Measuring the Delay Time Distribution of Binary Neutron Stars. I. Through Scaling Relations of the Host Galaxies of Gravitational-wave Events

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Abstract

The delay time distribution of (DTD) of binary neutron stars (BNSs) remains poorly constrained, mainly by the small known population of Galactic binaries, the properties of short gamma-ray burst host galaxies, and inferences from *r*-process enrichment. In the new era of BNS merger detections through gravitational waves (GWs), a new route to the DTD is the demographics of the host galaxies, localized through associated electromagnetic counterparts. This approach takes advantage of the correlation between star formation history (SFH) and galaxy mass, such that the convolution of the SFH and DTD impacts the BNS merger rate as a function of galaxy mass. Here we quantify this approach for a power-law DTD governed by two parameters: the power-law index (Γ) and a minimum delay time (t_{\min}). Under the reasonable assumption that electromagnetic counterparts are likely only detectable in the local universe, accessible by the current generation of GW detectors, we study how many host galaxies of massive galaxies (stellar mass of $M_* \gtrsim 10^{10.5} M_{\odot}$, comparable to the host galaxy of GW170817). Taking account of relevant uncertainties we find that $\mathcal{O}(10^3)$ host galaxies are required to constrain the DTD; for a fixed value of t_{\min} , as done in previous analyses of the DTD, $\mathcal{O}(10^2)$ host galaxies will suffice. Such a sample might become available within the next two decades, prior to the advent of third-generation GW detectors.

Key words: gravitational waves – stars: neutron

1. Introduction

The delay time distribution (DTD) of binary neutron stars (BNSs) is currently only weakly constrained, mainly by the statistics of the small known sample of Galactic BNSs (e.g., Vigna-Gómez et al. 2018), from arguments related to r-process enrichment (e.g., Matteucci et al. 2014; Safarzadeh et al. 2018), and from the properties of short gamma-ray burst (SGRB) host galaxies (Zheng & Ramirez-Ruiz 2007; Leibler & Berger 2010; Fong et al. 2013; Berger 2014). The DTD is generally expected to follow a power-law distribution based on the following reasoning. After the formation of the BNS, the binary's orbit decays through the emission of gravitational waves (GWs) on a timescale that depends on the binary's separation as $t \propto a^4$, where a is the semimajor axis of the binary at formation (Peters 1964). The distribution of the merging times, therefore, depends on the distribution of the initial orbital separation modeled as $dN/da \propto a^{-\beta}$. The initial distribution of the O/B star (the progenitors of the NSs) is assumed to follow a power law $dN/da \propto a^{-1}$. If the binary goes through a common envelope phase, then the distribution of the separation becomes steeper and approaches $dN/da \propto a^{-3}$. Therefore, the expected merger times follow $dN/dt_{\text{merge}} \propto t^{\Gamma}$, where $\Gamma \equiv -\beta/4 - 3/4$. For those two limiting cases, Γ ranges from -1.5 to -1 (e.g., Belczynski et al. 2018). The various weak observational constraints are roughly in agreement with these values.

Separate from the slope of the power-law distribution, the minimum timescale for BNS mergers (t_{min}) is another parameter that is equally important in determining the merger rate across cosmic time. From population synthesis models t_{min} could be as short as a few Myr (Dominik et al. 2012), but various effects could serve to set a minimum initial separation that will increase the value of t_{min} . Observationally, the two key DTD parameters are approximately degenerate with each other

in that it is not trivial to distinguish between a DTD with a steep slope but larger $t_{\rm min}$ and a DTD with shallow slope but shorter $t_{\rm min}$. Recent simulations have shown that fast-merging channels are needed to explain the fraction of all the metal-poor stars that are *r*-process enriched (Matteucci et al. 2014; Safarzadeh et al. 2018), as well as for *r*-process enrichment of ultra-faint dwarf galaxies (Safarzadeh & Scannapieco 2017; Safarzadeh et al. 2019). We note that most previous attempts to determine the DTD have assumed a small value of $t_{\rm min}$, leaving Γ as the only free parameter. Such an assumption is also made in the case of the DTD of Type Ia supernovae (e.g., $t_{\rm min} \sim 40$ Myr based on the minimum lifetime of stars that produce white dwarfs; Maoz et al. 2012, 2014), leading to better constraints on the power-law index.

