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Machine Interference Problem with Reliable Server under Multiple Vacations Policy

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Abstract

The machine interference problem with reliable server under multiple vacations policy is considered. There are M similar machines that are subject to breaks down with a single server who is responsible for repairing the failed machines under multiples vacations. The failed machines arrive for service according to Poisson distribution with rate λ. The service time distributions of the failed machines are assumed to be exponentially distributed with state dependent service rate μ_n, where n is the number of failed machines. The differential difference equations obtained for the reliable server is solved through in MATLAB to obtain transient probability for the system. The transient probabilities are used to compute the operational measures of performance for the systems. The effects of failure rate, service rate and vacation length for the system were studied. We show that with the same service rate μ, failure rate λ and vacation length θ, as the number of operating machine in the system increases the variance also increases. We also found that the variance under multiple vacations system is slightly less than that of single vacation. This means that the multiple vacations models may be preferred to the single vacation. The result also shows that the CPU time for the machine interference problem with reliable server under single vacation is slightly lower than that of machine interference problem with an unreliable server under single vacation policy.

Keywords: Transient solution; machine interference problem; reliable server; multiple vacations; ODE45 in MATLAB.

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1 Introduction

This article discusses the machine interference problem with a reliable server under multiple vacations. In the machine interference problem there are M group of machines under the supervision of a single repairer who attends to the failed machines when they break down under multiple vacations. In the machine interference problem the repairer (server) repairs the broken down machine to make it operational. If there are one or more broken down machines and the repairer is busy repairing broken down machine when another broken down machine needs service we say the machine interferes with each other's service [1].

In the reliable server with multiple vacations, if there is no failed machine waiting for service after a vacation, the server immediately leaves for another vacation. This pattern continues until he returns from vacation to find at least one failed machine waiting in queue for service.

Also [2] studied the reliability characteristics of a machine interference system with M identical machines and S warm standby machines with one removable server. The removable server operates an N-policy. Failed machines are allowed to renege (a failed machine may be removed from the queue without being serviced) when the servers are busy. Laplace transform techniques were used to derive the explicit expressions for both the reliability function and the mean time to system failure. Jain et al. [3] studied a similar system but used a recursive method to obtain steady state measures of performance. Jain [4] used a recursive method to study the multi-server machine interference problem. In the system examined, the number of servers changes depending on the queue length. Cost functions were derived based on the average number of customers in the system. Jain et al. [5] extended the study to include mixed standbys (either cold or warm standbys) balking and reneging. Two modes of failure of the machines were considered.

Further [5] studied the machine repair problem with regular and reserved servers. The system has both regular and standby machines so that failed machines are immediately re/placed by standbys if available. The standby machines consist of both warm and cold. The failed machines may balk or renege. The matrix geometric method was used to obtain transient state measures of performance and a procedure for determining the optimal number of warm and cold standby machines required for the desired level of quality of service was proposed. Jain et al. [5] obtained transient results for the machine repair problem with regular and reserved servers where failed machines can balk or renege. The system has provision for the use of warm and cold standby machines and the reserved servers are turned on following a threshold policy. Further, a fixed number of functioning machines are required for the system to function in normal model otherwise it will operate in short mode. Jain and Upadhyaya [6] obtained steady state measures of performance for a similar system.

Jain and Kumar [7] considered a machine repair problem consisting of two heterogeneous servers and mixed spares (warm and cold) in the system. The two repairmen can go on vacation using two different N policies. Further, the two repairmen are used under different conditions. Failed machines are immediately replaced by spare machines (either a cold or a warm spare). A bi-level control policy was used to introduce the servers into the system. Recursive method was applied to derive steady state measures of performance.

Ojobor [8] considered machine interference problem with a reliable server under single vacation. The server in his system is reliable, that is the server is always active. He shows that the breakdown and repair rate of server in [9] have slight effect on the expected number of failed and operating machines.

Ojobor [10] considered transient solution of machine interference problem with an unreliable server under multiple vacations policy. Their server is unreliable, that is when the server is active it can break down. Anytime the server breaks down it is immediately repaired. The server goes on multiple vacations. These motivate us to examine the machine interference problem with a reliable server under multiple vacations.

The purpose of this paper is first to produce transient probability for the machine interference problem with reliable server under multiple vacations. The transient probabilities obtain are used to find various operational measures of performance for the system. The second is to compute the CPU time for obtaining the transient solution for the multiple vacations policy. The third is that, apart from finding the expected number of failed and operating machine we also obtain the variance and standard deviation of the number of failed and operating machines in the system which previous authors did not obtain. This problem was suggested by [11].

2 Mathematical Formulations

We shall use the same notation given by [12].

The state of the system is described at epoch t by two variables namely: the number of failed machine in the system and the state of the server. Considering the transitions that occur in the system, the recurrence relations for the state probabilities for the multiple vacations is derived. To implement the relations, we assume that the numbers of failed machines in the system are finite.

A computer program which is implementable in MATLAB ([13]) is written to provide transient results for the machine interference problem with reliable server that can go on multiple vacations. Various performance measures for the machine interference problem with reliable server are then derived.

2.1 ASSUMPTIONS AND NOTATIONS

(i) Let the state of the system at epoch t be denoted by (i, n); $i=0, 1; 0 \leq n \leq M$; where i is the state of the server, and n is the number of failed machines in the system. When the server is on vacation i=0, when the server is active $i=1$
(ii) The machines fail or arrive for service according to Poisson distribution with rate λ.
(iii) The failed machines are service (repaired) according to exponential distribution with rate $\mu_{n.}$
(iv) When there is no failed machines queueing for service the server goes on multiple vacations of random length. The vacation length is exponentially distributed with parameter θ.
(v) Lastly, the number of breaks down machine in the system is finite.

