1), and NL AR with bootstrapping (AR_2). The bootstrapping method was used to fix WL ambiguities for each PPP AR mode. Two different cutoff angles (7° and 30°) were chosen to investigate the performance of the methods under different satellite geometries. To assess the accuracy, reference coordinates of the stations were taken from the IGS weekly solutions.

The epoch availability of each RINEX file was checked using in-house software. The data availability threshold was set to 98% and, RINEX files below this threshold were discarded from the processing. PRIDE PPP-AR open-source software was used to process each PPP. The processing parameters are given in Table 1.

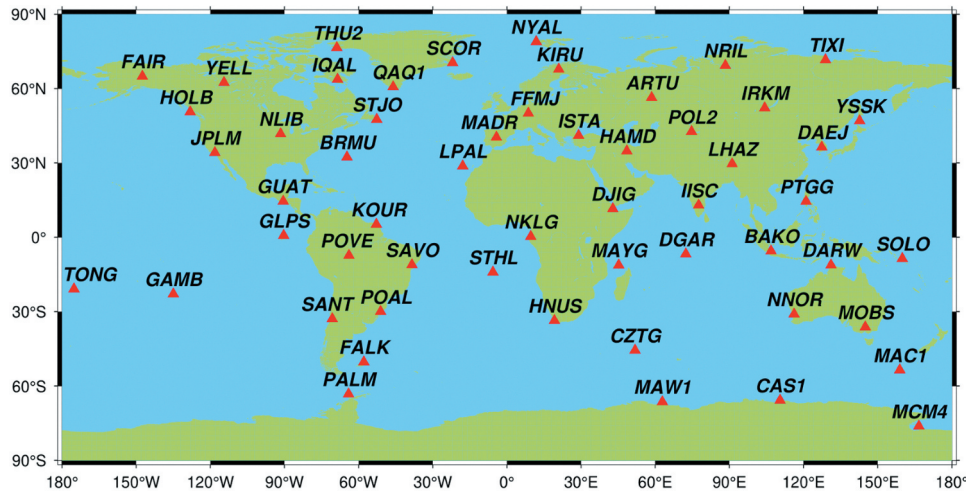


Figure 1. Geographic distribution of the selected IGS stations.

Table 1. PPP processing parameters.

GNSS system	GPS
Orbit and clock	CODE final orbit and estimated integer phase clock
AR products	WUHAN phase/clock bias products
Adjustment model	Least squares adjustment, smoothed with forward and backward filtering (Vaclavovic and Dousa 2015)
Epoch interval	30 s
Elevation cutoff angle	7°/30°
Weighting strategy	$W = 1$ for $e > 30^\circ$; $W = 4 * \sin^2$ for $e < 30^\circ$ where W is the weight scaling and e is the satellite elevation
Satellite/receiver phase center	Up-to-date IGS14.atx
Ionospheric effect	Removed by IF linear combination
Phase Ambiguities	Float/AR_1/AR_2
AR validation	LAMBDA AR: Ratio test with 3.0 threshold Bootstrapping AR: No validation applied. Bias rounding criterion: 0.15 cycle with 0.15 sigma threshold in cycle
Intra-Frequency Bias	GPS C1-P1 code bias was corrected using monthly DCB file.
Troposphere	GMF model with piece-wise constant with Saastomoinen model
Zenith wet delay estimation	Piece-wise constant
Horizontal delay gradients estimation	Piece-wise constant
Phase windup	Corrected
Solid earth, ocean tide loading and polar tides	IERS conventions, 2003

5. Results and analysis

For convenient visualization, PPP results using float ambiguity estimates for each cutoff angle and observation session are provided in Table 2. In order to investigate the performance of NL AR using the bootstrapping and LAMBDA methods, accuracy improvements in north, east and up components from the two AR modes with respect to the float PPP solutions are given in Figures 3 and 4 for each observation session and cutoff angle. The threshold of the outliers was determined as 10 cm for each PPP processing and each station. If the error of any of the coordinate components (N , E , U) with respect to the reference solution was greater than 10 cm, it was discarded from

Table 2. RMSEs computed from the float PPP (unit: cm).

Sessions (h)	Cutoff: 7°			Cutoff: 30°		
	N	E	U	N	E	U
24	0.3	0.4	1.4	0.4	0.6	2.4
12	0.4	0.6	1.5	0.6	0.8	2.8
8	0.4	0.7	1.6	0.6	1.0	3.1
4	0.6	1.0	1.9	0.8	1.5	3.5
2	0.8	1.7	2.4	1.3	2.5	4.1
1	1.2	2.7	3.1	1.9	3.4	4.8
0.5	1.8	3.3	4.0	2.6	3.9	5.2

the root mean square error (RMSE) computation. Figure 2 shows the outlier percentages for the PPP float and the AR_1 and AR_2 solutions for each cutoff angle.

