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F Theory Compactifications and Central Charges of BPS States

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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ABSTRACT

F-theory, as Theory of everything is compactified on Calabi-Yau threefolds or fourfolds. Using Batyrev's toric approximation and mirror symmetry of Calabi-Yau manifolds it is possible to present Calabi-Yau in the form of dual integer polyhedra. With the help of Gelfand, Zelevinsky, Kapranov algorithm are calculated the numbers of BPS-states in F-theory, and by application of Tate's algorithm are determined the enhanced symmetries. As the result, any integral dual polyhedron representing a Calabi-Yau manifold, is characterized by its own set of topological invariants - the numbers of BPS-states, whose central charges are classified by enhanced symmetries.

Keywords: F-theory; Calabi-Yau manifold; BPS-states; enhanced symmetries; central charge.

1. INTRODUCTION

F-theory or "theory of everything" (Theory of everything, abbr. TOE) - hypothetical combined physical and mathematical theory that describes all known fundamental interactions. During the twentieth century, it was proposed a lot of "theories of everything", but none of them could go through experimental testing. The main problem of construction the scientific "theory of everything" is that quantum mechanics and General Theory of Relativity (GTR) have different

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areas of their application. Quantum mechanics is mainly used to describe the microcosm, and general relativity applies to the macrocosm. Directly the combination of quantum mechanics and special relativity in single formalism (quantum relativistic field theory) leads to divergence problem - the lack of final results for experimentally testable variables. To solve this problem was used the idea of renormalization. For some models renormalization mechanism allows to build a very good working theory, but the addition of gravity (ie the inclusion of the theory of general relativity as the limiting case of small fields and large distances) leads to divergences that still can not be removed. But it does not mean that such a theory can not be constructed.

Currently, the main candidate for a “theory of everything” is an F-theory, which operates with a large number of measurements. The impetus for this has become the Kaluza Klein theory, which allows us to see that the addition of extra dimension to general theory of relativity leads to Maxwell’s equations. Thanks to the ideas of Kaluza and Klein it was possible to create theories that operates with large dimensions. Use of the extra dimensions proposed the answer to the question of why the action of gravity is much weaker than other types of interactions. The conventional answer is that gravity exists in additional dimensions, so its effect as the observable become weaker.

F-theory string twelve-dimensional theory defined on the energy scale of the order of 10^{19} GeV [1]. F-theory compactification leads to a new type of vacuum, so for study SUSY we must compactify F-theory on Calabi-Yau manifold. Since there are a lot of Calabi-Yau manifolds, we are dealing with a large number of new models implemented in low-energy approximation. A singular manifold Calabi-Yau determines the physical characteristics of the topological solitonic states that are interpreted as particles in high energy physics. Essential for us is to present threefold Calabi-Yau in the form of an elliptic fibration with singular layers, that enables to use Kodairas classification of singularities for elliptic bundles. To the type of singularities correspond the sets of particles classified by enhanced symmetry, for which it is possible to find BPS states [2]. BPS states appear in Seiberg and Witten’s derivation, which leads to a formula for mass spectrum. A crucial role is played by the central charge, which appears in the BPS mass inequality. The masses of BPS states are determined by the central charge, which depends on the vevs and masses that parameterize the moduli space of F-theory vacua. So, the purpose of this paper is to study the properties of Calabi-Yau manifold as the space, in which is encoded the information about the central charge. Interpretation of these BPS states for the fiber bundles is presented in our paper. The purpose of the article is the following. It is known that Calabi-Yau manifolds can be represented as dual reflexive polyhedron with integer vertices. To such manifold, on the one hand, you can associate a set of topological invariants - BPS states, calculated by application Gelfand, Zelevinsky, Kapranov algorithm [3], and on the other hand - the enhanced symmetry obtained by applying Tate algorithm. Thus, BPS states can be definitely characterized by a set of enhanced symmetries what is important to further searches for new physics at collider experiments in future. The singularities of elliptic fibration are classified by enhanced groups and, at the same time, by the number of BPS states, defined by central charges. Therefore, with points on polyhedra of enhanced groups can be associated the central charges by analogy with electro-magnetic charge grid in Maxwell’s electrodynamics. Another important purpose of the paper is to understand the central charge of BPS state with the help of analogy with usual electric and magnetic charges. The picture becomes more complicated by the complex structure of Calabi-Yau manifold with singularities. Let’s consider in more detail the compactification of F-theory to Calabi-Yau threefols.