M number of operating machine

λ failure rate of an operating machine

μ_n State dependent service rate of a failed machine, here we use $\mu_n = 1 + \frac{n}{5}$. That means if we have five failed machines in the system, the service rates will be $\mu_1 = 1.2, \mu_2 = 1.4, \mu_3 = 1.6, \mu_4 = 1.8 \; and \; \mu_5 = 2.0$.

θ Length of vacation of server

$P_{0,n}$(t) The probability that there are n failed machines in the system when the server is on vacation at time t

$P_{1,n}$(t) The probability that there are n failed machine in the system when the server is active at time t

$N(t)$ The exact number of failed machines in the system at time t

$Y(t)$ The server state at time t

where $Y(t) = \begin{cases} 0 \text{ server on vacation} \\ 1 \text{ server is active} \end{cases}$

2.2 Mathematical Formulation

The process $(\{Y(t), N(t) : t \geq 0\}$ *is a bivariate process*. It is a continuous time Markov process on a state space

$s = \{(0,n): n = 0,1,2,\ldots,M\} \cup \{(1,n): n = 0,1,2,\ldots,M\}$, State 1,0 is not admissible because the system is never active when there is no failed machine.

We define the probabilities

$$P_{0,n}(t) = prob\{Y(t) = 0, N(t) = n\}$$

and

$$P_{1,n}(t) = prob\{Y(t) = 1, N(t) = n\}$$

3 Reliable Servers with Multiple Vacations

On the machine interference problem with reliable server we shall derive transient probability under multiple vacations policy. The probability that there is no failed machine when the server is on multiple vacations in the interval [t, t+h] is obtained as follows: consider the state of the system between t and t+h, first possibility is that at epoch t the system is on multiple vacations, one failed machine arrive and no service completion during the interval t and t+h. This has probability $P_{0,0}(t)[1 - M\lambda h]$.

The second possibility is that at epoch t the system is active with one failed machine serviced. There is service completion during the interval t and t+h. This has probability$P_{1,1}(t)[\mu_1 h]$.

Hence $P_{0,0}(t+h) = P_{0,0}(t)[1 - M\lambda h] + P_{1,1}(t)[\mu_1 h]$

From which we obtain

$$P'_{0,0}(t) = -(M\lambda)P_{0,0}(t) + \mu_1 P_{1,1}(t) \tag{1}$$

The probability that there is n failed machines when the server is on multiple vacations in the interval [t, t+h] is obtained as follows: consider the state of the system between t and t+h, the first possibility is that at epoch t the system is on multiple vacations with n failed machines and no service completion during the interval t and t+h. This has probability

$$P_{0,n}(t)[1 - (M-n)\lambda h][1 - \theta h].$$

The second possibility is that at epoch t the system is on multiple vacations with one server, one failed machine arrive and no service completion during the interval t and t+h. This has probability$P_{0,n-1}(t)[(M - n+1)\lambda h[1-\theta h]$.

Hence $P_{o,n}(t+h) = P_{0,n}(t)[1 - (M-n)\lambda h][1 - \theta h] + P_{0,n-1}(t)[(M-n+1)\lambda h][1 - \theta h]$

From which we obtain

$$P'_{0,n}(t) = -(\theta + (M-n)\lambda)P_{0,n}(t) + (M-n+1)\lambda P_{0,n-1}(t) \tag{2}$$

$$1 \leq n \leq M-1$$

The probability that there are *M* failed machines when the server is on multiple vacations in the interval [t, t+h] is obtained as follows: consider the state of the system between t and t+h, the first possibility is that at epoch t the system is on multiple vacations with *M* failed machines and no service completion during the interval t and t+h. This has probability$P_{0,M}(t)[1-\theta h]$.

The second possibility is that at epoch t the system is on multiple vacations with one server, one failed machine arrive and no service completion during the interval t and t+h. This has probability $P_{0,M-1}(t)\lambda h$.

Hence $P_{0,M}(t+h) = P_{0,M}(t)[1-\theta h] + P_{0,M-1}(t)\lambda h$

From which we obtain

$$P_{0,M}^{'}(t) = -\theta P_{0,M}(t) + \lambda P_{0,M-1}(t) \tag{3}$$

The probability that there is one failed machine when the server is active serving failed machine in the interval [t, t+h] is obtained as follows: consider the state of the system at time t and t+h, the first possibility is that at epoch t the system is active and one failed machine is serviced during the interval t and t+h. This has probability $P_{1,1}(t)[1-[(M-1)\lambda + \mu_1]h]$.

The second possibility is that at epoch t the system is active, the second failed machine is serviced and server did not break down during the interval t and t+h. This has probability $P_{1,2}(t)\mu_2 h$.

The third possibility is that at epoch t the system leaves multiple vacations to attend to the failed machine in the system during the interval t and t+h. This has probability $P_{0,1}(t)\theta h$.

Hence $P_{1,1}(t+h) = P_{1,1}(t)[1-[(M-1)\lambda + \mu_1]h] + \quad P_{1,2}(t)\mu_2 h + P_{0,1}(t)\theta h$

From which we obtain

$$P_{1,1}{'}(t) = P_{1,1}(t)\big[-[(M-1)\lambda + \mu_1]\big] + \quad P_{1,2}(t)\mu_2 + P_{0,1}(t)\theta \tag{4}$$

The probability that there is n failed machine when the server is active in the interval [t, t+h] is obtained as follows: consider the state of the system between t and t+h, the first possibility is that at epoch t the system is active with n failed machines and no service completion during the interval t and t+h. This has probability $P_{1,n}(t)[1-[(M-n)\lambda + \mu_n]h]$.