As expected, the number of outliers using a 30° cutoff angle is significantly higher compared with a 7° cutoff angle. A high degree of cutoff angle significantly restricts the redundancy of the GPS-only PPP. After excluding the outliers for each PPP mode, only the coordinate solutions belonging to identical times between the three PPP modes were used. In this way, RMSEs were computed using the estimated solutions from three PPP modes sharing the same observation conditions.

As can be seen from the results of the improvements, NL fixing with LAMBDA (AR_1) produces more accurate coordinates for short observation times compared with NL fixing with bootstrapping (AR_2). As expected, improvements in east components are much bigger than north and up components. The results show that the bootstrapping method produces much more accurate results for observation sessions longer than 2 h, while the LAMBDA method performs much better than bootstrapping for observation sessions equal to and shorter than 2 h for the 7° cutoff angle. For the 30° cutoff angle, the performance of the LAMBDA method is much greater than the bootstrapping method for observation sessions of 4 h and less. As the cutoff angle increases, the number of ambiguities decreases so the search space of LAMBDA

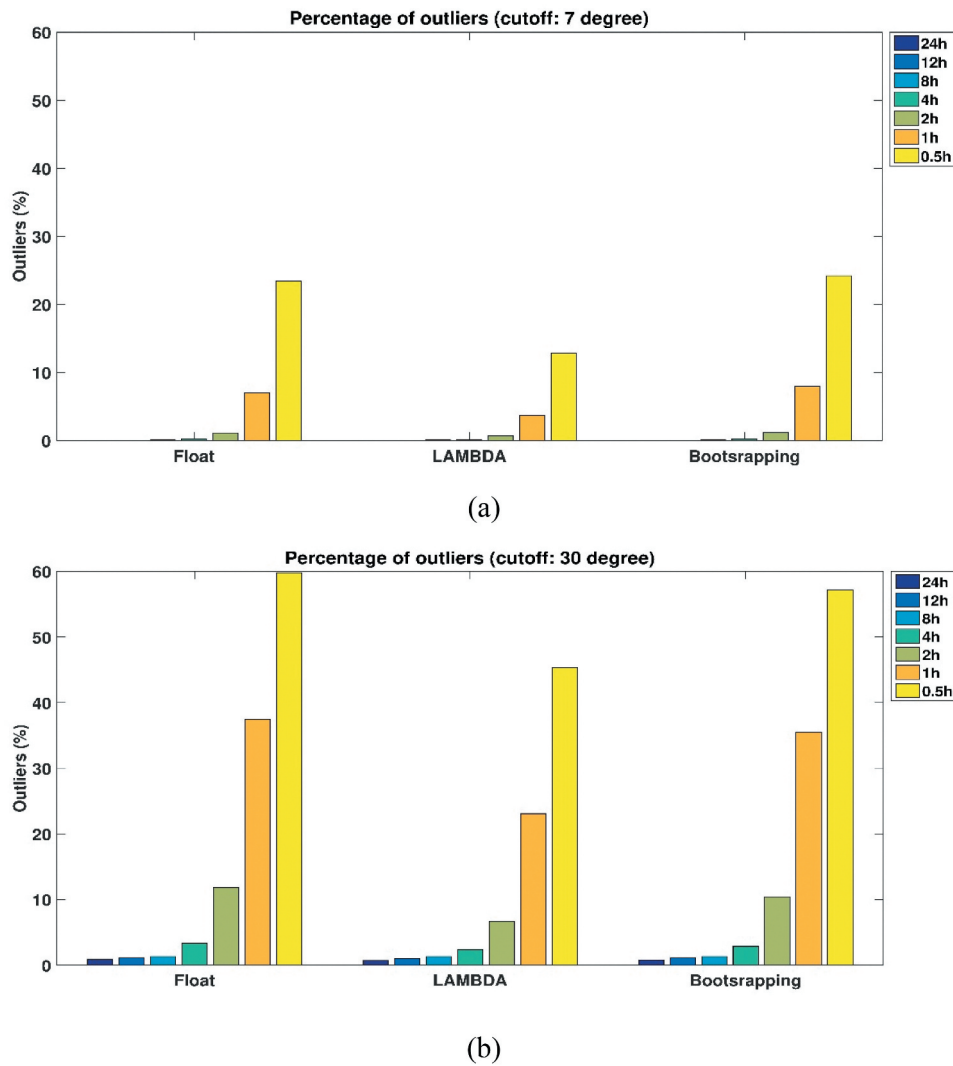


Figure 2. Outlier percentages for the PPP float and AR modes for the cutoff of 7° (a) and 30° (b).