The shape of the DTD is also imprinted in the demographics of the galaxies that host BNS mergers in the local universe, manifested either as the host galaxies of SGRBs (Berger 2014), or as the host galaxies of GW events that can in turn be pinpointed through the detection of associated electromagnetic (EM) counterparts (e.g., kilonovae; Abbott et al. 2017; Coulter et al. 2017; Soares-Santos et al. 2017). This is because the star formation histories (SFHs) of galaxies are determined by their masses, and the convolution of the SFH with the DTD will therefore impact the mass distribution of BNS merger host galaxies (Zheng & Ramirez-Ruiz 2007; Leibler & Berger 2010; Fong et al. 2013; Artale et al. 2019). The detection of the BNS merger GW170817, and the identification of its host galaxy, pave the way for utilizing this approach to constrain the DTD.

Here we use galaxy scaling relations to explore the impact of the DTD on the distribution of BNS merger host galaxies, and explore the number of events required to constrain the DTD. The upcoming observing campaigns with Advanced Laser Interferometer Gravitational-Wave Observatory (LIGO)/Virgo and the upcoming detectors Kamioka Gravitational Wave



Detector (KAGRA) and the Indian Initiative in Gravitationalwave Observations (IndIGO) are expected to yield BNS merger samples of $\mathcal{O}(10)$, $\mathcal{O}(100)$, and $\mathcal{O}(1000)$ within the next year, ~5 yr, and ~20 yr, respectively, before the advent of thirdgeneration GW detectors. The structure of this Letter is as follows: in Section 2 we demonstrate how the shape of the DTD affects the demographics of BNS merger host galaxies in the local universe; in Section 3 we explore the sample size required to constrain the shape of the DTD; and in Section 4 we discuss the caveats involved in this analysis. We adopt the Planck 2015 cosmological parameters (Planck Collaboration et al. 2016): $\Omega_M = 0.308$, $\Omega_{\Lambda} = 0.692$, $\Omega_b = 0.048$, and $H_0 = 0.678$ km s⁻¹ Mpc⁻¹.

2. Method

We can write the BNS merger rate for a galaxy with halo mass, M_h , at z = 0 as

$$\dot{n}(M_h) = \int_{z_b=10}^{z_b=0} \lambda \frac{dP_m}{dt} (t - t_b - t_{\min}) \psi(M_h, z_b) \frac{dt}{dz} (z_b) dz_b,$$
(1)

where

$$\frac{dt}{dz} = \frac{-1}{(1+z)E(z)H_0},$$
(2)

and

$$E(z) = \sqrt{\Omega_{m,0}(1+z)^3 + \Omega_{k,0}(1+z)^2 + \Omega_{\Lambda}(z)}.$$
 (3)

Here, $\psi(M_h, z)$ is the mean SFH of a galaxy with M_h at z = 0parametrized following Moster et al. (2013). We integrate the SFH from $z_b = 10$ to 0 (where the choice of maximum redshift has little impact on the calculation); t_b is the cosmic time corresponding to z_b ; λ is the BNS mass efficiency, assumed to be a fixed value of $10^{-5}M_{\odot}^{-1}$ independent of redshift or environment; dP_m/dt is the merger time distribution, which we parameterize to follow a power law, $\propto t^{\Gamma}$ with a minimum delay time, t_{\min} and not evolving with redshift. Although the DTD for binary black holes is likely highly dependent on metallicity, the DTD for BNS systems has been argued to be at most weakly dependent on metallicity (Dominik et al. 2012). The mass efficiency is assumed to be constant, although this parameter could be fit for in principle (Safarzadeh et al. 2019), due to the limited depth of Advanced LIGO (adLIGO) it acts as a normalization constant that could be ignored when studying the distribution of host galaxy masses in the local universe. We note that the delay time refers to the time since birth of the zero-age main-sequence (ZAMS) stars and not when the BNS is formed. We also impose a maximum delay time of 10 Gyr for our fiducial case, but our results are not sensitive for a longer maximum delay time.