The second possibility is that at epoch t the system is active, one failed machine arrives and no service completion during the interval t and t+h. This has probability $P_{1,n-1}(t)[M-n+1]\lambda h$.

The third possibility is that at epoch t the system is active and one failed machine is serviced during the interval t and t+h. This has probability $P_{1,n+1}(t)\mu_{n+1}\, h$.

The fourth possibility is that at epoch t the server leaves multiple vacations to attend to the failed machines during the interval t and t+h. This has probability $P_{0,n}(t)\theta h$.

Hence

$$P_{1,n}(t+h) = P_{1,n}(t)[1-[(M-n)\lambda + \mu_n]h] + P_{1,n-1}(t)[M-n+1]\lambda h \; + P_{1,n+1}(t)\mu_{n+1}h + P_{0,n}(t)\theta h$$

From which we obtain

$$P_{1,n}^{'}(t) = P_{1,n}(t)\big[-[(M-n)\lambda + \mu_n]\big] + P_{1,n-1}(t)[M-n+1]\lambda \; + P_{1,n+1}(t)\mu_{n+1} + P_{0,n}(t)\theta$$

$$2 \le n \le M-1 \tag{5}$$

The probability that there are *M* failed machines when the server is active in the interval [t, t+h] is obtained as follows: consider the state of the system between t and t+h, the first possibility is that at epoch t the system is active with M failed machines and no service completion during the interval t and t+h. This has probability $P_{1,M}(t)[1-(\mu_M)h]$.

The second possibility is that at epoch t the system is active, one failed machine arrives and no service completion during the interval t and t+h. This has probability $P_{1,M-1}(t)\lambda h$.

The third possibility is that at epoch t the server the leaves multiple vacations to attend M failed machines in the system during the interval t and t+h. This is the last failed machine in the system. This has probability $P_{0,M}(t)\theta h$.

Hence $P_{1,M}(t+h) = P_{1,M}(t)[1-(\mu_M)h] + P_{1,M-1}(t)\lambda h + P_{0,M}(t)\theta h$

From which we obtain

$$P_{1,M}^{'}(t) = P_{1,M}(t)[-(\mu_M)] + P_{1,M-1}(t)\lambda + P_{0,M}(t)\theta \quad (6)$$

For the machine interference problem with reliable server under multiple vacations policy the numbers of equation to be solved is *1+2M*.

The state transition diagram for the system is given in Fig. 1. Note that the system we considered is finite state space.

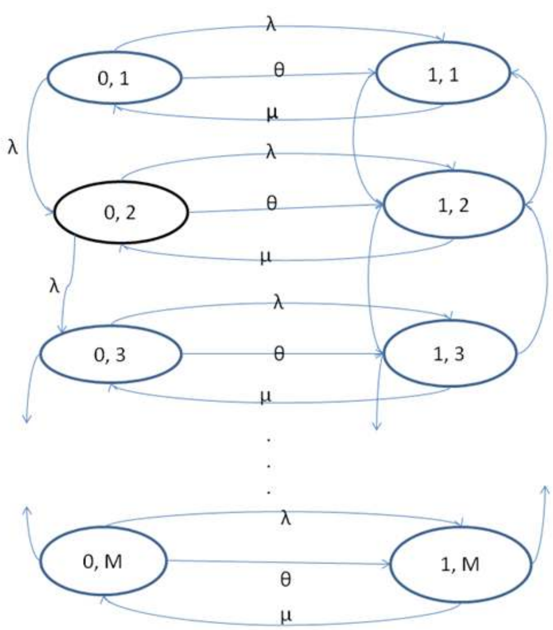

Fig. 1. The state transition diagram for the machine interference problem with reliable server under multiple vacations policy

4 Numerical Solutions

The transient solution $P_{i,n}(t); i = 0, 1\ and\ 0 \le n \le M$ (where $P_{1,0}$ is not an admissible state) for the machine interference problem with reliable server under multiple vacations is obtained by solving the set of transient state Chapman-Kolmogorov differential- difference equations (1) to (6). We use MATLAB program to find the numerical solutions of the differential- difference equations above. We set t=20 seconds.

The transient state results for the reliable server model and that of [12] steady state results are compared in Table 5. The number of equations, parameters and time spend on computation are reduced in our model compared to [12]. This shows that our model is good.

The expression for the expected number of failed machines E[F], expected number of operating machine E[O], expected length of vacation the server has E[V], expected idle period E[I] and the expected number of busy periods E[B] are calculated with the expressions given below:

$$E[F](t) = \sum_{n=0}^{M} nP_{0,n}(t) + \sum_{n=0}^{M} nP_{1,n}(t)$$

$$E[O](t) = M - E[F](t)$$

$E[I](t) = P_{1,0}(t)$ is always zero because state 1,0 is not admissible and therefore its probability equals to zero.

$$E[V](t) = \sum_{n=0}^{M} nP_{0,n}(t)$$

$$E[B](t) = 1 - E[I](t) - E[v\}(t)$$

Machine availability $M.A. = 1 - \frac{E[F](t)}{M}$

Operative utilization $O.U. = E[B](t)$

Variance: The variance of the number of broken down machine and the number of operating machines are calculated by using the expression

$$\sigma^2(t) = \sum_{n=0}^{M} n^2 P_{0,n}\,(t) + \sum_{n=0}^{M} n^2 P_{1,n}(t) - [E(F(t))]^2$$

Where

$$E(F(t)) = \sum_{n=0}^{M} nP_{0,n}\,(t) + \sum_{n=0}^{M} nP_{1,n}(t)$$

4.1 Transient Results for Machine Interference Problem with Reliable Server under Multiple Vacations

Tables' 1-4 show results from MATLAB for the operational measures of performance for the system. Following from [10] the model is running for sufficient time t. The results are presented to four decimal places.

