Teaching Algebraic Word Problems through Constructivism: The Real Classroom Evidence

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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ABSTRACT

Aim: Due to constant effort by educators to find lasting solution to student's poor performance in classrooms, there is the need to elaborate on the systematic processes that can be adopted and employed in mathematics classroom when teaching algebraic word problems to bring about meaningful learning in improving performance. Therefore, the study gathered data and analyzed it in the end to find out the effect of teaching through constructivism.
Study Design: Action research design was used for the study.
Methodology: The main instrument used for the study was test items (pre-test and post-test). The sample size for the study comprised of forty-six (46) students of which nineteen (19) were female students, and twenty-seven (27) were male students.
Results: In the pre–test (see Table 1), forty- two (42) out of the forty- six (46) students scored marks which are either less than half or half of the 30 marks for the test representing 91.3% of the total number of students as compared to the frequency distribution of the post – test scores (Table 2), where out of the forty six (46) students who took part in the test, thirty-eight (38) of them

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obtained more than half of the total mark of 30 for the test representing 82.7% of the total students number.

Conclusion: The findings from the results of the study confirmed that the use of constructivist approach of teaching and learning will assist the students to have the opportunity of using their own experience to create their conceptual understanding (ability and capability), which will consequently improve their academic performance. Therefore, it is recommended that regular professional development should be organized for mathematics teachers at all levels to refresh them on practical approaches to teaching mathematics through constructivism.

1. INTRODUCTION

Algebra is conceived as a branch of mathematics concerned with, and operating within, the symbolization and generalization of numerical relationships and mathematical structures [1]. Van de Walle [2] also stated that algebraic reasoning requires describing patterns and regularity in all fields of mathematics, generalizing them and formalizing them. Among many other processes, doing mathematics involves describing mathematical problems in various forms, researching, formalizing patterns and regularities, making generalizations and solving mathematical problems. This suggests that learning algebra is central to the ability of the students to do mathematics. Many students with learning difficulties in mathematics may have challenges in Algebra, not because their challenges is based on mathematics, but because it is focused on language, slowing their assimilation and understanding of mathematics instruction [3].

A student cannot solve a word problem without first understanding what the problem is before he or she is asked to find it. Generally, students have difficulty identifying mathematical or cognitive demand and understanding the word problem in their context [4]. From Chamot et al. [5], learning to understand well enough to decode concepts found within a word problem is now a prerequisite for mathematical problem solving. 'Mathematical word problems, or story problems, have long been a common aspect of school mathematics,' according to [6]. Burton [7] stated that transformation of word problems into arithmetic or algebra creates considerable difficulty for many students, and from a psychological point of view a number of studies have discussed the linguistic and mathematical origins of that difficulty.

It is believed that the fundamental purpose of mathematics education is to help children to understand, reason, and mathematically communicate and solve problems in their daily lives [8]. For contemporary education, along with the related academic studies, mathematics education is the process of teaching and studying mathematics. Mathematics education researchers are primarily concerned with the tools, methods, and approaches which facilitate practice or practice analysis. Recently, however, research in the field of mathematics pedagogy has grown into a wide area of study with its own principles, theories, methods, national and international organizations, conferences, and literature. The current mathematics curriculum in Ghana and many countries across the world stipulates the use of constructivist approach of teaching and learning. The same curriculum suggested problem-centered teaching approach which selects constructivism as its learning approach. Considering Andam et al. [6] 'Constructivism is a philosophy that describes how knowledge is formed in the human being when information comes into contact with established knowledge that has been developed through experiences.' They further indicated that Constructivists assume that learners can create awareness through active participation in the learning process, rather than being deposited into the minds of the learner [6]. Constructivism is not a method, but it is an information and learning philosophy which should inform practice but not recommend practice [9].

Successful learning of mathematical concepts and skills is a feature of the teachers' methods and techniques in their teaching. In a large extent, the manner in which mathematics is taught is informed by the teachers' views of the subject and of what they consider in be effective teaching. Asiedu-Addo and Yidana [10] claim that mathematics is the means to sharpen the mind of individuals, form their capacity to think and grow their personality, hence their immense contribution to the general and fundamental education of the world's population.