We compute the merger rate per galaxy as a function of halo mass for a grid of nine joint choices of $\Gamma = [-1.5, -1.0, -0.5]$ and $t_{\min} = [10, 100, 1000]$ Myr. In Figure 1 we show the predicted merger rate (\dot{n}) as a function of halo mass and stellar mass for the nine DTDs. We find that the key difference between the various DTDs is apparent at $M_h \gtrsim 10^{12} M_{\odot}$, corresponding to $M_* \gtrsim 10^{10.5} M_{\odot}$. This is primarily because on average galaxies of lower masses have fairly flat SFHs that

are therefore not sensitive to convolution with the different DTDs. The high-mass galaxies, on the other hand, have SFHs that peak at progressively earlier cosmic time with larger mass. Therefore, we find that DTDs that favor long merger timescales (e.g., $\Gamma = -1/2$ and $t_{\min} = 1$ Gyr) lead to a higher representation of massive host galaxies.

To determine the observed mass distribution of BNS merger host galaxies we need to rescale \dot{n} with the halo mass function, computed following Press & Schechter (1974):

$$\phi(M_h) \equiv \frac{dn}{dM_h} = \frac{\bar{\rho}}{M_h} f(\nu) \frac{d\nu}{dM_h},\tag{4}$$

where *n* is the number density of haloes, ν is the peak height of perturbations, $\bar{\rho}$ is the average density of the universe, and the first crossing distribution, $f(\nu)$ (Bond et al. 1991), is obtained from the ellipsoidal collapse model as

$$\nu f(\nu) = A \sqrt{\frac{a\nu}{2\pi}} [1 + (a\nu)^{-p}] e^{-a\nu/2},$$
(5)

with A = 0.322, p = 0.3, and a = 0.75 (Sheth & Tormen 2002). Here, the peak height, ν , is defined as $\nu \equiv \delta_{c,0}^2 \sigma_{\chi}(R, z)^{-2}$, with $\delta_{c,0} = 1.686$. The variance is $\sigma_{\chi}^2(M, z) = \sigma_{\chi}^2(M, 0)D(z)^2$, with

$$\sigma_{\chi}^{2}(M, 0) = \sigma_{\chi}^{2}(R, 0) = \int_{0}^{\infty} \frac{dk}{2\pi^{2}} k^{2} P_{\chi}(k) w^{2}(kR)$$
(6)

where $M = 4\pi R^3 \Omega_M \rho_c/3$, $w(kR) \equiv 3j_1(kR)/(kR)$, with $j_1(x) \equiv (\sin x - x \cos x)/x^2$, and D(z) is the linear growth factor

$$D(z) \equiv \frac{H(z)}{H(0)} \int_{z}^{\infty} \frac{dz'(1+z')}{H^{3}(z')} \left[\int_{0}^{\infty} \frac{dz'(1+z')}{H^{3}(z')} \right]^{-1}.$$
 (7)

We compute the cumulative fraction of the BNS merger host halos with mass above M_h as

$$f(>M_{h}) = \frac{\int_{M_{h},\max}^{M_{h,\max}} \phi(M_{h}')\dot{n}(M_{h}')dM_{h}'}{\int_{M_{h,\min}}^{M_{h,\max}} \phi(M_{h}')\dot{n}(M_{h}')dM_{h}'},$$
(8)

where we consider halos in the range $M_{h,\min} = 10^{11}$ to $M_{h,\max} = 10^{14} M_{\odot}$. In Figure 1 we show the cumulative distribution function (CDF) for BNS merger host galaxies as a function of M_h and M_* for the nine DTDs. We find that there is a difference of about 0.5 dex in the median value of M_h for the range of DTDs, and about 0.7 dex in the value of M_h for the top 20% most massive galaxies; at the low-mass end the CDFs converge. Similarly, in terms of stellar mass we find a nearly order of magnitude spread in the median value of M_* . In both CDFs we again find a clear degeneracy between Γ and t_{\min} , such that a DTD with a shallower power-law index and small value of t_{\min} is similar in shape to one with a steeper power-law index and a large t_{\min} .