Table 1 shows the expected number of operating machines E (O), the expected number of failed machines E(F), expected number of vacations the server has E(V), expected idle period E(I), the machine availability (M.A.) and operative utilization (O.U.) for the machine interference problem with a reliable server under multiple vacations policy with the following parameters λ=0.15, θ=1, μ_1=1.1, M=10. We run the model for sufficient time t (until transient state probabilities no longer varies with time), after some time the successive values of the measures of performance were no longer varying; this means that the transient results were close to the steady state results. The results are presented to four places of decimal in Table 1.
Similarly, varying the service rate λ and the numbers of machines M in the system we also observed after some time that the successive values of the measures of performance were no longer varying; which means

that the transient results were close to the steady state results. The results are presented to four places of decimal in Tables 2-4.

Table 1. Some performance measures for different values of t for the multiple vacations policy with λ*=0.15,* θ*=1,* μ_1*=1.1, M=10*

t	E(O)	E(F)	E(V)	M.A.	O.U.
0	10.0000	0.0000	0.0000	1.0000	1.0000
1	8.5105	1.4895	0.5947	0.8510	0.4053
2	7.7708	2.2292	0.3664	0.7771	0.6336
3	7.4148	2.5852	0.2695	0.7415	0.7305
4	7.2436	2.7564	0.2249	0.7244	0.7751
5	7.1630	2.8370	0.2037	0.7163	0.7963
6	7.1258	2.8742	0.1934	0.7126	0.8066
7	7.1094	2.8906	0.1883	0.7109	0.8117
8	7.1025	2.8975	0.1858	0.7103	0.8142
9	7.1001	2.8999	0.1846	0.7100	0.8154
10	7.0995	2.9005	0.1840	0.7099	0.8160
11	7.0996	2.9004	0.1838	0.7100	0.8162
12	7.1000	2.9000	0.1837	0.7100	0.8163
13	7.1004	2.8996	0.1836	0.7100	0.8164
14	7.1007	2.8993	0.1837	0.7101	0.8163
15	7.1009	2.8991	0.1837	0.7101	0.8163
16	7.1010	2.8990	0.1837	0.7101	0.8163
17	7.1011	2.8989	0.1837	0.7101	0.8163
18	7.1012	2.8988	0.1837	0.7101	0.8163
19	7.1012	2.8988	0.1837	0.7101	0.8163
20	7.1012	2.8988	0.1837	0.7101	0.8163

Table 2. Some performance measures for different values of t for the multiple vacations policy with λ*=0.2,* θ*=1,* μ_1*=1.1, M=9*

T	E(O)	E(F)	E(V)	M.A.	O.U.
0	9.0000	0.0000	0.0000	1.0000	1.0000
1	7.2936	1.7064	0.5619	0.8104	0.4381
2	6.4664	2.5336	0.3273	0.7185	0.6727
3	6.0747	2.9253	0.2263	0.6750	0.7737
4	5.8958	3.1042	0.1798	0.6551	0.8202
5	5.8177	3.1823	0.1581	0.6464	0.8419
6	5.7853	3.2147	0.1478	0.6428	0.8522
7	5.7730	3.2270	0.1429	0.6414	0.8571
8	5.7690	3.2310	0.1406	0.6410	0.8594
9	5.7683	3.2317	0.1396	0.6409	0.8604
10	5.7688	3.2312	0.1391	0.6410	0.8609
11	5.7695	3.2305	0.1390	0.6411	0.8610
12	5.7702	3.2298	0.1389	0.6411	0.8611
13	5.7707	3.2293	0.1389	0.6412	0.8611
14	5.7711	3.2289	0.1389	0.6412	0.8611
15	5.7713	3.2287	0.1390	0.6413	0.8610
16	5.7715	3.2285	0.1390	0.6413	0.8610
17	5.7716	3.2284	0.1390	0.6413	0.8610
18	5.7716	3.2284	0.1390	0.6413	0.8610
19	5.7716	3.2284	0.1390	0.6413	0.8610
20	5.7716	3.2284	0.1390	0.6413	0.8610

Table 3. Some performance measures for different values of t for the multiple vacations policy with ***λ=0.35, θ=1, μ_1=1.1, M=8***

t	E(O)	E(F)	E(V)	M.A.	O.U.
0	8.0000	0.0000	0.0000	1.0000	1.0000
1	5.6260	2.3740	0.4895	0.7032	0.5105
2	4.5525	3.4475	0.2420	0.5691	0.7580
3	4.1088	3.8912	0.1355	0.5136	0.8645
4	3.9436	4.0564	0.0891	0.4930	0.9109
5	3.8852	4.1148	0.0687	0.4857	0.9313
6	3.8653	4.1347	0.0598	0.4832	0.9402
7	3.8591	4.1409	0.0559	0.4824	0.9441
8	3.8578	4.1422	0.0542	0.4822	0.9458
9	3.8581	4.1419	0.0535	0.4823	0.9465
10	3.8587	4.1413	0.0532	0.4823	0.9468
11	3.8594	4.1406	0.0531	0.4824	0.9469
12	3.8598	4.1402	0.0530	0.4825	0.9470
13	3.8601	4.1399	0.0530	0.4825	0.9470
14	3.8603	4.1397	0.0530	0.4825	0.9470
15	3.8605	4.1395	0.0530	0.4826	0.9470
16	3.8605	4.1395	0.0530	0.4826	0.9470
17	3.8606	4.1394	0.0530	0.4826	0.9470
18	3.8606	4.1394	0.0530	0.4826	0.9470
19	3.8606	4.1394	0.0530	0.4826	0.9470
20	3.8606	4.1394	0.0530	0.4826	0.9470

The results in Tables 1-4 are compared with those of [12] in Table 5.