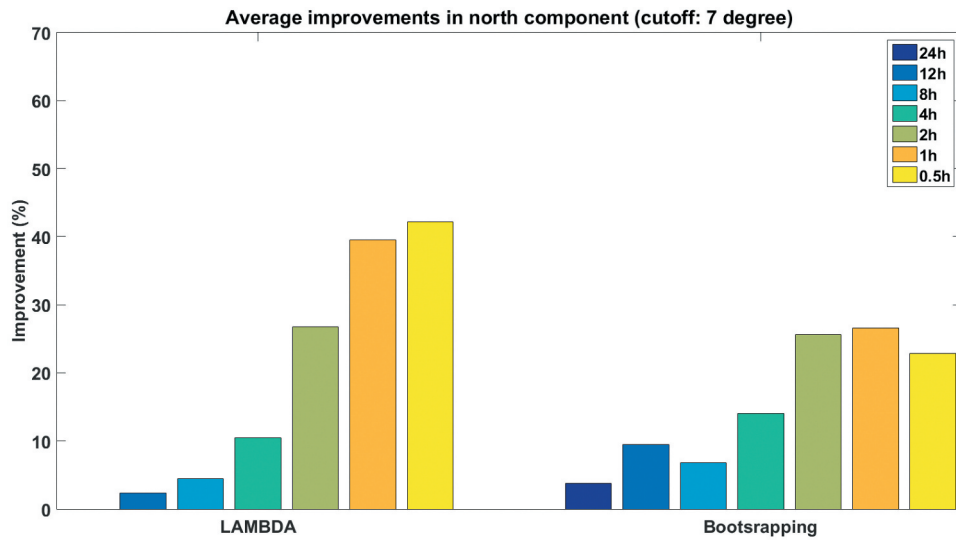
does not extend as much as the 7° cutoff angle. It can also be observed that the LAMBDA method leads the slight accuracy degradation with daily data for the 7° cutoff angle. It is related that some ambiguities that belong to the identical epoch time could not be fixed correctly by the LAMBDA method (Ogutcu 2020). When the RMSEs were computed without the outlier removal, accuracy improvement in all coordinate components with respect to the float solution was seen for the LAMBDA method. In Figure 3(a), the improvement percentage for the north component of the LAMBDA for 24 h session, and in Figure 3(c), the improvement percentage for the vertical component of the LAMBDA method for 12 h and 8 h are very close to zero therefore, they are not visible in Figure 3 (a,c).

The outlier results indicate that the LAMBDA method produced a smaller number of outliers than the bootstrapping method, especially for short observation sessions. The wrong fixing of integer candidates is investigated for each AR technique. A wrong fix solution from LAMBDA and bootstrapping methods is validated if any fixed solution (after AR validation)

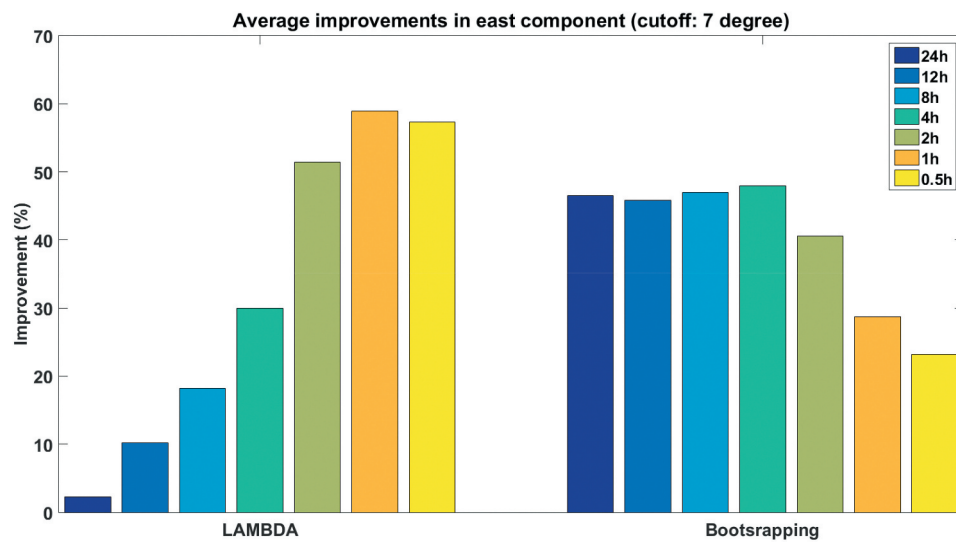
is within the outliers but its float counterpart is not an outlier solution. Figure 5 shows the percentages of wrong fixing for each AR method and cutoff angle.

When the wrong fixing results are examined, it is seen that the number of wrong fixes is significant for observation sessions of 0.5 h and 1 h for the 7° cutoff angle, and the number of wrong fixes for the 30° cutoff angle is significant for 4 h and shorter observation sessions for each AR method. As satellite visibility decreases, the arch length of the satellites shortens, and this leads to unreliable AR.