2. LITERATURE REVIEW

2.1 Concepts of Mathematical Word Problems

Word problems in mathematics were contained in Egyptian papyri 4000 years ago; in ancient Chinese and Indian manuscripts; and the Treviso (Italy) arithmetic textbook from 1478 (Swetz, 1987; as cited in [11]). There is no accepted description of Word Problems. Typically, people use terms like text problems, verbal problem, story problem, and so on to construct a description for word problems. Verschaffel, Greer and De Corte [11] offer the following definition:

Word problems can be described as verbal explanations of the problem situation where one or more questions are posed, the answer to which can be obtained by applying the mathematical operations to the numerical data available in the problem statements. In its most common form, word problems take the form of a brief text explaining the fundamentals of a situation in which certain quantities are specifically specified and others are not specified and in which the solver, usually a student facing the problem in the sense of a mathematics lesson or mathematics exam, is expected to give a numerical answer to a particular question by making it clear.

Verschaffel, Greer and De Corte [11] propose that word problems serve a number of functions and they explain as follows:

- ✓ **Application function:** The application of knowledge required to solve a word problem that is placed into a certain situation.
- ✓ **Motivation function:** Since word problems are presented in a certain context, they engage students in their solution because the context convinces the students that mathematics is necessary for the life out of school.
- ✓ **Selection Function:** To evaluate the new generations mathematical competencies that in the future will guide and serve society.
- ✓ **Thought provoking function:** To develop students' creative thinking and problem-solving competencies.
- ✓ **Concept formation function:** Word problems with their advantage of having context can help students to construct new mathematical concepts.

2.2 Concept of Algebra

Wheeler [12] defines algebra as a symbolic system (its existence is understood by symbols), a calculus (its use in computing numerical solutions to problems), and also as a representational method (it plays an important role in mathematizing situations and experience). Algebraic understanding is key to student success in higher-level mathematics classes, but many students are struggling with algebraic understanding and comprehension. Algebra is also regarded as the gatekeeper to higher education [13]. Most students fail as they move from arithmetic to algebra, because classes in elementary mathematics still do not train students for algebraic thought [14]. Too often, students learn to operate and manipulate algebraic symbols without understanding the meaning behind important concepts like coefficients, constants, variables and the equal sign. In fact, students still fail to understand the sense and importance of algebra in their everyday lives [15].

Recommendations presented by the NCTM [16] standards, suggested and stressed the importance of algebra and mathematical modeling. For this, NCTM [16] proposed that students should be able to represent mathematical situations and structures using algebraic symbols and analyze them.

2.3 Concept of Algebraic Linear Equation

Equations are mathematical statements which mean equality between two expressions, according to James and James Mathematics Dictionary. Equations express identities or relationships that are conditional between numbers and/or variables. A linear equation is one that is of the first degree in its variable. Linear-equation graphs form straight lines.

According to a study by [17], equations are such an important part of mathematics, particularly algebra, due to the many uses that they have. Equations may describe functions; express one variable in terms of the other; or provide information about a property used to calculate extreme points for functions when a particular quantity is maximized or minimized. Learning linear equation consists of formulating equations from contextual problems, solving the equations and eventually presenting solutions to the original problems. As Kaput [18] puts it, solving an equation has always meant using the available syntactic methods to turn the

expression into an equation since the early days of algebra as we know it (since Viete), before the resulting representation makes the roots of the equation cognitively accessible.

Solving linear equations is a process of finding a numerical value for the unknown (usually expressed by a letter), or of making the unknown subject in the given or formulated equation. In certain cases this process is preceded by the formulation for the given situation of an algebraic equation, which is then accompanied by the solution process. The effectiveness of the linear equations solving process depends on the logical, procedural and conditional knowledge of the solver and thus on his/her understanding of linear equations.