Table 4. Some performance measures for different values of t for the multiple vacations policy with λ=0.3, θ=5, μ_1=1.1, M=6

t	E(O)	E(F)	E(V)	M.A.	O.U.
0	6.0000	0.0000	0.0000	1.0000	1.0000
1	4.3860	1.6140	0.2678	0.7310	0.7322
2	4.0004	1.9996	0.1709	0.6667	0.8291
3	3.8402	2.1598	0.1471	0.6400	0.8529
4	3.7622	2.2378	0.1375	0.6270	0.8625
5	3.7224	2.2776	0.1330	0.6204	0.8670
6	3.7018	2.2982	0.1307	0.6170	0.8693
7	3.6910	2.3090	0.1295	0.6152	0.8705
8	3.6853	2.3147	0.1289	0.6142	0.8711
9	3.6823	2.3177	0.1286	0.6137	0.8714
10	3.6808	2.3192	0.1284	0.6135	0.8716
11	3.6799	2.3201	0.1283	0.6133	0.8717
12	3.6795	2.3205	0.1283	0.6133	0.8717
13	3.6793	2.3207	0.1282	0.6132	0.8718
14	3.6792	2.3208	0.1282	0.6132	0.8718
15	3.6791	2.3209	0.1282	0.6132	0.8718
16	3.6791	2.3209	0.1282	0.6132	0.8718
17	3.6790	2.3210	0.1282	0.6132	0.8718
18	3.6790	2.3210	0.1282	0.6132	0.8718
19	3.6790	2.3210	0.1282	0.6132	0.8718
20	3.6790	2.3210	0.1282	0.6132	0.8718

Table 5. Comparing system characteristics of [12] results with our transient results for the reliable server multiple vacations policy II

α=0.05 β=10	Ke [12] results	Transient results	Ke [12] results	Transient results	Ke [12] results	Transient results	Ke [12] results	Transient results
(λ,θ)	(0.15,1.0)	0.15,1.0)	(0.2,1.0)	(0.2,1.0)	(0.35,1.0)	(0.35,1.0)	(0.3,5.0)	(0.3,5.0)
M	10	10	9	9	8	8	6	6
E(F)	2.8974	2.8988	3.2358	3.2283	4.1560	4.1394	2.3678	2.3210
E(o)	7.1026	7.1012	5.7642	5.7717	3.8440	3.8606	3.6322	3.6790
E(v)	0.1937	0.1837	0.1434	0.1390	0.0525	0.0530	0.1370	0.1282
E(D)	0.0040	0.0000	0.0043	0.0000	0.0047	0.0000	0.0043	0.0000
M.A.	0.7103	0.7101	0.6405	0.6413	0.4805	0.4826	0.6054	0.6132
O.U.	0.8023	0.8163	0.8523	0.8610	0.9427	0.9470	0.8587	0.8718
Var		0.4754		0.5831		0.9251		0.2689
STD		0.6895		0.7636		0.9618		0.5186

Where Var is variance and STD is standard deviation.

4.2 Discussion

Table 5 shows the steady state measures of performance obtained using the methodology proposed in this article and the steady state results of [12] for the multiple vacations policy. The result shows that there are slight differences in the values obtained by the proposed method and those obtained using [12]. This error is attributed to the inherent accuracy of the ODE45. However, the number of equations, parameters and time spent on our computation is reduced in our machine interference problem with reliable server compared to that of [12]. This shows that our model is good. We also produce results for the variance and standard deviation for the machine interference problem which [12] did not produce. We also found that the failure rate of machines affect the variance of the expected number of failed machines in the system. As the failure rate of machines increases the variance also increase. Observe also that E(D) is zero for the transient results for all parameters in Table 5. This is so because our server does not break down while [12] does.

For the machine interference problem with reliable server under multiple vacations policy there are*1+2M* equations in the system, we observe that for $M \leq 100$, the CPU time is also less than 2 seconds Table 6. We also observe that there is a relationship between the number of machines in the system and the CPU time. The actual CPU times observed for different number of machines in the system for the multiple vacations policy is inputted into linear regression in EXCEL package to compute the predicted CPU time for the system. We found that the predicted $CPU\ time\ = a + bM,\ where\ a\ and\ b$ are constants and M is the number of machines. We observe that the predicted CPU time is an indication of the actual CPU time. From linear regression the values of $a = 1.1044\ and\ b$ =0.00354

It is also observed that as the number of machine in the system increases with the same parameters, the variance of the expected number of operating machines also increases. This is true since it is a single server, as the number of machine increases the server cannot cope with the repair of failed machines in the system.

The figures below correspond to some of the results presented in Tables 1-6 above.

Figs. 2-7 below show the graph of the expected number of failed and operating machines in the system at time t with respect to the following parameters λ, μ and θ for the multiple vacations policy.

Figs. 2 and 4 show the effect of failure and service rate of broken down machines on the expected number of failed machines in the system. We found that as the failure rate of operating machine increases the expected number of failed machines increases. We also found that the breakdown and repair rates of server do not affect the expected number of operating machines. This is so because the results are closed with slight difference.

