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Wind Speed Equation of Circular Cyclone

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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Short Research Article

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Abstract

This paper uses a point model of cylindrical box wrapped by zero-weighted membrane to derive a wind speed equation of circular cyclone by method of section based on Boyle's law, Charles' law and Newton's laws. The obtained equation is a non-linear partial differential equation (PDE) with two unknown functions, seldom seen in literatures. The obtained vertical motion is upward which has been confirmed by a recorded video; and it also shows that the mass of air approaches to zero at isothermal layer where the temperature keeps unchanged for any height in the layer.

Keywords: Wind speed equation; Newton's law; Boyle's law and Charles' law; ideal gas law; cyclone; steady flow; D'Alembert's principle.

1 Introduction

There are two kinds of methods: one of them belongs to the "certain type", where the problem is expressed by differential equation(s) (integral equation, etc.) with boundary value(s), its solution is certain; the other

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belongs to the "uncertain type", where the problem (usually involves random, or stochastic factor) is reduced to expression based on statistic, probability, etc. its solution is uncertain, or the rule it obtained is certain only under the sense of statistic.

In the field of atmosphere, most methods used are belonging to the uncertain type. For examples, Woodten used statistical analysis to obtain the relationship between wind speed, pressure and temperature [1]. The obtained relationship among wind speed, pressure and temperature in [1] must not be expressed by certain equation, because the method he used is based on statistic. Calif and Schmit modeled wind speed sequence using lognormal continuous stochastic equation [2], they can not get results expressed by certain formula, but by numerical results, since the method they used is based on probability. Zhang et al used Poisson-Gumbel distribution to wind speed calculation for the southeast coastland of China [3], Chu et al used regression method for climate prediction [4]. Duan et al used optimization method for numerical studies of atmospheric and oceanic sciences [5]. Stull provided the mathematical equations needed for higher level of understanding of meteorology [6], etc.

Different from the above traditional method, the author derived a wind speed equation of a point in air, expressed in PDE, based on Boyle's law, Charles' law and Newton's law [7]. The solution of that wind speed equation was obtained in [8].

A tropical cyclone is a rapidly rotating storm system. According to its strength, it is classified as hurricane, typhoon, tropical storm, cyclonic storm, tropical depression, and simple cyclone. This paper studies the simple cyclone, in which lesser scientific laws are related. The purpose of this paper is to find the relationship among wind speed, pressure and temperature of a point in uniform circular motion (UCM) of steady flow (independent of time t) in a simple cyclone.

In Section 2, a point model is constructed. The wind speed equation of a point in UCM of a cyclone of a steady flow is derived, in which the components of velocity u_r and u_θ are related each other, but independent with u_z .

In section 3, a comparison between using the combination of Boyle's law and Charles' law and the ideal gas law [9] is studied. It shows that the ideal gas law belongs to micro-level study. If one tries to use a micro-level law with assumptions of random motion etc., to a macro-level problem, then, even if one got a result, the result must belong to uncertain type. We use the combination of Boyle's law and Charles' law lead the result to certainty.

Finally, a conclusion is made.

2 Wind Speed Equation of a Point in UCM of Steady Flow of Cyclone

2.1 The calculated model

The air has no shape, no boundary and no volume, thus, how to set up an equation? To solve the problem, The author used a point model, in which the air is wrapped by a zero-weighted membrane of a given shape. Such a wrapping is equivalent to the air wrapped by nothing, it does not change the actually stress field but has important use, since the wrapped air has shape, volume and thus one can use scientific laws to set up equations.

The wind speed equation of a point in air had been derived in Cartesian coordinates for straight line motion [8]. Here, we construct a cylindrical box for circular motion.

A "cylindrical box" N (r, θ, z, t) wrapped by zero-weighted membrane, with six surface areas $A_{r+\Delta r} = (r + \Delta r)\Delta\theta\Delta z$, $A_r = r\Delta\theta\Delta z$, $A_{\theta+\Delta\theta} = A_\theta = \Delta r\Delta z$, $A_{z+\Delta z} = A_z \approx r\Delta\theta\Delta r$, normal to r, θ, z respectively, and volume $V \approx r\Delta\theta\Delta r\Delta z = r\Delta\theta A_\theta = \Delta r A_{r+\Delta r/2} = \Delta z A_z$ is studied. Where (r, θ, z) is the cylindrical coordinates, t is time. The tangent of θ takes "cyclonic turn" as its direction. The z-axis sets on sea-level as z = 0, and up-ward for z > 0.

When $\Delta r \to 0, \Delta s \to 0, \Delta z \to 0$, the "box" reduced to a "point" and in the following it is denoted by a "point" or a "box".