2.4 Concept of Constructivism

According to the constructivism principle, students are not only passively acquiring information but are actively generating new knowledge based on previously acquired knowledge in combination with new experiences, thereby enhancing their performance [6]. Constructivism has changed the concept of teaching and learning to a more progressive way through which fresh ideas of the learners are brought to class, understood and strengthened by a range of teaching and learning strategies that actively involve them [6]. That suggests, therefore, that a constructivist approach to learning builds on the learner's natural innate abilities [19]. Generally, the constructivist classes have an environment like training sessions where students learn from each other and teach each other [20]. It is based on this perspective that Eggen and Kauchak [21] have argued that the learner is seen as actively building understanding by using authentic resources and social interaction. It is known that the lecture/rote learning method promotes the transmission of large quantities of knowledge within a given period, and the students tend to memorize the content, but this type of learning does not empower students to think critically and solve problems [22]. Therefore, the emphasis is on cognitive growth and deep understanding in which learning is nonlinear and students are encouraged to look for solutions openly and actively. It also goes beyond how the brain stores and retrieves knowledge, but promotes sense making based on personal experience of the students [23].

Duffy and Jonassen [24] take the view that constructivism can also be viewed as a dialog between learning theorists and instructional designers, but often the coordination between them is missing and, thus, the curriculum is not built according to the needs of the learners. The constructivist theory of teaching and learning was widely discussed in a number of mathematics education researches [25]. While constructivist learning theory doesn't tell us how to teach mathematics, a teacher with a constructivist background can promote knowledge building by applying different constructivist teaching methods consistent with this theory of learning. This kind of teaching of mathematics forms the basis of this research. It is based on this idea that Good and Brophy [26] posit that learners build their own understanding in the constructivist approach; new learning depends on current understanding; learning is facilitated through social interaction, and meaningful learning occurs through the use of authentic learning tasks. Ndon [27] further proposed that "a instructor as facilitator can have a variety of learning contexts, interactions and activities by integrating incentives for collaborative research, problem solving and meaningful tasks" p.253. Within the constructivist classroom the teacher focuses on the learning of students rather than on the success of teachers.

3. PURPOSE OF THE STUDY

New college students are supposed to demonstrate a good understanding of algebraic word problems in their everyday math-related dealings. During a preparatory test organized for first year college students, it was discovered that most candidates ignored questions about word problems and the few who attempted them were unable to solve the problems correctly because they did not know the approach necessary to solve the problem. And due to continuous attempts by educators to find permanent solutions to the weak performance of students in our classrooms, there is a need to elaborate the systemic processes that can be implemented and employed in our mathematics classroom while teaching algebraic word problems in order to introduce practical learning to improve performance. The research thus seeks to collect data and eventually examine it to find out the impact of teaching through constructivism.

4. RESEARCH QUESTIONS

The questions below serve as guide for the study:

1. To what extent does teaching through constructivism help to improve student performance of the algebraic word problems?
2. How does teaching and learning through constructivism increase the ability and capability of students in solving algebraic word problems?

5. METHODOLOGY

5.1 Research Design

This study's design is an action research, as it aims to find solutions to the weakness of students to successfully solve problems concerning algebraic word problems. However, an action research is aimed at exploring ways to solve practical issues for practitioners. An action research is viewed as a process in which practitioners attempt to examine their challenges scientifically in order to guide, modify and examine their decisions and behaviors [28]. Action research is deemed to be primarily tailored to remedy a problem in the operation of classrooms [29]. In addition, action research is chosen in this context because it deals with a small-scale intervention specific to one classroom situation in which the researcher conducted the study.

5.2 Population and Sampling

The study was conducted at Wiawso College of Education in the Sefwi Wiawso municipality. The school has a population of one thousand and two hundred (1200) students. Three hundred and ninety-six (396) of the students are in the level 100 (first year). Purposive sampling method was employed for the sample selection. The research was conducted in 1A1, a level 100 class which has a population of forty-six (46) students. In that class there were nineteen (19) female students, and twenty-seven (27) male students. The average class age was twenty (20) years, and the students came from various regions in Ghana.

5.3 Instrumentation

Pre-test and post-test were used to collect information on the successes of the learners in solving mathematics questions concerning algebraic word issues. The instrument was well built to assist in simple data collection, presentation, analysis, and organization. Furthermore, the pre- and post-tests were used to evaluate the performance of the students before and after the intervention.