Table 6. Effect of M on the machine availability and CPU Time for sufficient value of t for the multiple vacations policy with $\lambda = 0.15, \mu_1 = 1.1, \theta = 1.$

M	**10**	**20**	**30**	**40**	**50**	**100**
E(F)	2.8988	7.9884	13.9289	19.8891	25.8499	55.6552
E(O)	7.1012	12.0116	16.0711	20.1109	24.1501	44.3448
E(v)	0.1837	0.0059	0.0000	0.0000	0.0000	0.0000
E(I)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
M.A.	0.7101	0.6006	0.5357	0.5028	0.4830	0.4434
O.U.	0.8163	0.9941	1.0000	1.0000	1.0000	1.0000
Var of E(O)	0.4754	4.1909	13.6964	28.8326	49.5632	237.2544
ACTUAL CPU TIME (secs)	1.1431	1.1792	1.1900	1.2581	1.2842	1.4577
PREDICTED CPU TIME (secs)	1.1399	1.1753	1.2107	1.2461	1.2815	1.4586

Figs. 3 and 5 below show the effect of failure and service rate of broken down machines on the expected number of operating units in the system for the multiple vacation policy. We found that the rate at which the machine fails and are serviced affect the expected number of failed and operating machines in the system.

We also found that as the failure rate of the machines increases, the expected number of operating machines decreases. We found that the breakdown and repair rates of server do not also affect the expected number of operating machines.

We also found that the variance under multiple vacations is slightly lower than that of single vacation.

The effect of vacation length on the expected number of operating machines and failed machines is shown in Figs. 6 and 7 below. We found that as the vacation length increases, the expected number of operating machines increases while the expected number of failed machines decreases.

Fig. 2 shows the graph of the effect of failure rate on expected number of failed machines for the machine interference problem with reliable server that can go on multiple vacations at time t with respect to the following parameters $\theta = 1$, *M=10* $\mu_1 = 1.1$. We observe that as the failure rate of machine increases the expected number of failed machines also increases.

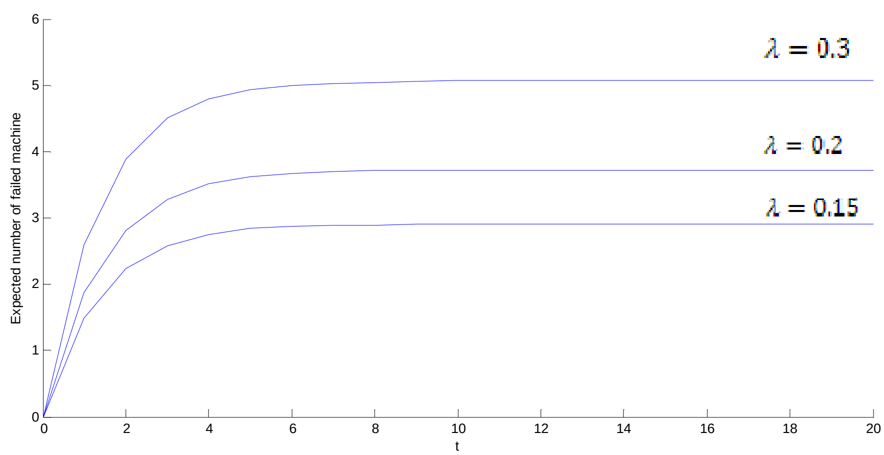

Fig. 2. Effect of failure rate of machines on the expected number of failed machines in the system at time t when $\theta = 1$, *M=10*, $\mu_1 = 1.1$

Fig. 3. shows the graph of the effect of failure rate on expected number of operating machines for the machine interference problem with reliable server that can go on multiple vacations at time t with respect to the following parameters $\theta = 1$, *M=10* $\mu_1 = 1.1$. We observe that as the failure rate of machine decreases the expected number of operating machines increases.

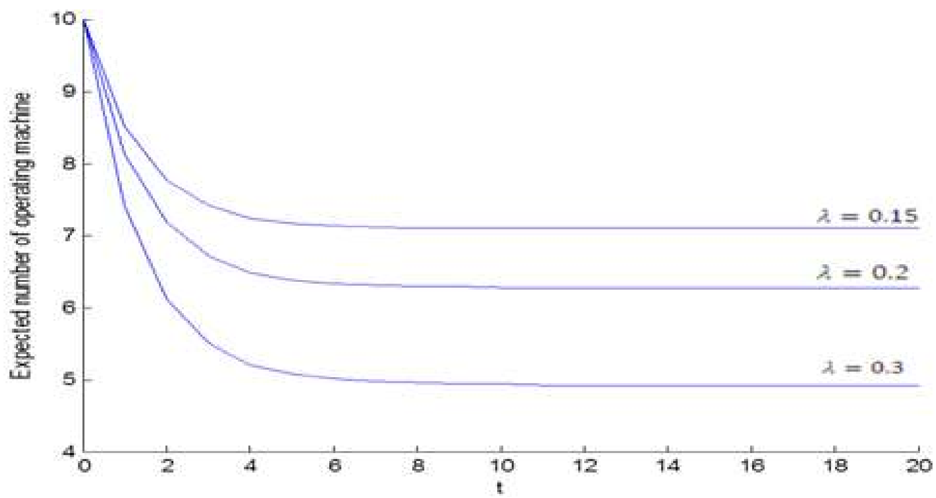

Fig. 3. Effect of failure rate of machines on the expected number of operating machines in the system at time t when $\theta = 1$, *M=10*, $\mu_1 = 1.1$

Fig. 4 shows the graph of the effect of service rate on expected number of failed machines for the machine interference problem with reliable server that can go on multiple vacations at time t with respect to the following parameters *λ=0.15*, $\theta = 3$, *M=10.*

We observe that as the service rate of machine decreases the expected number of failed machines increases.