2.1.1 Why we study a point model?

The concept of "point force", "point mass" had already familiar with engineering circle and the solution of a "point load" plays an important role in analysis. For example, the solution of force at a point in the interior of an elastic space (Kelvin's solution), and the solution of force at a point in the interior of an elastic half space (Mindlin's solution) form a basic solution for engineering analysis. So the solution of a point in air should have uses in a defined range.

2.1.2 What is the features of a point model?

An assumption for a point-force or point-mass is that the force (or mass) on (or in) the point has volume $V \approx \Delta r \Delta s \Delta z$ such that as $\Delta r, \Delta s, \Delta z \to 0$, the volume V has a limit. Note that V is not a constant. V is variable, V approaches to zero and is not equal to constant zero. So that V is a function of (r, θ, z) (time t is neglected for a steady flow). The size of shape of the box can be change while the volume can keep unchanged, this is shown in the following (2-3).

A vector **u** in Cartesian coordinates is expressed by its components:

$$\mathbf{u} = u_x \mathbf{i} + u_y \mathbf{j} + u_z \mathbf{k}, \tag{2-1}$$

Where **i, j, k,** are unit vectors in x, y, z direction respectively. In the following, vectors are shown by **boldface**.

The relation between the unit vectors $\mathbf{e_r}, \mathbf{e_\theta}, \mathbf{e_z}$ of cylindrical coordinates cohered with the surface of the cylinder central at (0,0,0) and **i, j, k** is:

$$\begin{Bmatrix} \mathbf{e_r} = \cos\theta\, \mathbf{i} + \sin\theta\, \mathbf{j} \\ \mathbf{e_\theta} = -\sin\theta\, \mathbf{i} + \cos\theta\, \mathbf{j} \\ \mathbf{e_z} = \mathbf{k} \end{Bmatrix}, \tag{2-2}$$

At first, we show that

$$\frac{\partial V}{\partial r} = \lim_{\Delta r \to 0, \Delta\theta \to 0, \Delta z \to 0} \frac{\Delta V}{\Delta r} = \frac{V(r+\Delta r, \theta, z) - V(r, \theta, z)}{\Delta r} = 0 \,, \tag{2-3}$$

Similarly, we have: $\frac{\partial V}{\partial \theta} = 0, \frac{\partial V}{\partial z} = 0.$

To show that V is at maximum or minimum, the sign of higher order derivatives is needed to check.

$$\lim_{\Delta r \to 0, \Delta\theta \to 0. \Delta z \to 0} \frac{\partial^2 V}{\partial (\Delta r)^2} = \frac{\partial^2 V}{\partial (\Delta \theta)^2} = \frac{\partial^2 V}{\partial (\Delta z)^2} = 0 \,,$$

In order to know the sign of the third order of derivatives, a cube with length c = min $\{\Delta r, r\Delta\theta, \Delta z\}$, volume $V_c = c^3 \le V$, inserted in V is constructed. Then, we have:

$$\lim_{\Delta r \to 0, \Delta\theta \to 0, \Delta z \to 0} \frac{\partial^3 V}{\partial (\Delta r)^3} \ge \lim_{c \to 0} \frac{\partial^3 V_c}{\partial c^3} = 6 > 0, \tag{2-4}$$

(2-4) shows that the volume V of the box is at minimum, for all $\Delta r, \Delta s, \Delta z \to 0$.

2.2 The scientific laws

The combination of Boyle's Law and Charles' Law for the point is:

$$pV = RT, \tag{2-5}$$

where $p = p(r, \theta, z)$, $V = V(r, \theta, z)$, and $T = T(r, \theta, z)$ are pressure, volume, and temperature respectively. R is a constant.

Note that: The common (or engineering) unit dimensional system (m, s, N) is used here for（$\text{length}, \text{time}, \text{force}(N = \text{Newton})$).

The dimension of pressure p is expressed by $(\dim p = N.m^{-2})$, similarly, $(\dim V = m^3)$, $(\dim R = N.m^2 \text{deg} K^{-1})$, $(\dim T = \deg K)$ where $T = T_{°C} - T_K = T_{°C} + 273.15℃ \geq 0$, $T_K = -273.15℃$ is the absolute temperature K. $T_{°C}$ is the temperature in Celsius' degree.

Differentiating both sides of (2-5) respect to r, θ, and z, respectively, we have

$$\frac{\partial p}{\partial r} V = R \frac{\partial T}{\partial r}, \ \frac{\partial p}{\partial \theta} V = R \frac{\partial T}{\partial \theta}, \ \frac{\partial p}{\partial z} V = R \frac{\partial T}{\partial z}, \tag{2-6}$$

2.3 The calculating method --- method of section

The method of section is used. That is: according to D'Alembert's principle, a box subjected to stresses on surfaces and inertial body force must be in equilibrium.

Thus, we get three equilibrated equations of components of applied force at $r-$, $\theta-$, and z-direction, respectively. i.e.,$\sum F_r = 0$, $\sum F_\theta = 0$, $\sum F_z = 0$.