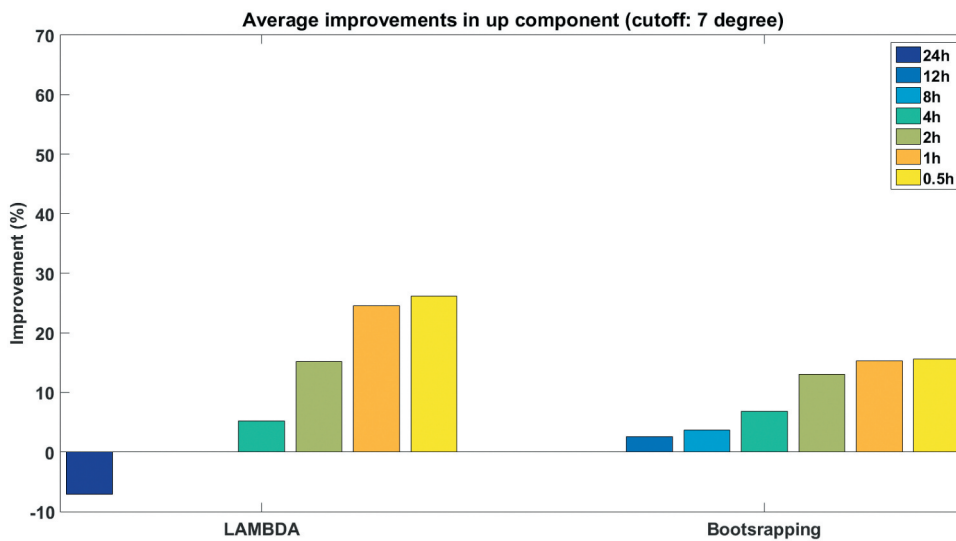
The wrong fixing results are similar to the overall performance of the AR methods. For short observation times, the LAMBDA method produces fewer numbers of wrong fixes compared with the bootstrapping method. For the 7° cutoff angle, the number of wrong fixes of the AR methods is almost the same for 24, 12, and 8 h observation sessions. For observation sessions of 4 h and shorter, the LAMBDA method produces fewer numbers of wrong fixes compared with the bootstrapping method. For the 30° cutoff angle, the number of wrong fixes is similar among the methods from the 24 h to 4 h observation sessions,



(a)



(b)



(c)

Figure 3. Accuracy improvements of AR_1 and AR_2 modes with a 7° cutoff angle.