Finally, to determine the sample size needed to constrain the DTD we draw from the constructed CDF of a given pair of $[\Gamma, t_{min}]$, and perform a Kolmogorov–Smirnov (KS) test against other possible CDFs constructed in a 10 × 10 interpolated 2D plane of Γ – t_{min} . We use a threshold P < 0.01 to consider the CDFs as being drawn from different underlying distributions. For each case we repeat the KS test 10 times and determine the median value of P.



Figure 1. Top row: the BNS merger rate (\dot{n}) as a function of halo mass (left panel) and stellar mass (right panel) at z = 0 for 9 different DTDs, with $\Gamma = [-3/2, -1, -1/2]$ and $t_{min} = [10, 100, 1000]$ Myr. DTDs with a long delay time (e.g., $\Gamma = -1/2$ and $t_{min} = 1000$ Myr) lead to a high merger rate in massive galaxies. Bottom row: the cumulative distribution function of BNS merger host galaxies in terms of halo mass (left panel) and stellar mass (right panel) for the same DTDs. These have been constructed by taking into account the halo mass function.

We note that the overall approach requires an identified EM counterpart and host galaxy; with only two examples to date (GW170817 and likely S190425z; Hosseinzadeh et al. 2019) it is difficult to assess the counterpart identification success rate. We show in Safarzadeh et al. (2019) how future third-generation GW detectors could enable a measurement of the DTD without the need for EM counterparts. Moreover, we focused on BNS systems formed in the field, while there are other proposed mechanism for their formation, although those

are expected to be a minor contributor (e.g., Grindlay et al. 2006; Lee et al. 2010).

3. Results

In Figure 2 we plot the value of $\log P$ for three different injected DTDs and three different sample sizes (30, 100, and 1000 BNS merger host galaxies) in the plane of Γ - t_{min} . This figure uses the halo mass CDFs shown in Figure 1. We find that

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Figure 2. Example KS test *P*-value maps as a function of the number of observed BNS merger host galaxies, in the space of halo mass. In each row the injected model is shown as a yellow circle, and the grayscale indicates the *P*-value in the full range of considered Γ - t_{min} parameter space (darker color indicates lower *P*-value). The columns are for sample of 30, 100, and 100 BNS merger host galaxies. We find that a sample size of $\mathcal{O}(10^3)$ is required to well constrain both Γ and t_{min} , but that some intrinsic degeneracy remains.

a sample size of $\mathcal{O}(10^2)$ is required to begin to rule out portions of the DTD parameter space. With a sample size of $\mathcal{O}(10^3)$ a significant portion of the parameter space can be ruled out, but with a possible remaining degeneracy between Γ and t_{\min} depending on the true values of the parameters.

So far we have cast our results in terms of M_h , but observationally we determine the stellar mass (M_*) based on modeling a galaxy's spectral energy distribution. While there is uncertainty involved in estimating M_* , the connection to M_h ,

based on abundance matching techniques (Behroozi et al. 2013) is likely to dominate. For example, Blanchard et al. (2017) determined the stellar mass of the host galaxy of GW170817 (NGC 4993) to be $\log(M_*/M_{\odot}) = 10.90^{+0.03}_{-0.03}$ by reconstructing its SFH using ultraviolet (UV) to infrared (IR) data. However, the corresponding halo mass is estimated to be $\log(M_h/M_{\odot}) =$ $13.0^{+0.3}_{-0.3}$, with the much larger uncertainty due to the dispersion in the M_h - M_* relation (Behroozi et al. 2013). To accommodate this uncertainty into our analysis, we disperse the halo masses by

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Figure 3. The same as Figure 2, but in the space of stellar mass, in which we add an additional 0.3 dex uncertainty in the halo mass-stellar mass relation of galaxies. The required host galaxy sample size is largely unchanged.

0.3 dex when sampling from a given CDF, and then follow the same procedure as above to determine *P*-values. The results of including this systematic uncertainty of 0.3 dex are equivalent to the uncertainty on the SFH of an observed galaxy with a given stellar mass. The inclusion of this additional uncertainty reduces the constraining power of the observed host galaxy sample on the DTD. However, in Figure 3 we show that a sample size of $\mathcal{O}(10^3)$ can still provide significant constraints on the DTD even when this systematic uncertainty is included.