$\sum F_r = 0$, We have:

$$p(r + \Delta r)[A_{r+\Delta r}] - p(r)[A_r] - ma_r \approx 0, \tag{2-7}$$

Where m is the mass of the box, a_r is the acceleration component in r-direction, and $p(r) = p(r, \theta, z)$ emphasis on that only r is varied, $(\theta, \text{and } z \text{ keep unchange})$ for simply. Expanding $p(r + \Delta r)$ into Taylor expansion and keeps two terms, i.e.,

$$p(r + \Delta r) = p(r) + \Delta r \frac{\partial p(r)}{\partial r}, \tag{2-8}$$

Substituting (2-8) into (2-7), we have

$$\frac{\partial p(r)}{\partial r} V + p(r) \frac{V}{r} - ma_r = 0, \tag{2-9}$$

Substituting (2-9) into (2-6), we have:

$$m\, a_r = R \left(\frac{\partial T}{\partial r} + \frac{T}{r} \right), \tag{2-10}$$

$\sum F_\theta = 0$, we have

$$p(\theta + \Delta\theta) A_{\theta+\Delta\theta} \cos\Delta\theta - p(\theta)\, A_\theta - ma_\theta = 0, \tag{2-11}$$

Where a_θ is the acceleration component in $\theta-$direction, and $p(\theta) = p(r,\theta,z)$ emphasizes on that only θ is varied (r, z keep unchange) for simple. Expanding $p(\theta+\Delta\theta)$ into Taylor expansion and keeps two terms, i.e.,

$$p(\theta+\Delta\theta) = p(\theta) + \Delta\theta\frac{\partial p(\theta)}{\partial\theta}, \tag{2-12}$$

Substituting (2-12) into (2-11) and neglecting the smaller terms, we have

$$\frac{\partial p}{\partial\theta} V\frac{1}{r} = m\, a_\theta , \tag{2-13}$$

Substituting (2-13) into (2-6), we have:

$$ma_\theta = \frac{R}{r}\frac{\partial T}{\partial\theta}, \tag{2-14}$$

$\sum F_z = 0$, we have

$$p(z+\Delta z)A_{z+\Delta z} - p(z)A_z - mg = 0 , \tag{2-15}$$

Where g is the acceleration of gravity. Expanding $p(z+\Delta z)$ into Taylor expansion and keeps two terms, i.e.,

$$p(z+\Delta z) = p(z) + \Delta z\frac{\partial p}{\partial z}, \tag{2-16}$$

Substituting (2-16) into (2-15), we have:

$$\frac{\partial p}{\partial z}V - mg = 0 , \tag{2-17}$$

Substituting (2-17) into (2-6), we have

$$mg = R\frac{\partial T}{\partial z}, \tag{2-18}$$

For the case of the box in UCM within a horizontal plane, the components of velocity u_r and u_θ are related each other, which can be obtained from vector triangle (vector addition of u_θ, u_r and $u_{\theta+\Delta\theta}$), of a box in circular motion, i.e.,

$$u_\theta + u_r = u_{\theta+\Delta\theta} , \tag{2-19}$$

Where $\mathbf{u_r} = u_r\mathbf{e_r}$, $\mathbf{u_\theta} = u_\theta\mathbf{e_\theta}$, $\mathbf{u_{\theta+\Delta\theta}} = u_{\theta+\Delta\theta}\mathbf{e_{\theta+\Delta\theta}}$.

Decomposition of $\mathbf{u_{\theta+\Delta\theta}}$ into $\mathbf{e_r}$ and $\mathbf{e_\theta}$, we have

$$u_{\theta+\Delta\theta} = u_{\theta+\Delta\theta}\sin\Delta\theta\, e_r + u_{\theta+\Delta\theta}\cos\Delta\theta\, e_\theta, \tag{2-20}$$

Substituting (2-20) into (2-19), by equilibrium, i.e., let the terms of $\mathbf{e_r}$ and $\mathbf{e_\theta}$ be zero, we have:

$$u_r = -u_{\theta+\Delta\theta}\sin\Delta\theta \approx -u_\theta\Delta\theta = -u_\theta\frac{\Delta s_\theta}{r}, \tag{2-21}$$

$$u_\theta = u_{\theta+\Delta\theta}\cos\Delta\theta = u_{\theta+\Delta\theta}, \text{(as } \Delta\theta\to 0) \tag{2-22}$$

Where $u_r = -$ indicates its direction opposite to the r-direction (towards to the center of the circle) no matter how is the direction of $u_\theta = +, \Delta\theta = +$ (cyclonic turn) or $u_\theta = -, \Delta\theta = -$ (anti-cyclonic turn).