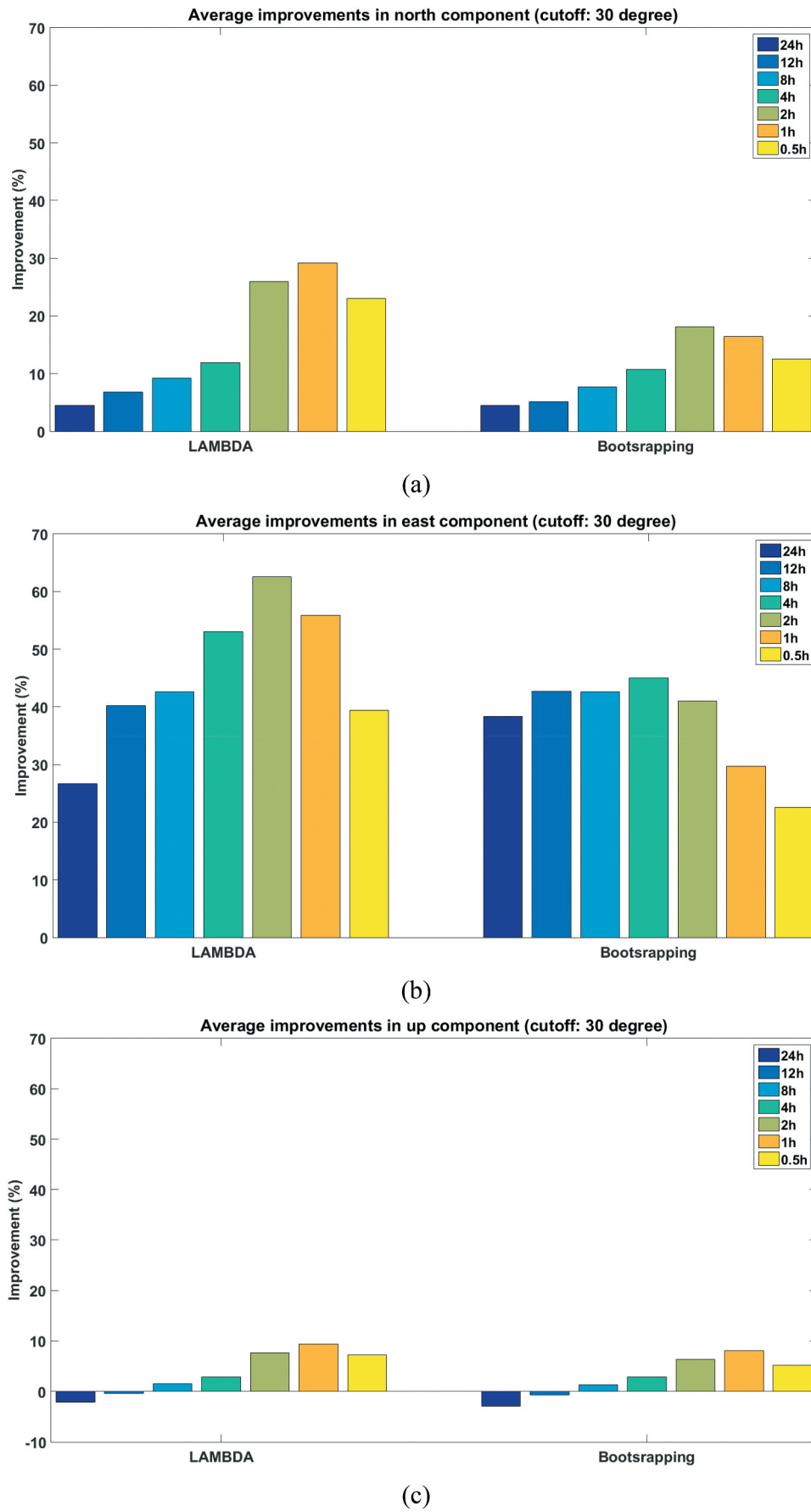


Figure 4. Accuracy improvements of AR_1 and AR_2 modes with a 30° cutoff angle.

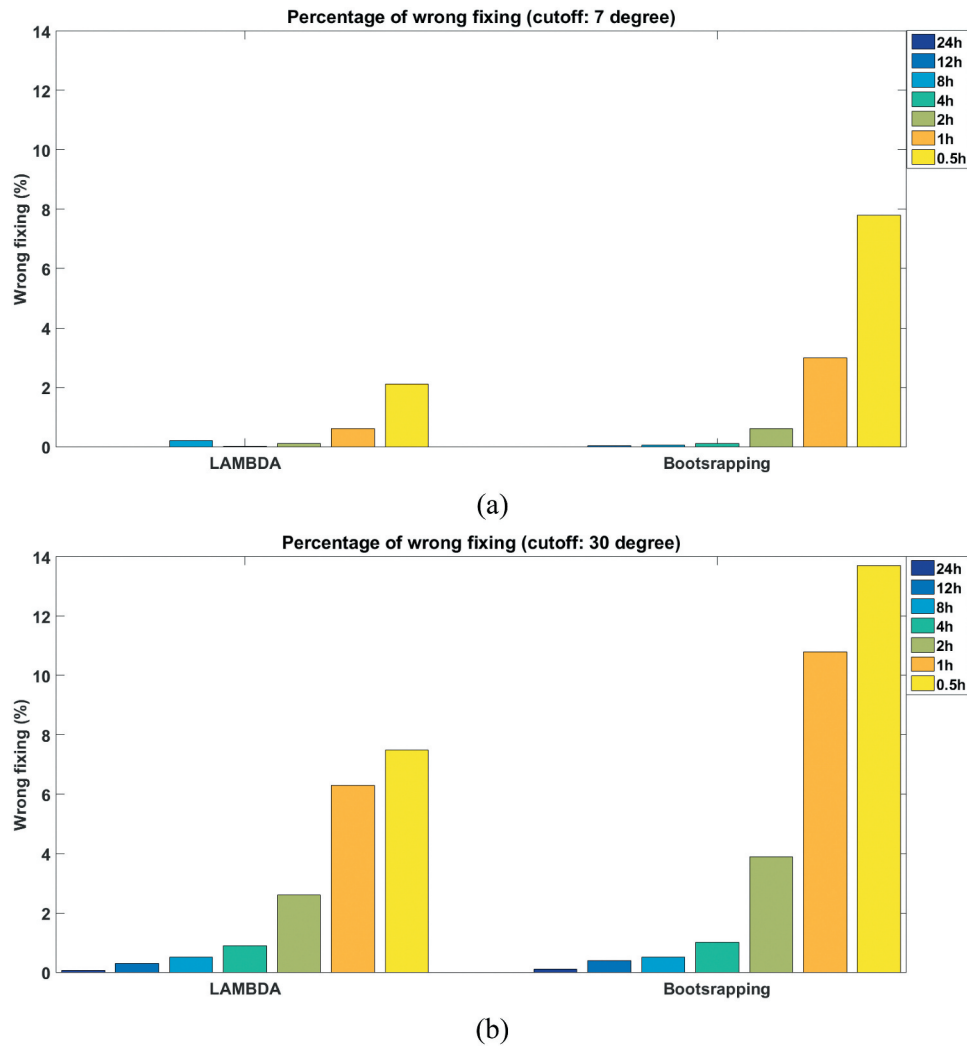


Figure 5. Percentage of wrong fixing for each AR method and cutoff angle.

while for observation sessions equal to 2 h and shorter, the performance of the LAMBDA method is significantly better compared with the bootstrapping method. LAMBDA is generally a higher success rate compared to the bootstrapping method. Therefore, it is more likely to find the correct integer candidate using LAMBDA compared to bootstrapping, especially for short observation sessions. This performance difference is expected because the ambiguity validation of narrow-lane AR is employed for LAMBDA method, whereas no ambiguity validation test is employed for the bootstrapping method.

