

# The Generalised Newton's Gravitational Field Equation Based upon Riemann's Great Metric Tensors

G. G. Nyam<sup>1\*</sup>, F. O. Adeyemi<sup>1</sup> and N. E. J. Omaghal<sup>2</sup>

<sup>1</sup>Department of Physics, University of Abuja, Nigeria.

<sup>2</sup>Department of Physics, University of Jos, Nigeria.

## Authors' contributions

*This work was carried out in collaboration among all authors. Authors GGN and FOA designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript.*

*Author NEJO managed the analyses of the study and managed the literature searches.*

*All authors read and approved the final manuscript.*

## Article Information

DOI: 10.9734/CJAST/2019/v38i630463

### Editor(s):

(1) Dr. João Miguel Dias, Associate Professor, Department of Physics, CESAM, University of Aveiro, Portugal.

(2) Dr. Koji Nagata, Department of Physics, Korea Advanced Institute of Science and Technology, Daejeon, South Korea.

### Reviewers:

(1) Pasupuleti Venkata Siva Kumar, Vallurupalli Nageswara Rao Vignana Jyothi Institute of Engineering and Technology, India.

(2) P. A. Murad, USA.

(3) Igor Bulzhenkov, Lebedev Physical Institute, Russia.

(4) A. Ayeshamariam, Khadir Mohideen College, India.

(5) Jun Steed Huang, Southern University of Science and Technology, China.

Complete Peer review History: <http://www.sdiarticle4.com/review-history/50788>

**Received 05 November 2019**

**Accepted 09 January 2020**

**Published 24 January 2020**

**Original Research Article**

## ABSTRACT

The existing theories of gravitation are founded mostly on the Euclidean theoretical Physics in which the influence of gravitational field is ignored. In this paper we derive the Newton's dynamical gravitational field equation based upon the great metric tension in which the influence of gravitation is consider.

**Keywords:** Great metric tensor; Newton's gravitational field equation; Riemann's great metric tensors.

## 1. INTRODUCTION

It is well known that gravitational force is one of the fundamental forces in nature alongside electromagnetic and nuclear forces [1].

Gravitational force governs the motion of the planet, moon and the galaxies in their respective orbits [2].

The two major theories of gravitation include that of Newton dynamical theory of gravitation which provides the simple Newton's gravitational field equation given by

$$\Delta\phi = 4\pi G_n\rho \quad (1)$$

For the gravitational potential  $\phi$  given a mass density  $\rho$  and  $G_n$  is the gravitational coupling constant, [3]. Where  $\Delta$  is the Laplacian operator.

Newton's theory explains the manifestation of all interactions in nature through a force and it was successful in explaining the gravitational phenomena in the surface of the earth and experimental facts of the solar system [2].

And the Einstein geometric theory of gravitation published in 1916 called the General Relativity which generalized special relativity and Newton's law of universal gravitation, providing a unified description of gravity as a geometric property of space and time or space time. Einstein's showed through a system of partial differential equations (called Einstein field equation) how the curvature of space time is directly related to energy and momentum of whatever matter and radiation are present [4].

These equations are given as

$$R_{ab} - \frac{1}{2}Rg_{ab} = \frac{8\pi G}{c^4}T_{ab} \quad (2)$$

The left hand side of the equation (2) is the Einstein tensor, (a specific divergence-free) combination of the Ricci tensor  $R_{ab}$  and the metric tensor  $g_{ab}$ . On the right hand side,  $T_{ab}$  is the energy-momentum tensor and  $\frac{8\pi G}{c^4}$  is the proportionality constant with G the gravitational constant and C the speed of light.

## 2. THEORY

Based upon the Riemannian's geometry the Great Metric Tensor in all gravitational field in the Einstein's spherical coordinate  $(r, \theta, \phi, x^0)$  is given by Howusu, [5], Nyam, et al. [6].

$$g_{11}(r, \theta, \phi, x^0) = (1 + \frac{2}{c^2}f)^{-1} \quad (3)$$

$$g_{22}(r, \theta, \phi, x^0) = r^2 \quad (4)$$

$$g_{00}(r, \theta, \phi, x^0) = -(1 + \frac{2}{c^2}f) \quad (5)$$

$$g_{\mu\theta}(r, \theta, \phi, x^0) = 0 ; otherwise \quad (6)$$

Where f is the gravitational scalar potential given as

$$f = -\frac{K}{R}; \quad r = R \quad (7)$$

Which is;

- a function of not only the radial coordinate r, as in the well-known Schwarzschild's metric tensor, but also more generally of the two angle  $\Theta$  and  $\phi$  as well as the coordinate  $x^0$ ;
- assumed to reduce to the corresponding pure Newtonian gravitational scalar potential in the order of  $x^0$  (correspondence principle) and

$$x^0 = ict \quad (8)$$

The generalized Great Metric Tensor satisfies the following criteria:

- Contains the phenomenon of gravitational space contraction for which there is experimental evidence.
- Contains the phenomenon of gravitational time dilation for which there is experimental evidence.
- Contain the phenomenon of singularity in gravitational fields in nature for which there is experimental evidence.
- Reduces to the pure Euclidean metric tensor in all space time without gravitational fields in all orthogonal curvilinear coordinate.
- Contains Schwarzschild metric tensor in the space time exterior to all static homogenous spherical distribution of mass in spherical polar coordinate.
- Makes the three space parts of the Riemann's tensorial Geodesic Equation of motion, for particle of non zero rest masses in gravitational fields in nature, in all orthogonal curvilinear coordinate to reduce to the corresponding pure Newton's equation of the motion in the limit of  $c^0$  [7].

### 3. THE GENERAL LAPLACIAN OPERATOR

The general Riemann's Laplacian operator is giving by Nyam, et al. [6]

$$\nabla_R^2 = \frac{1}{r^2} \frac{\partial}{\partial r} \left[ r^2 \left( 1 + \frac{2}{c^2} f \right) \frac{\partial}{\partial r} \right] + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial r} \left[ \sin \theta \frac{\partial}{\partial \theta} \right] + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \theta^2} - \frac{1}{v_p^2} \frac{\partial^2}{\partial t^2} \quad (9)$$

Where  $c$  is the speed of light in vacuum  
 $f$  is the gravitational scalar potential  
 $t$  is coordinate time

$\nabla_R^2$  can transform into any other coordinate according to the transformation equation [Speigel 1974, [5,6]

$$\nabla_R^2 = \frac{1}{\sigma_g x^\alpha} \left[ \sqrt{g} g^{-\mu\beta} \frac{\partial}{\partial x^{\alpha\beta}} \right] \quad (10)$$

The post Newtonian gravitational field equation based on Riemann's Theoretical Physics is giving by

$$\nabla_R^2(f) = 4\pi G \rho_0(r, t) \quad (11)$$

Substituting equation (9) in (10), we can write explicitly that

$$\left( \frac{1}{r^2} \frac{\partial}{\partial r} \left[ r^2 \left( 1 + \frac{2}{c^2} f \right) \frac{\partial}{\partial r} \right] + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial r} \left[ \sin \theta \frac{\partial}{\partial \theta} \right] + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \theta^2} - \frac{1}{v_p^2} \frac{\partial^2}{\partial t^2} \right) f = 4\pi G \rho_0(r, t) \quad (12)$$

Equation (12) is the Riemann's Generalized Newtonian gravitational field equation in spherical coordinates in pure gravitation field.

### 4. CONCLUSION

The Euclidean theoretical Physics has been of great importance to the world of theoretical science especially in Physics and it influence cannot be over emphasis. In the research we have be able to formulate the Riemann's Generalized Newtonian gravitational field

equation in spherical coordinates in pure gravitation field equation (12).

### COMPETING INTERESTS

Authors have declared that no competing interests exist.

### REFERENCES

1. Wikipedia Contributors; 2019. Available: [https://en.wikipedia.org/wiki/Fundamental\\_interaction](https://en.wikipedia.org/wiki/Fundamental_interaction) (Accessed 2019)
2. Nyam GG, Howusu SXX, Izam MM, Jwanbot DI. Generalised linear accelerated and linear velocity for a particle of non-zero mass in a static homogeneous spherical distribution of mass. International Journal of Basic and Applied Science. 2015;4(3): 157-159.
3. Matthias Blau. Lecture note on general relativity. Albert Einstein Center for Fundamental Physics Institute for Theoretical Physik Universital Bern Switzerland; 2012.
4. Wikipedia Contributors; 2019. Available: [https://en.wikipedia.org/wiki/General\\_relativity](https://en.wikipedia.org/wiki/General_relativity) (Accessed 2019)
5. Howusu SXX. The Metric Tensors For Gravitational Fields and The Mathematical Principles of Riemannian Theoretical Physics, Jos University Press Ltd. 2009; 1-44.
6. Nyam GG, Howusu SXX, Adeyemi OF. The generalized riemmanian schrodinger wave equation for hydrogen atom. IOSR Journal of Applied Physics (IOSR-JAP). 2017;9(4):III:32-34. e-ISSN: 2278-4861 Available: [www.iosrjournals.org](http://www.iosrjournals.org)
7. Howusu SXX. Riemannian Revolutions in Physics and Mathematics (Reprint) Jos University press Ltd. 2013;69-106.

© 2019 Nyam et al.; This is an Open Access article distributed under the terms of the Creative Commons Attribution License (<http://creativecommons.org/licenses/by/4.0>), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

*Peer-review history:*  
 The peer review history for this paper can be accessed here:  
<http://www.sdiarticle4.com/review-history/50788>