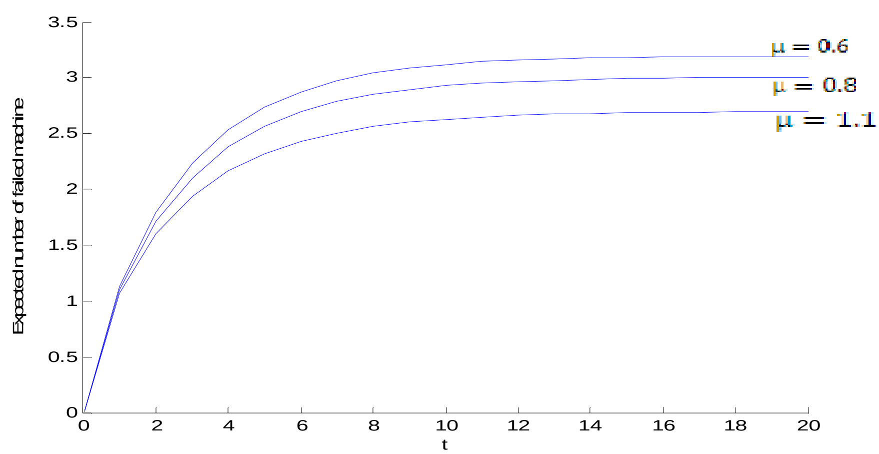

Fig. 4. The effect of service rate of machines on the expected number of failed machines in the system at time t when *λ=0.15*, $\theta = 3$, *M=10*

Fig. 5 shows the graph of the effect of service rate on expected number of operating machines for the machine interference problem with reliable server that can go on multiple vacations at time t with respect to the following parameters *λ=0.15*, $\theta = 3$, *M=10.*

We observe that as the service rate of machine increases the expected number of operating machines increases.

From Figs. 4 and 5, we found that between times 0 to 2, there is no variation in the expected number of failed and operating machines in the system as the service rate increases, but as the time varies from 2 to 20 the expected number of operating machines increases with increase in service rate (Fig. 5). In a similar manner with decrease in service rate the expected number of failed machines increases (Fig. 4).

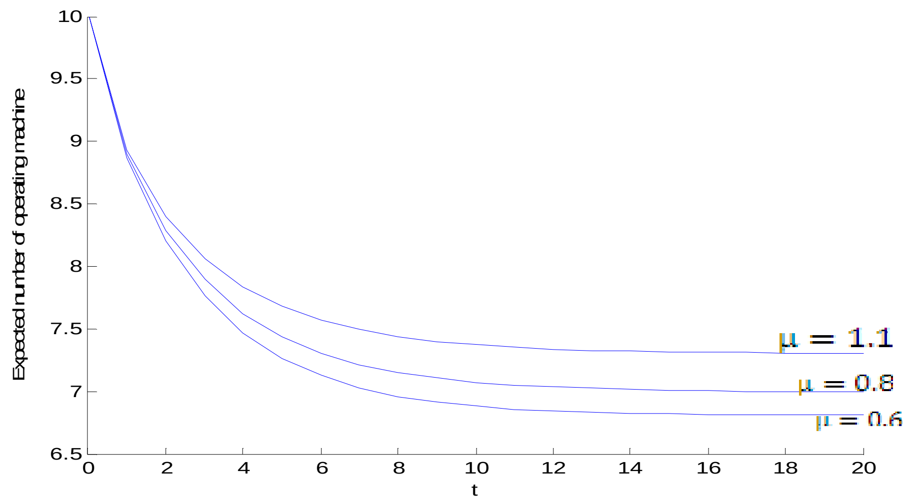

Fig. 5. The effect of service rate of machines on the expected number of operating machines in the system at time t when *λ=0.15,* $\theta = 3$, *M=10*

Fig. 6 shows the graph of the effect of vacation length on expected number of failed machines for the machine interference problem with reliable server under multiple vacations policy at time t with respect to the following parameters *λ=0.15,* $\theta = 3$, M=10.

We observe that as the vacation length of machine decreases the expected number of failed machines increases.

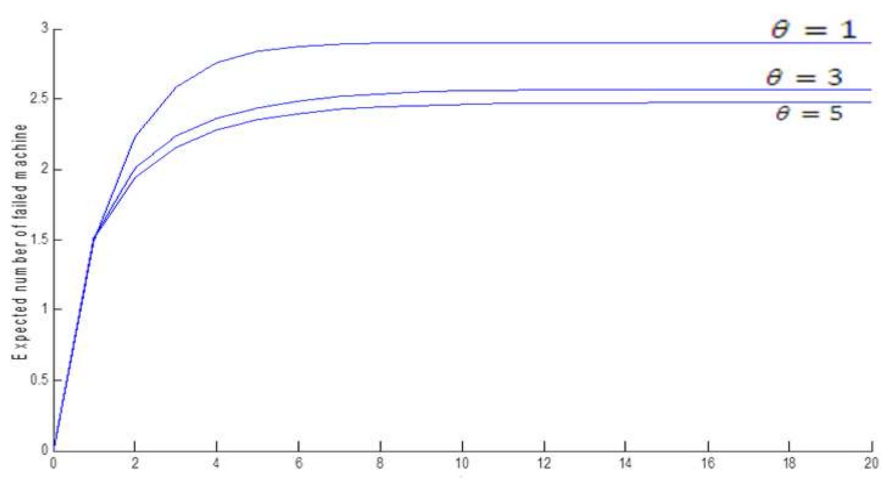

Fig. 6. Effect of vacation length on the expected number of failed machines in the system at time t when *λ=0.15, M=10,* $\mu_1 = 1.1$

Fig. 7 shows the graph of the effect of vacation length on expected number of operating machines for the machine interference problem with reliable server under multiple vacations policy at time t with respect to the following parameters λ=*0.15*, $\theta = 3$, *M=10*.