5.4 Intervention Strategy

Schifter [30] argues that while the teaching of mathematics in a constructive way can interrupt the routine of regular classroom learning, it makes the mathematics classroom a place of inquiry which influence learning in a positive way. A classroom atmosphere encouraging of learning must be one where students feel comfortable enough to share their formative thoughts [31]. Yelon [32] outlines ten instructional principles that instructors can implement in their classroom teaching and are as follows;

- ✓ **Meaningfulness:** The instructor must inspire students by helping them link the subject matter to be studied with their past and present experiences.
- ✓ **Prerequisites:** The instructor is expected to evaluate the level of knowledge and skills of the students and also to change instructions where necessary.
- ✓ **Open Communication:** Ensure the students figure out what they need to know so they can concentrate on what they can learn.
- ✓ **Organized Essential Ideas:** Help students concentrate on the relevant ideas and organize them to be able to learn and to remember those ideas where necessary.
- ✓ **Learning Aids:** Support students on using apps for fast and easy learning.
- ✓ **Novelty:** Vary the progress of instruction to keep the students attentive.
- ✓ **Modeling:** Show the students how to remember, think, behave and solve problems.
- ✓ **Active Practice:** Provide practical problem-solving, recalling, analyzing and practicing opportunities so that students contribute and make their learning better.
- ✓ **Pleasant Conditions and Consequences:** Make learning enjoyable, so that students incorporate comfort with what they learn.
- ✓ **Consistency:** Provide clear goals, assessments, practice, content and clarification. This will help students to learn what they need and use what they have learned outside the educational environment.

In addition, Crawford and Cobb [33] conclude that the teacher should form effective groups,

assign appropriate tasks, be keenly observant during group activities, quickly diagnose problems, and provide direction or information necessary to keep all groups moving forward. With this the group discussions have to be based on mathematics critical thinking issues.

Mathematics and problem solving are everywhere, no matter your age or occupation, and at some stage you are expected to face the task of solving word problems. The prevalent stumbling block most people face is the translation of word problems from English into actual mathematical equations. However, once you establish the equation, it's fairly simple to solve for the answer.

In Andam et al. [6], they suggested some interventional instructional activities that can be implemented by mathematics instructors to assist students in understanding algebraic word problems. The instructions for the activities were given as follows:

1. Thoroughly read the problem to understand what you're solving. List all the unknowns in the question, and assign for each unknown a variable. For example, if there are two unknowns, you need two variables, like x and y. When there are three unknowns, three variables are needed, such as x, y and z. In the word problem the number of unknowns also indicates the number of equations required. It can help to name the variables so they represent the unknowns that you solve. For example, if you solve a problem dealing with an unknown number of apples and pears, use "a" as the apple variable, and use "p" as the pear variable.

2. Translate the problem into a system of equations, using key words to define the necessary operations. Terms like "increased by," "total of," "more than," "combined together," "sum," "added to," etc. signal operations that involve **ADDITION**. Phrases like 'decreased by', 'difference between', 'less than', 'fewer than', 'reduced by', 'difference of', etc. means the operations involve **SUBTRACTION**. Words and phrases such as 'of', 'product of', 'times', 'multiplied by', etc. suggests operations that require **MULTIPLICATION**. Terms such as 'per', 'out of', 'ratio of', 'quotient of', 'percent', etc. suggests operations that require **DIVISION**. When words like 'is' or 'will be'

appear in a word problem, this implies the quantity of unknown terms must be **EQUAL**.

3. Solve the equations using the suitable or acceptable methods.

4. Verify the solution you have suggested by plugging the answers into each equation. If the two sides of each equation are identical, the solution is true. If one side of the equation isn't equal to the other, you might need to review your work and redo the problem.

5.4.1 Step one

In the first step, the researchers guided the student through series of activities on how they will translate or convert English sentences into mathematical statements and expressions. With this, the researchers explained to the students to look out for the key words that are contained in the sentences for them to be able to translate them into mathematical statements and expressions. To make this understandable to the students, the researchers guided the students through a list of instructions, stated above, that need to be followed in order to arrive at their results. This will serve as a guide for the students when they come across such problems and also be able to come out with correct answers to the questions involved. Here are examples that the researchers went through with the students in the first step.