2. COMPACTIFICATION ON CALABI-YAU THREEFOLDS AND TORIC REPRESENTATION OF THREEFOLDS

Twelve-dimensional space, describing space-time and internal degrees of freedom is represented as following:

$$R^6 \times X^6,$$

where R^6 - six-dimensional space-time, on which acts conformal group SO (4, 2) and X^6 - compact threefold, three-dimensional complex manifold Calabi-Yau. Let’s consider the weighted projective space defined as follows:

$$P^4_{\omega_1,\ldots,\omega_5} = P^4 / Z_{\omega_1} \times \ldots \times Z_{\omega_5},$$
$$X_d(\varpi_1,\ldots,\varpi_5), \quad W(\varphi_1,\ldots,\varphi_5) = 0,$$

where P^4 - four-dimensional projective space, Z_{ω_i} - cyclic group of order ω_i. On a weighted projective space $P^4_{\omega_1,\ldots,\omega_5}$ is determined quasihomogeneous polynomial

$$W(\varphi_1,\ldots,\varphi_5),$$

called superpotential, which satisfies the homogeneity condition

$$W(x^{\omega_1}\varphi_1,\ldots,x^{\omega_5}\varphi_5)=x^d W(\varphi_1,\ldots,\varphi_5),$$

where $d=\sum_{i=1}^5 \omega_i$ and $\varphi_1,\ldots,\varphi_5 \in P^4_{\omega_1,\ldots\omega_5}$. The set of points $p \in P^4_{\omega_1,\ldots,\omega_5}$, satisfying $W(p)=0$ forms Calabi-Yau threefold $X_d(\omega_1,\ldots,\omega_5)$ [4].

2.1 Toric Manifolds as an Extensions of Weighted Projective Spaces

The simplest examples of toric varieties are projective spaces P^2 and $P^{(2,3,1)}$, where P^2 is defined as follows

$$P^2=\frac{C^3\setminus\{0\}}{C\setminus\{0\}}$$

where division into $C\setminus\{0\}$ means the identification of points in complex space *C*, connected by equivalence relation

$$(x,y,z)\sim(\lambda x,\lambda y,\lambda z)$$

$$\lambda\in C\setminus\{0\}$$

x, y, z are called homogeneous coordinates. Elliptic curve in P^2 is described by the Weierstrass equation

$$y^2z=x^3+axz^2+bz^3.$$

A similar description can be given for $P^{(2,3,1)}$, which in contrast to P^2 is represented by the following equivalence relation:

$$(x,y,z)\sim(\lambda^2x,\lambda^3y,\lambda z)$$
$$\lambda\in C\setminus\{0\}$$

and Weierstrass equation has the form

$$y^2 = x^3 + axz^4 + bz^6.$$

The elliptic Calabi-Yau manifold can be described by Weierstrass form

$$y^2 = x^3 + xf(z) + g(z),$$

which describes an elliptical fibration (parameterized by *(y, x)*) over the base, where *f(z)*, *g(z)* - functions on the basis [1]. 24 parameters on P^1 associated with the functions *f (z), g(z)* are specified by zeros of the discriminant. Then in some divisors D_i the layer is degenerated. Such divisors are the zeros of the discriminant

$$\Delta=4f^3+27g^2.$$

Table 1. Kodaira's classification of singularities of elliptic fibrations

$ord(\Delta)$	Fiber type	Singularity type
0	smooth	no
n	I_n	A_{n-1}
2	II	no
3	III	A_1
4	IV	A_2
$n+6$	I_n^*	D_{n+4}
8	IV^*	E_6
9	III^*	E_7
10	II^*	E_8

Singularities of Calabi-Yau manifold - are singularities of its elliptic fibration. These singularities are encoded in the polynomials *f, g* and their type determines the gauge group and matter content of the compactified F-theory. Classification of singularities of elliptic fibrations was given by Kodaira [5] and presented in Table 1. Classification of the fibers of an elliptic fibrations is presented in Fig. 1.

P^2 and $P^{(2,3,1)}$ may be represented by diagrams with vectors v_x, v_y, v_z in some lattice, such that

$$q_xv_x + q_yv_y + q_z v_z = 0,$$

where q_x, q_y, q_z are exponents.

Since the possible singular sets of Calabi-Yau manifold may be the points, which are singularities of type C^3/Z_{Ns} or curves - singularities of type C^2/Z_{Ns}, both types of singularities and their blow-up can be described by methods of toric geometry [6]. To describe toric variety $P^4_{\omega_1,\dots,\omega_5}$, let's consider integer polyhedron $\Delta \in R^n$. In this case, we can determine a simplicial reflexive polyhedron

$$\Delta(\vec{\varpi}) := \left\{ (x_1,\dots,x_{n+1}) \in R^{n+1} \mid \sum_{i=1}^{n+1} \varpi_i x_i = 0,\ x_i \geq -1 \right\}$$