We observe that as the vacation length of machine increases the expected number of operating machines increases.

Figs. 6 and 7 show the effect of vacation length on the expected number of failed and operating machines, we found that from 0 to 1 there is no variation in the expected number of failed and operating machines in the system as the vacation length increases, but as the time varies from 1 to 20 the expected number of operating machines increases with increase in vacation length (Fig. 7). In a similar manner with decrease in vacation length the expected number of failed machines increases (Fig. 6).

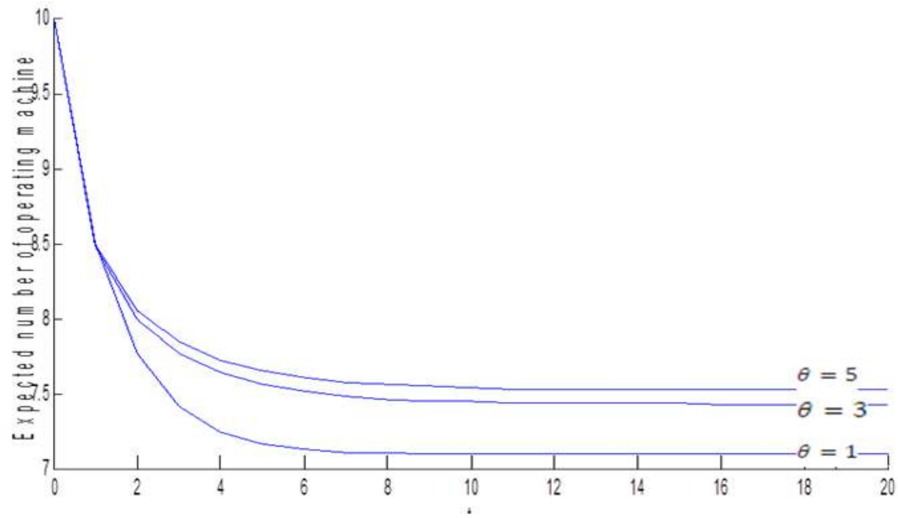

Fig. 7. Effect of vacation length on the expected number of operating machines in the system at time t when λ=*0.15*, *M=10*, $\mu_1 = 1.1$

5 Conclusions

In this paper, we considered transient analysis of machine interference problem with a reliable server under multiple vacations policy. We adopt the elementary probability argument birth-death process to formulate the Chapman-Kolmogorov differential equations for the machine interference problem with reliable server vacation The differential difference equations derived were solved using ODE45 (Runge-Kutta algorithm for solving ordinary differential equations) in MATLAB programming language. From the transient solutions we obtained various performance measures like the expected number of operating machine, expected number of failed machine, expected idle period, expected number of vacation the server has for the machine interference problem with reliable server under multiple vacations. We showed numerical results for the effect failure rate of machines, service rate of failed machines and the number of operating machines. It is observed that as the failure rate of operating machine increases the expected number of failed machines also increases. It is also notice that the breakdown and repair rate of the server in [8] have slight effect on the expected number of failed and operating machines.

We also investigate the effect of CPU time for the machine interference problem with reliable server under multiple vacation policy.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] Cox DR, Smith WL. Queues. Chapman and Hall, London; 1961.

[2] Jain M, Rakhee, Singh M. Bilevel control of degraded machining system with warm standbys, setup and vacation. Applied Mathematical Modelling. 2004;28(12):1015-1026.

[3] Jain M, Rakhee, Maheshwari S. N-policy for a machine repair system with spares and reneging. Applied Mathematical Modelling. 2004;28(6):513-531.

[4] Jain M. Finite capacity M/M/r queueing system with queue-dependent servers, Computers & Mathematics with Applications, 2005;50(1–2):187-199.

[5] Jain M, Sharma GC, Singh N. Transient analysis of M/M/R machining system with mixed standbys, switching failures, balking, reneging and additional removable repairmen. IJE Transactions B: Basics. 2007;20(2):169-182.

[6] Jain M, Upadhyaya S. Threshold N-policy for degraded machine system with multiple type spares and multiple vacations. Quality Technology & Quality Management. 2009;6(2):185-203.

[7] Jain M, Kumar S. Bi-Level control policy for redundant repairable multicomponent system with reneging. Set Up and Vacation. IJE Transactions A: Basics. 2009;22(2):159-168.

[8] Ojobor SA. Machine interference problem with reliable server under single vacation. Journal of Mathematics and System Science. 2014;4:732-742.

[9] Ojobor SA, Omosigho SE. Transient solution of machine interference problems with unreliable server under single vacation. The Journal of Nigeria Institution of Production Engineers. 2014;17:47-62.

[10] Ojobor SA. Transient solution of machine interference problem with an unreliable server under multiple vacations policy. British Journal of Mathematics and Computer Science. 2015;6(4):351-369.

[11] Haque L, Armstrong MJ. A Survey of the machine interference problem. European Journal of Operational Research. 2007;(l):179:469-482.

[12] Ke JC. Vacation policies for machine interference problem with an un-reliable server and state dependent service rate. Journal of the Chinese Institute of Industrial Engineers. 2006;23(2):100-114.

[13] Dormand JR, Prince PJ. A family of embedded Runge-Kutta formulae. J. Comp. Appl. Math. 1980;l. 6:19-26.

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