I. The sum of a number and seven.
   With this question, the teacher asked the student to first read and understand this statement. The researchers then ask them to freely represent that number with any variable of their choice. Most students are familiar with x and y, and hence most of them used it. The teacher then asked them to look out for the key word in the sentence. They all responded that 'sum' is the key term. Hence, the expression for the sentence will be y + 7. This process was repeated for the rest of questions at this step

II. The difference between a number and five.
   Key word: difference between, meaning subtraction.
   Answer: $y - 5$.

III. The product of six and a number.
   Key word: product of, meaning multiplication.

Answer: $6y$

IV. The quotient of eight and a number.
Key word: quotient of, which means division.
Answer: $\frac{8}{y}$

5.4.2 Step two

In the second step, the activities were done to teach students how to translate words into numbers, variables and mathematical operations using constructivist approach of teaching and learning. In this activity, the researchers guided the students to write an algebraic linear expression for each problem. This was followed by an explanation of the problems with the students after observing their answers.

These are some of the problems the researchers went through with the students. With each of them, the students were asked to write the phrases as an algebraic expression;

I. A number is increased by six.

With this question, the students were able to identify the key word in the sentence, i.e. 'increased by', which stands for the addition operation. Most of them used the variable x and y to represent that particular number. Majority of the students had the answer correct. The answer they agreed on was x + 6. The teacher explained to them that, in all cases, they are at liberty to select any variable to represent the number in the question.

II. The quotient of a number and five.

Most of the students were able to write the answer correctly. The students were able to represent the number with a variable and then identified the key word in the problem. In this case, it was 'quotient' and they gave the operation as division, as required of them. There were few who for one reason or another, instead of writing $\frac{y}{5}$, they wrote $\frac{5}{y}$. The researchers explained to them that they should always try as much as possible to follow what the problem demands before they write their respective expressions.

III. The difference of three times a number and seven.

In this question, most of the students were found wanting. They found it difficult to write the correct expression. Majority of them wrote $7 - 3y$, while others too wrote $3(y - 7)$ instead of 3y – 7. The researchers tried to construct the expression from the problem with the students. The researchers asked the students to identify the key word in the problem. They gave the respond as 'difference' and they also added the meaning to it as 'subtraction'. The researchers then asked the students to write an expression for the phrase 'three times a number', given y as the number. They all responded with 3y as the answer, which was correct. The researchers again asked them to write an algebraic expression for the phrase 'the difference between 3y and 7'. They all gave the answer as 3y – 7 and they had it correct.

IV. The quotient of a number and six, increased by twelve.

It was observed here that, the explanation given in the previous questions helped the majority of the students to successfully write the correct expression for this question. However, the researchers guided the students who were unable to write the expression correctly. The researcher asked them to first of all, write the expression for the phrase 'the quotient of a number and six'. With the understanding from the instructions and the previous examples, the students responded with $\frac{x}{6}$ as the answer.

The researcher then asked them to write the expression of the phrase '$\frac{x}{6}$ increase by 12' to complete the question. The students gave the response as $\frac{x}{6} + 12$

At this point of the activity, the researchers realized that most of the students began showing interest and understanding of the concept of translating word problems into algebraic expressions. This was a result of the students adopting the constructivist approach of learning, where the learner will have the chance of using their experience to create their understanding rather than delivered to them in an already organized form from the teacher.

There were other examples that the researcher went through with the students, where they gave their responds as follows;

V. A number added to twenty.

Students respond: k + 20. (In this case, k represents the number)

VI. A number decreased by ten.

Students respond: p – 10. (p is the number)

VII. The product of seven and a number

Students respond: $7 \times t = 7t$. (With t as the number)

VIII. A number increased by five.

Students respond: q + 5. (q is used as the number)

IX. A number out of ten.

Students respond: $\frac{u}{10}$ (u represents the number)

X. The sum of a number and nine times the same number.

Students: y + 9y. (y is used as the number in this case)

After going through the activities in step 2, the researchers realized that some students were still having problems in modeling algebraic word problems. With this at hand, the researchers carried out another routine activity with the said students in step 3 to help them catch up with other members of the class.