Complex d-dimensional toric variety is defined by combinatorial data Δ, called fan. A finite non-empty set Δ, called a fan, is determined by a combination of convex rational polyhedral cones σ in R^{n+1}

$$\sigma = R_{\geq 0}\overrightarrow{n_1} + \dots + R_{\geq 0}\overrightarrow{n_r}\ .$$

1. If each face of a cone in Δ belongs to Δ;
2. The intersection of any two cones in Δ is a face of each.

Integer polyhedron Δ is called reflexive polyhedron [7] if the corresponding dual polyhedron ∇

$$\nabla = \left\{ (y_1,\dots,y_{n+1}) \mid \sum_{i=1}^{n+1} x_i y_i \geq -1, \quad (x_1,\dots,x_{n+1}) \in \Delta \right\}$$

is also integer. This property of polyhedra is connected with mirror symmetry of Calabi-Yau manifolds [8]. The vertices of a simplicial reflexive polyhedron $\Delta(\omega_i)$ are determined by the weights of $P^4(\omega_i)$, since the degree d of Calabi-Yau threefold $X_d(\omega_1,\dots,\omega_5)$ satisfies the condition $d = \sum_{i=1}^{5} \omega_i$ .Examples of construction of reflexive polyhedra through the Calabi-Yau weights are given in [9].

Let's consider the holomorphic three-form *Ω(ψ)* of threefold Calabi-Yau *X* as a function of ψ_i - coordinates on the complex Calabi-Yau space. Their derivatives are elements of group $H^3(X)$. After the integration of elements in $H^3(X)$, we get linear differential equations for the periods Π, the Picard-Fuchs equations, which allows us to calculate the Yukawa couplings. In terms of Mori generator $l(\theta)$, satisfyin $\sum_i l_i \omega_i = 0$ and according to Gelfand, Kapranov and Zelevinsky algorithm [10], thanks to mirror symmetry between Kahler and complex Calabi-Yau manifols, we can write Picard-Fuchs equation with periods Π(x) as [9]:

$$\left\{ \prod_{l_j^{(k)} \succ 0} \left(\prod_{i=0}^{l_j^{(k)}-1} (\theta_j - i) \right) - \prod_{i=1}^{|l_0^{(k)}|} \left(i - |\, l_0^{(k)} \,| - \theta_0 \right) \prod_{\substack{l_j^{(k)} \prec 0 \\ j \neq 0}} \left(\prod_{i=0}^{|l_j^{(k)}|-1} \left(\theta_j + l_j^{(k)} - i \right) \right) x_k \right\} \tilde{\Pi}(x) = 0$$

$$\theta_j = \sum_{k=1}^{d} l_j^{(k)} \Theta_{x_k}$$

where $\theta_i = x_i d/dx_i$, x_i - are algebraic coordinates on the moduli space of the complex structure of Calabi-Yau manifold.

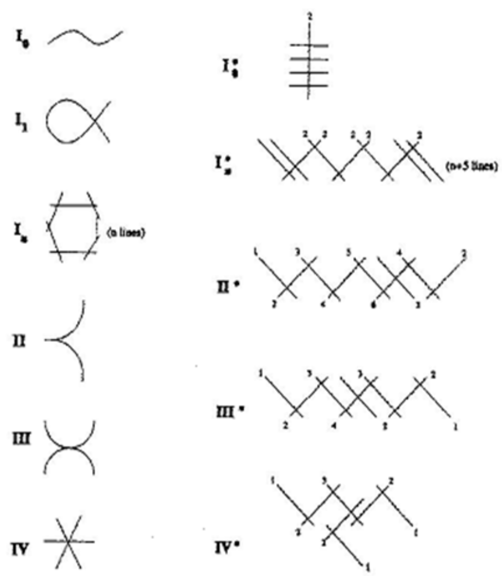

Fig. 1. Classification of elliptic fibers

The principal parts of the Picard-Fuchs operators could have, in particular, the form [11]:

$$\begin{aligned} L_1 &= 3\theta_1^2 - \theta_1\theta_2 + \theta_2^2, \\ L_2 &= \theta_2^2, \\ L_3 &= \theta_3^2, \\ L_4 &= \theta_2^2 + 4\theta_2\theta_3 + 4\theta_3^2 - 3\theta_2\theta_4 - 6\theta_3\theta_4 + 9\theta_4^2, \end{aligned}$$