6. Conclusions

In this paper, the performance of the LAMBDA and bootstrapping AR methods were comprehensively investigated for fixing PPP NL ambiguities with 24, 12, 8, 4, 2, 1, and ½-h sessions using two different cutoff angles (7° and 30°). Fifty-five IGS stations were processed over a one-month period in 2020 using three PPP modes – namely, float, NL AR with LAMBDA (AR₁), and NL AR with bootstrapping

(AR₂). The improvement in positioning accuracy with respect to the float PPP solutions was computed from AR₁ and AR₂ PPP modes. The number of wrong fixes using the AR₁ and AR₂ PPP modes was also computed for each data set.

Based on the results, it was found that the improvement in positioning accuracy from AR₁ PPP mode was much higher than AR₂ PPP mode when observation sessions were equal to and shorter than 2 h for the 7° cutoff angle. For the 30° cutoff angle, the performance of AR₁ mode outperformed the AR₂ mode for observation sessions equal to and shorter than 4 h. For restricted satellite visibility, the number of ambiguities decreases such that the search space of the LAMBDA also decreased. Due to this fact, the LAMBDA method produced much more accurate coordinate results for longer observation sessions with a 30° cutoff angle compared with a 7° cutoff angle. When the correlations between the ambiguities are high (for short observation sessions), the performance of the LAMBDA method is much greater than the bootstrapping method because the bootstrapping method does not take into account all the correlations

between the NL ambiguities. When the observation sessions become longer, the correlation between the NL ambiguities decreases as a result; more reliable AR can be maintained using the bootstrapping method. It should be emphasized that no AR validation test was applied to the bootstrapping method in this study. Because the bootstrapping method yields a single ambiguity solution, the ratio-test cannot be applicable. Success probability of the bootstrapping method can be used as an ambiguity validation of NL AR using a predetermined threshold value. The performance of the bootstrapping could be improved when an AR validation test is applied. Theoretically, LAMBDA generally outperforms bootstrapping. But in practice, different AR validations are used with these two methods. The ratio-test is often used to validate the integer candidates from LAMBDA searching method while not able to be used in bootstrapping method. Impacted by the validation, two methods have a different ambiguity fixing rate. Therefore, the bootstrapping method obtained a higher ambiguity fixing for long observation sessions in this study.

Wrong fixing results based on AR_1 and AR_2 modes indicated that the number of wrong fixes was significant for short observation sessions (0.5 h and 1 h for the 7° cutoff, and 4 h and shorter for the 30° cutoff) for each AR method. Similar to the accuracy performance of each AR method, the LAMBDA method produced fewer numbers of wrong fixes compared with the bootstrapping method for the short observation sessions. Ambiguity validation differences between the methods are mainly responsible for the number of wrong fixes differences in short observation sessions. Since no validation test employed for the bootstrapping method, the LAMBDA method with the ratio-test of ambiguity validation produced a higher performance for short observation sessions. For the long observation sessions, the number of wrong fixes was comparable to each AR method.

Based on this study, a flexible approach using the LAMBDA and bootstrapping methods with regard to the observation sessions and number of the ambiguities is recommended for PPP-AR.

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Data availability statement

The data that support the findings of this study are available on e-mail request from the corresponding author sermetogutcu@erbakan.edu.tr. The rinex data and orbit/clock products used in this study can be downloaded from <https://cddis.nasa.gov/archive/gnss/data/dailyandftp://igs.gnsswhu.cn,respectively>

References

- Alkan, R. M., V. İlçi, I. M. Ozulu, and M. H. Saka. 2015. "A Comparative Study for Accuracy Assessment of PPP Technique Using GPS and GLONASS in Urban Areas." *Measurement* 69: 1–8. doi:10.1016/j.measurement.2015.03.012.
- Banville, S., J. Geng, S. Loyer, S. Schaer, T. Springer, and S. Strasser. 2020. "On the Interoperability of IGS Products for Precise Point Positioning with Ambiguity Resolution." *Journal of Geodesy* 94 (1): 10. doi:10.1007/s00190-019-01335-w.
- Capilla, R. M., J. L. Berné-Valero, and A. Hermosilla-Rodrigo. 2017. "Impact of Multi-constellation Products and Ambiguity Resolution in Precise Point Positioning for Real-time Measurements." *Measurement* 100: 183–193. doi:10.1016/j.measurement.2016.12.047.
- Collins, P., S. Bisnath, F. Lahaye, and P. Héroux. 2010. "Undifferenced GPS Ambiguity Resolution Using the Decoupled Clock Model and Ambiguity Datum Fixing." *Navigation-Journal of the Institute of Navigation* 57 (2): 123–135. doi:10.1002/j.2161-4296.2010.tb01772.x.