5.4.3 Step three

In this routine activity, the researchers grouped the students in four separate groups which were evenly distributed. Each group had students who had developed their interest and competence in the topic through the previous activities. The idea behind this was that, the students will cooperatively help each other, especially for those who were still having problems.

The constructivist approach of teaching and learning emphatically make extensive use of cooperative learning. With this, students will be more comfortable and easily discover and comprehend difficult concepts if they communicate with each other about the problem at hand. These questions were given to the groups to solve:

Write the algebraic expression for each word problem.

I. The difference of one – third of a number and six.
II. Eight less than the product of a number and three.
III. The sum of nine and one – fifth of a number
IV. Four times the difference of a number and eight.

During the cause of solving the questions, the researchers went round providing assistance to the groups that were encountering difficulties. The students responded to the problems above as shown below;

I. Let x be the number: $\frac{1}{3}x - 6$
II. Let y be the number: $3y - 8$
III. Let k be the number: $9 + \frac{1}{5}k$
IV. The students in the groups found it difficult in writing this expression correctly. Some of the different answers given were; 4y – 8, 8 – 4y, 4(8 – y).

The researchers then tried to go through the solution with the students. First of all, the researchers asked the students to represent the number in the problem with a variable, which they all settled on 'y'. The researchers then asked them to write the expression for the phrase 'the difference of y and 8' and enclosed the answer in a bracket.

Students answer: (y – 8).

The teacher again asked the students to write the expression for the phrase '4 times   (y – 8), i.e the difference of y and 8'. Students respond: 4(y – 8).

5.4.4 Step four

At this step, students were guided through some solution processes. The researchers guided the students to model algebraic word problems into linear equations and also how to use the constructivist approach of teaching to solve the modeled linear equations. The researchers went

through these processes with the students and afterwards went through some examples with the students.

1. Carefully read the problem, and find out what the problem is seeking from you to find. Usually the information is found at the end of the question.
2. Assign a variable to the quantity or number you are instructed to find. There are no limits on variable preference. Students are at liberty to use any variable of their preference. Most students are familiar with 'x' and 'y'.
3. Read the problem again, and write an equation for the quantities given in the problem.
4. Solve the equation using whatever approach you choose.
5. Check the solution to the problem.

At this point, most of the students were abreast with the requisite concept and knowledge in modeling algebraic word problems into linear equations and solving the linear equations as well. The researchers then went through the following examples before giving the students examples to try on their own.

I. 68 less than 5 times a number is equal to the number. Find the number.

Let x be the number. Note that "68 less than 5 times the number" translates to the expression $5x - 68$ and not $68 - 5x$. Solving the problem becomes;

$5x - 68 = x$
$5x - 68 + 68 = x + 68$ (add 68 to both sides of the equation)
$5x = x + 68$
$5x - x = x - x + 68$ (subtract x from both sides of the equation)
$4x = 68$
$\frac{4x}{4} = \frac{68}{4}$ (divide both sides by 4)
$x = 17$

All the students participated in solving this question and the researchers then asked them to check the answer by substituting $x = 17$ into the equation.
That is: $5(17) - 68 = 17$
$17 = 17$

II. When 142 is added to a number, the result is 64 more than 3 times the number. Find the number.

Let u be the number and in solving the problem, students are to note that the problem is in two phases. The students were able translate both phases as 142 + u and 3u + 64 respectively.

$142 + u = 3u + 64$
$142 - 64 = 3u - u$ (group like terms, with variables on one side and numbers on the other side)
$78 = 2u$
$\frac{78}{2} = \frac{2u}{2}$ (divide both sides by 2)
$u = 39$

The students again check the validity of the answer by substituting the answer into both sides of the initial equation.

That is: $142 + 39 = 3(39) + 64$
$181 = 181$

III. In a mathematics quiz, the lowest mark was 42 less than the highest mark. If the sum of the two marks is 138, find the highest mark.