The overall statistics of the fraction of the Γ - t_{min} parameter space that can be ruled out with different sample sizes is shown in Figure 4. Here we inject 50 random [Γ , t_{min}] pairs, perform

the same KS test analysis, and determine the fraction of parameter space with P < 0.01 (i.e., the excluded fraction). We find that the median excluded fraction of the parameter space is about 10%, 25%, and 65% for sample sizes of 30, 100, and 1000 BNS merger host galaxies, respectively.

We note that our analysis uses a DTD with two free parameters, while generally only Γ is used as a free parameter, and $t_{\rm min}$ is fixed at a small value (~10 Myr). This simplifying assumption is based on the notion that some binaries can merge as soon as the second neutron star is formed. This approach has also been used in analyses of the DTD of Type Ia supernovae, in which a value of $t_{\rm min} \sim 40$ Myr is often assumed (e.g., Maoz



Figure 4. Fraction of the Γ - t_{min} parameters space that can be excluded as a function of the number of observed BNS merger host galaxies (N_{obs}). This is evaluated for 50 random [Γ , t_{min}] pairs following the examples shown in Figure 2 and using a threshold of P < 0.01. The median values of the excluded fraction of the 2D parameters space are 10%, 25%, and 65% for the sample sizes of 30, 100, and 1000 BNS merger host galaxies, respectively.



Figure 5. Histograms of *P*-value in the simplified case in which we fix the value of t_{min} (to 10 Myr) and use only Γ as a free parameter of the DTD. The injected values are shown as yellow vertical lines and we explore host galaxy sample size of 30, 100, and 300. We find that a constraint on Γ to about 30% can be achieved with a sample of about 300 host galaxies.

et al. 2012, 2014), leaving only Γ as a free parameter. In Figure 5 we show the resulting constraints on Γ if we fix $t_{\min} = 10$ Myr and repeat our analysis. We find that in this simplified model the value of Γ can be determined with $\approx 30\%$ uncertainty with a reduced sample size of ~ 300 BNS merger host galaxies. Such a sample can be accumulated in about one-third of the time compared to the requirement when both Γ and t_{\min} are free parameters.

4. Summary

We showed how the DTD of BNS systems can be constrained through the demographics of the host galaxies of BNS mergers detected in GWs and pinpointed through EM observations. We focused on the case of a DTD parameterized as a power law, although other possible DTD shapes have been proposed in the literature (e.g., Simonetti et al. 2019). Our analysis is similar to that of Zheng & Ramirez-Ruiz (2007), proposed in the context of SGRB host galaxy demographics, but with the difference that we model the DTD with two parameters, while those authors used just Γ and fixed t_{min} .

Our results show that $\mathcal{O}(10^3)$ host galaxies are needed to constrain a two-parameter DTD, although some degeneracy between Γ and t_{\min} is intrinsically difficult to resolve even with a large sample of events. On average, about two-thirds of the Γ - t_{\min} parameter space can be ruled out with such a sample. In the case when only Γ is a free parameter, a sample size of about 300 BNS merger host galaxies is sufficient for a 30% uncertainty on Γ . The current range of BNS merger rates from AdLIGO/ Virgo Observing Runs 1 and 2 is 110–3840 Gpc⁻³ yr⁻¹ (The LIGO Scientific Collaboration et al. 2018), indicating that a sample of $\mathcal{O}(10^3)$ events might be achieved within a couple of decades at a design sensitivity of 200 Mpc, or potentially even faster in the case of A+ (Barsotti et al. 2018), with an expected factor of 2 increase in the BNS merger detection range. Thus, a constraint on the DTD using the demographics of BNS merger host galaxies can be achieved before the advent of the thirdgeneration GW detectors. We note that a similar approach using SGRB host galaxies is likely to take longer given an identification rate of only a few events per year (Berger 2014).

In an upcoming paper we will investigate a related approach to constraining the DTD, using the measured individual SFH of BNS merger host galaxies (rather than the mean scaling relations used here). This is similar to the approach used by Blanchard et al. (2017) for the host galaxy of GW170817, and by Maoz & Graur (2017) in the context of Type Ia SNe.

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