- Feng, S., and A. Jokinen. 2017. "Integer Ambiguity Validation in High Accuracy GNSS Positioning." *GPS Solutions* 21 (1): 79–87. doi:10.1007/s10291-015-0506-9.
- Ge, M., G. Gendt, M. Rothacher, C. Shi, and J. Liu. 2008. "Resolution of GPS Carrier-phase Ambiguities in Precise Point Positioning (PPP) with Daily Observations." *Journal of Geodesy* 82 (7): 389–399. doi:10.1007/s00190-007-0187-4.
- Geng, J., X. Chen, Y. Pan, S. Mao, C. Li, J. Zhou, and K. Zhang. 2019b. "PRIDE PPP-AR: An Open-source Software for GPS PPP Ambiguity Resolution." *GPS Solutions* 23 (4): 91. doi:10.1007/s10291-019-0888-1.
- Geng, J., X. Chen, Y. Pan, and Q. Zhao. 2019a. "A Modified Phase Clock/bias Model to Improve PPP Ambiguity Resolution at Wuhan University." *Journal of Geodesy* 93 (10): 2053–2067. doi:10.1007/s00190-019-01301-6.
- Geng, J., F. N. Teferle, C. Shi, X. Meng, A. H. Dodson, and J. Liu. 2009. "Ambiguity Resolution in Precise Point Positioning with Hourly Data." *GPS Solutions* 13 (4): 263–270. doi:10.1007/s10291-009-0119-2.
- Hatch, R. 1982. "The Synergism of GPS Code and Carrier Measurements." *Proceedings of the third international symposium on satellite Doppler positioning at Physical Sciences Laboratory of New Mexico State University* 2: 1213–1231.
- Katsigianni, G., F. Perosanz, S. Loyer, and M. Gupta. 2019. "Galileo Millimeter-level Kinematic Precise Point Positioning with Ambiguity Resolution." *Earth, Planets and Space* 71 (1): 76. doi:10.1186/s40623-019-1055-1.
- Labib, B., J. Yan, J. Barriot, F. Zhang, and P. Feng. 2019. "Monitoring Zenithal Total Delays over the Three Different Climatic Zones from IGS GPS Final Products: A Comparison between the Use of the VMF1 and GMF Mapping Functions." *Geodesy and Geodynamics* 10 (2): 93–99. doi:10.1016/j.geog.2018.11.005.
- Laurichesse, D., F. Mercier, J. P. Berthias, P. Broca, and L. Cerri. 2009. "Integer Ambiguity Resolution on Undifferenced GPS Phase Measurements and Its Application to PPP and Satellite Precise Orbit Determination." *Navigation-Journal of the Institute of Navigation* 56 (2): 135–149. doi:10.1002/j.2161-4296.2009.tb01750.x.
- Leick, A., L. Rapoport, and D. Tatarnikov. 2015. *GPS Satellite Surveying*. 4th ed. Hoboken: Wiley.
- Li, P., and X. Zhang. 2015. "Precise Point Positioning with Partial Ambiguity Fixing." *Sensors* 15 (6): 13627–13643. doi:10.3390/s150613627.
- Li, P., X. Zhang, X. Ren, X. Zuo, and Y. Pan. 2016. "Generating GPS Satellite Fractional Cycle Bias for Ambiguity-fixed Precise Point Positioning." *GPS Solutions* 20 (4): 771–782. doi:10.1007/s10291-015-0483-z.
- Li, T., and J. Wang. 2012. "Some Remarks on GNSS Integer Ambiguity Validation Methods." *Survey Review* 44 (326): 230–238. doi:10.1179/1752270611Y.0000000027.
- Li, X., G. Dick, C. Lu, M. Ge, T. Nilsson, T. Ning, J. Wickert, and H. Schuh. 2015. "Multi-GNSS Meteorology: Real-time Retrieving of Atmospheric Water Vapor from BeiDou, Galileo, GLONASS, and GPS Observations." *IEEE Transactions on Geoscience and Remote Sensing* 53 (12): 6385–6393. doi:10.1109/TGRS.2015.2438395.
- Li, X., X. Li, Y. Yuan, K. Zhang, X. Zhang, and J. Wickert. 2018. "Multi-GNSS Phase Delay Estimation and PPP Ambiguity Resolution: GPS, BDS, GLONASS, Galileo." *Journal of Geodesy* 92 (6): 579–608. doi:10.1007/s00190-017-1081-3.
- Loyer, S., F. Perosanz, F. Mercier, H. Capdeville, and J. Marty. 2012. "Zero-difference GPS Ambiguity Resolution at CNES–CLS IGS Analysis Center." *Journal of Geodesy* 86 (11): 991–1003. doi:10.1007/s00190-012-0559-2.
- Melbourne, W. G. 1985. "The Case for Ranging in GPS-based Geodetic Systems." *Proceedings First International Symposium on Precise Positioning with the Global Positioning System*: 373–386. Rockville, Maryland.