Yukawa couplings are:

$$K_{\tilde{t}_i\tilde{t}_j\tilde{t}_k}(\tilde{t}) = \frac{1}{\varpi_0\left(x(\tilde{t})\right)^2} \sum_{l,m,n} \frac{\partial x_l}{\partial \tilde{t}_i} \frac{\partial x_m}{\partial \tilde{t}_j} \frac{\partial x_n}{\partial \tilde{t}_k} K_{x_l x_m x_n}\left(x(\tilde{t})\right)$$

and could be overwritten by a variable $q_i = e^{\tilde{t}_i}$

$$K_{\tilde{t}_i\tilde{t}_j\tilde{t}_k} = K^0_{ijk} + \sum_{n_i} \frac{N(\{n_i\})n_i n_j n_k}{1 - \prod_l q_l^{n_l}} \prod_l q_l^{n_l}$$

where $\tilde{t}_i$ - Kahler space coordinates x_i - coordinates of complex mirror manifold. Here $n_i = \int_C h_i$ is an integer and not necessarily positive, in particular, for singular varieties. That is, the solution of the Picard-Fuchs equations makes it possible to calculate the Yukawa coupling constants, which are expressed in terms of the numbers n_i. n_1 - is the number of rational curves of degree 1, n_2 - the number of rational curves of degree 2 etc. In general, n_l- numbers of BPS-states through which is determined the central charge and the mass of the solitonic objects. Thus, knowing Mori generators we can find the principal part of the Picard-Fuchs operators, through which are calculated numbers of BPS-states.

In summary, it must be stressed that toric presentation of Calabi-Yau manifolds makes it possible to calculate the topological invariants - BPS-states.

3. ENHANCED SYMMETRY IN F-THEORY

F-theory allows geometrical and physical interpretation of solitonic states in terms of geometrical singularities and enhanced symmetry [12]. Tate et al. [12,13] proposed algorithm which allows to extract the enhanced symmetry from toric description of elliptic Calabi-Yau manifold. This algorithm allows to read off the Dynkin diagram from the dual polyhedron ∇ that realizes toric description of elliptic Calabi-Yau manifold according to toric Batyrevs approximation [7]. Using the technique of [12], dual polyhedron ${}^3\nabla$, representing Calabi-Yau is divided by triangle ${}^2\nabla$ on the top and bottom

$$\nabla = \nabla^H_{bot} \cup \nabla^{k=1}_{top},$$

where ∇^H_{bot} depends on enhanced gauge group H and $\nabla^{k=1}_{top}$ depends on k.

For fourfolds of type

$$X_{18k+18}(1, 1, 1, 3k, 6k+6, 9k+9)$$

The gauge groups are written in the following way [14]:

$H \times SU(1)$	*for*	$k = 1$,
$H \times SO(8)$	*for*	$k = 2$,
$H \times E_6$	*for*	$k = 3$,
$H \times E_7$	*for*	$k = 4$,
$H \times E_8$	*for*	$k = 5$,
$H \times E_8$	*for*	$k = 6$.

Thus, solitonic states, characterized by BPS-states, as singularities of Calabi-Yau manifolds may be classified by enhanced symmetry as to each type of Calabi-Yau, presented in the form of dual polyhedron, corresponds its enhanced symmetry.

4. CONCLUSION

We have given the definition of Calabi-Yau hypersurfaces in weighted projective spaces through their weights and presented Kodairas classification of singularities of elliptic fibrations. Application of Batyrev's toric approach, and Gelfand, Kapranov, Zelevinsky algorithm made it possible to calculate the number of BPS-states characterizing the solitonic objects in the F-theory. The presence of singularities of elliptic fibrations and the technique of blowing-up give us the possibility to describe Calabi-Yau manifolds by methods of toric geometry for understanding the nature of central charges. For simplicity of toric treatment of Calabi_Yau manifold were considered projective spaces P^2 and $\mathrm{P}^{(2,3,1)}$, which may be represented by diagrams with vectors v_x, v_y, v_z in some lattice. The description of Calabi-Yau manifold was realized in terms of simplicial reflexive polyhedron and corresponding dual polyhedron. Our purpose was to connect this integer polyhedron with central charges of Calabi-Yau in analogy with electric and magnetic charges, determined in some two-dimensional lattice [15].

Consideration of Calabi-Yau using Tate's algorithm enables to associate solitonic states with enhanced symmetries of F-theory. Thus, toric presentation of Calabi-Yau through Batyrev's toric approximation enables, on the one hand, to calculate BPS-states, and on the other, to calculate the enhanced symmetry of the polyhedron, describing massless solitonic states in F-theory. The main result of the article is reduced to the conclusion that we get an adequate treatment of central charges of the BPS-states as elements on the polyhedron connected with the enhanced symmetries. As we study gauge symmetry in F-theory in light of enhanced group, we can derive the matter contents of solitonic states. Thus, from the one hand, we can investigate central charges and masses, connected with the existence of BPS states and, from the other hand, we can correspond to each solitonic state its matter content. This fundamental conclusion is of importance because of practical use for future experiments at high energy colliders.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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