$u_r \ll u_\theta$. And $u_\theta = u_{\theta+\Delta\theta}$ means that **the tangent wind speed u_θ is uniform for all θ.** This is a rotation with uniform velocity, then we have:

$$a_\theta = \frac{\partial u_\theta}{\partial t} = \lim_{\Delta t \to 0} \frac{u_{\theta+\Delta\theta} - u_\theta}{\Delta t} = 0, \tag{2-23}$$

$$a_r = \frac{\partial u_r}{\partial t} = \frac{\partial}{\partial t}\left(u_\theta \frac{\Delta s_\theta}{r}\right) = \frac{\Delta s_\theta}{r}\frac{\partial u_\theta}{\partial t} + u_\theta \frac{\partial \Delta s_\theta}{r\,\partial t} = \frac{\Delta s_\theta}{r} a_\theta + \frac{u_\theta^2}{r} = \frac{u_\theta^2}{r}, \tag{2-24}$$

Substituting (2-23) into (2-13) and (2-14), we have:

$$\text{p and T are independent with } \theta. \tag{2-25}$$

Substituting (2-24) into (2-9) and (2-10), we have

$$m\frac{u_\theta^2}{r} = \frac{\partial p}{\partial r}V + p(r)\frac{V}{r}, \tag{2-26}$$

$$m\,\frac{u_\theta^2}{r} = R\left(\frac{T}{r} + \frac{\partial T}{\partial r}\right), \tag{2-27}$$

Summing up (2-9), (2-17) and (2-25), we have:

$$\rho(r,z)u_\theta^2(r,z) = p(r,z) + r\frac{\partial p(r,z)}{\partial r}, \tag*{(2-28)$_A$}$$

$$\rho(r,z)g = \frac{\partial p(r,z)}{\partial z}, \tag*{(2-28)$_B$}$$

where $\rho = \rho(r,z) = m(r,z)/V(r.z)$ is the density of mass.

Summing up (2-10), (2-18) and (2-26), we have

$$u_\theta^2(r,z) = g\left[\frac{\partial T(r,z)}{\partial z}\right]^{-1}\left[T(r,z) + r\frac{\partial T(r,z)}{\partial r}\right], \tag*{(2-29)$_A$}$$

$$mg = R\frac{\partial T(r,z)}{\partial z}, \tag*{(2-29)$_B$}$$

Now, we get the **two component forms of wind speed equations of a point in UCM of steady flow of cyclone, (2-28) and (2-29).**

(2-28) is a set of nonlinear partial differential equations. It shows the relationship between the major component of wind speed u_θ and pressure p. (2-29) shows the relationship between u_θ and T.

$(2\text{-}28)_B$ shows that **the vertical motion is upward**, since the direction of a positive gradient of vertical pressure (caused by a downward density of air) agrees with the z-axis. This have been confirmed by a recorded video of a tornado [9], and a description (The primary vertical motion is upward [10]).

$(2\text{-}29)_B$ shows that the gradient of temperature T respect to z is descent with z increasing. **This result agrees with the fact that the temperature is getting down with the height increasing**. $(2\text{-}29)_B$ shows that the mass of air approaches to zero at isothermal layer where the temperature keeps unchanged for any in the layer.

3 Comparing with the Ideal Gas Law

The ideal gas law was first state by Clapeyrong, E in 1834, as a combination of Boyle's law, Charles' law, and Avogadro's law [11], the common form of this law is:

$$pV = nRT, \tag{3-1}$$

where p, V, T is the same as above; n is the number of moles. R is gas constant, equal to the product of Bolzmann constant and Avogadro constant. (3-1) can be derived by empire or by kinetic theory under some assumptions including moles random motion etc. The element of the ideal gas law is mole or atom, its size belongs to micro-level. If one tries to use a micro-level law with random motion assumption to a problem of macro-level, then, even if one got a result, the result must belong to uncertain type.

We use combination of Boyle's law, and Charles' law, and link with Newton's law, no random factors has been involved, therefore, the result belongs to certainty. A certain analysis should be better than an uncertain analysis. This is the advantage of our method.

4 Conclusion

This paper uses a point model of cylindrical box wrapped by zero-weighted membrane to derive a wind speed equation of a point in a simple cyclone by method of section based on Boyle's law, Charles' law and Newton's law. The obtained equation is a non-linear PDE with two unknown functions, seldom seen in literatures [12]. It belongs to the certain type. Comparing with the traditional analytic method, most of them belong to uncertain type. Some results are mentioned: **the vertical motion is upward. The gradient of temperature T respect to height z is descent with z increasing. This result agrees with the fact that the temperature is getting down with the height increasing** and the description of cyclone (The primary vertical motion is upward [10]) and it is also confirmed by a recorded video of a tornado [11]. $(2\text{-}21)_B$ shows that the **mass of air approaches to zero at isothermal layer where the temperature keeps unchanged for any height in the layer.**

Competing Interests

Author has declared that no competing interests exist.

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