- Mohammed, J., T. Moore, C. Hill, R. M. Bingley, and D. N. Hansen. 2016. "An Assessment of Static Precise Point Positioning Using GPS Only, GLONASS Only, and GPS Plus GLONASS." *Measurement* 88: 121–130. doi:10.1016/j.measurement.2016.03.048.
- Ogutcu, S. 2020. "Performance Analysis of Ambiguity Resolution on PPP and Relative Positioning Techniques: Consideration of Satellite Geometry." *International Journal of Engineering and Geosciences* 5 (2): 73–93. doi:10.26833/ijeg.580027.
- Teunissen, P. 1998. "Success Probability of Integer GPS Ambiguity Rounding and Bootstrapping." *Journal of Geodesy* 72 (10): 606–612. doi:10.1007/s001900050199.
- Teunissen, P. 2005. "Integer Aperture Bootstrapping: A New GNSS Ambiguity Estimator with Controllable Fail-rate." *Journal of Geodesy* 79 (6–7): 389–397. doi:10.1007/s00190-005-0481-y.
- Teunissen, P., P. Joosten, and C. Tiberius. 2003. "A Comparison of TCAR, CIR and LAMBDA GNSS Ambiguity Resolution." In *Proceedings of the 15th International Technical Meeting of the Satellite Division of the Institute of Navigation (ION GPS 2002)*, Rockville, Maryland. 2799–2808.
- Teunissen, P. J. G. 1995. "The Least-squares Ambiguity Decorrelation Adjustment: A Method for Fast GPS Integer Ambiguity Estimation." *Journal of Geodesy* 70 (1–2): 65–82. doi:10.1007/BF00863419.
- Teunissen, P. J. G. 2007. "Influence of Ambiguity Precision on the Success Rate of GNSS Integer Ambiguity Bootstrapping." *Journal of Geodesy* 81 (5): 351–358. doi:10.1007/s00190-006-0111-3.
- Teunissen, P. J. G., P. J. De Jonge, and C. C. J. M. Tiberius. 1997. "Performance of the LAMBDA Method for Fast GPS Ambiguity Resolution." *Navigation-Journal of the Institute of Navigation* 44 (3): 373–383. doi:10.1002/j.2161-4296.1997.tb02355.x.
- Vaclavovic, P., and J. Dousa. 2015. "Backward Smoothing for Precise GNSS Applications." *Advances in Space Research* 56 (8): 1627–1634. doi:10.1016/j.asr.2015.07.020.
- Wen, Q., J. Geng, G. Li, and J. Guo. 2020. "Precise Point Positioning with Ambiguity Resolution Using an External Survey-grade Antenna Enhanced Dual-frequency Android GNSS Data." *Measurement* 157: 107634. doi:10.1016/j.measurement.2020.107634.
- Wübbena, G. 1985. "Software Developments for Geodetic Positioning with GPS Using TI-4100 Code and Carrier Measurements." In *Proceedings of First International Symposium on Precise Positioning with the Global Positioning System* 403–412.
- Xiao, G., P. Li, Y. Gao, and B. Heck. 2019. "A Unified Model for Multi-frequency PPP Ambiguity Resolution and Test Results with Galileo and BeiDou Triple-frequency Observations." *Remote Sensing* 11 (2): 116. doi:10.3390/rs11020116.
- Zhang, B., Y. Chen, and Y. Yuan. 2019. "PPP-RTK Based on Undifferenced and Uncombined Observations: Theoretical and Practical Aspects." *Journal of Geodesy* 93 (7): 1011–1024. doi:10.1007/s00190-018-1220-5.
- Zhang, B., P. Teunissen, and D. Odijk. 2011. "A Novel Un-differenced PPP-RTK Concept." *Journal of Navigation* 64 (S1): 180–191. doi:10.1017/S0373463311000361.

Zhao, Q., S. Pan, C. Gao, W. Gao, and Y. Xia. 2020. "BDS/GPS/LEO Triple-frequency Uncombined Precise Point Positioning and Its Performance in Harsh Environments." *Measurement* 151: 107216. doi:[10.1016/j.measurement.2019.107216](https://doi.org/10.1016/j.measurement.2019.107216).

Zumberge, J. F., M. B. Heflin, D. C. Jefferson, M. M. Watkins, and F. H. Webb. 1997. "Precise Point Positioning for the Efficient and Robust Analysis of GPS Data from Large Networks." *Journal of Geophysical Research* 102 (B3): 5005–5017. doi:[10.1029/96JB03860](https://doi.org/10.1029/96JB03860).