With this question, the students were able to identify that there are two quantities in the question, i.e. the lowest and highest marks.

Let w represent the highest mark, and hence the lowest mark then become w − 42.
i.e. highest mark = w
lowest mark = w − 42.

The sum of the two marks, given us 138 will now become,
$w + w - 42 = 138$
$w + w - 42 + 42 = 138 + 42$ (add 42 to both sides of the equation)
$2w = 180$
$\frac{2w}{2} = \frac{180}{2}$ (divide both sides by 2)
$w = 90$

Hence, the highest mark is 90 and that is the answer. With this at hand, the students went further to also find the lowest mark and this helped them to check their answers.

Lowest mark = $w - 42$
$= 90 - 42 = 48$
And by checking the answer,
Highest mark + lowest mark $= 90 + 48 = 138$

IV. The sum of a number and 9 is multiplied by -2 and the answer is -8. Find the number.

Let y be the number and in solving the problem with the students, they agreed that the equation will be;

$-2(y + 9) = -8$
$-2y - 18 = -8$ (expanding the bracket by multiplying -2)
$-2y - 18 + 18 = -8 + 18$ (add 18 to both sides of the equation)
$-2y = 10$
$y = -5$ (divide both sides by -2)

As the students did in the previous example, they checked the validity of the answer obtained by substituting y=-5 into the initial equation.

$-2(-5 + 9) = -8$
$-2(4) = -8$
$-8 = -8$

After these examples, students were given both group and individual assignments to try on their own. This is to measure the level of students understanding pertaining to the concept of algebraic word problems.

6. RESULTS AND DATA ANALYSIS

The results of the research obtained by the students in the pre-test were evaluated and discussed. The pre-test was based on testing the knowledge level of students' in solving questions involving algebraic word problems. The information gained from the pre-test acted as the researcher's guide in detailing suitable activities in the form of tasks to help the learners resolve their challenges. In total, the same test items, comprising of six questions were given to the students to answer in both the pre-test and the post-test. The questions were marked out of thirty (30) marks and were conducted for the forty six (46) students and were administered in a period of 40 minutes. Table 1 shows a frequency distribution with corresponding percentages of the scores obtained by the students in the pre – test.

After the administration of the pre – test, generally, the author observed very uninspiring performance after marking the student's scripts with most of them having marks which was below the average score. After a careful observation of their pre-test scripts, there was an indication that the students lack the understanding of the basic concepts of algebraic word problems and

therefore they were not able to use appropriate strategies and principles in finding solution to the mathematical problems.

Table 1. Frequency distribution of pre–test scores in percentage

Scores	Frequency	Percentage (%)
1 – 5	16	34.8
6 - 10	20	43.5
11 – 15	6	13.0
16 – 20	4	8.7
21 – 25	0	0
26 - 30	0	0
Total	**46**	**100**

To tackle the students' problems, a series of intervention exercises were designed by the authors for the students through the use of constructivism for teaching and learning as outlined in the intervention section, and they were given a post-test afterwards.

The post-test was also administered to the students in order to ascertain the effectiveness of the intervention and discussions made on algebraic word problems. However, the post-test consisted of the same test items used in the pre-test. The test, involving six questions were given to the students to answer in the post-test. The questions were marked out of thirty (30) marks and were conducted for the forty six (46) students and were administered in a period of 40 minutes. Table 2 is the frequency distribution with the corresponding percentages of the marks obtained by the students in the post–test.

Table 2. Frequency distribution of post–test scores in percentage

Scores	Frequency	Percentage (%)
1 – 5	0	0
6 - 10	2	4.3
11 – 15	6	13.0
16 – 20	18	34.8
21 – 25	14	30.5
26 - 30	8	17.4
Total	**46**	**100**

The post – test scores indicated a change in the performance of the students as compared to that of the pre–test scores. The author attributed the improvement in the students' performance to the use of constructivist approach of learning adopted during the intervention. With the introduction of the constructivist approach of learning, the students' were exposed to numerous activities during the intervention processes.
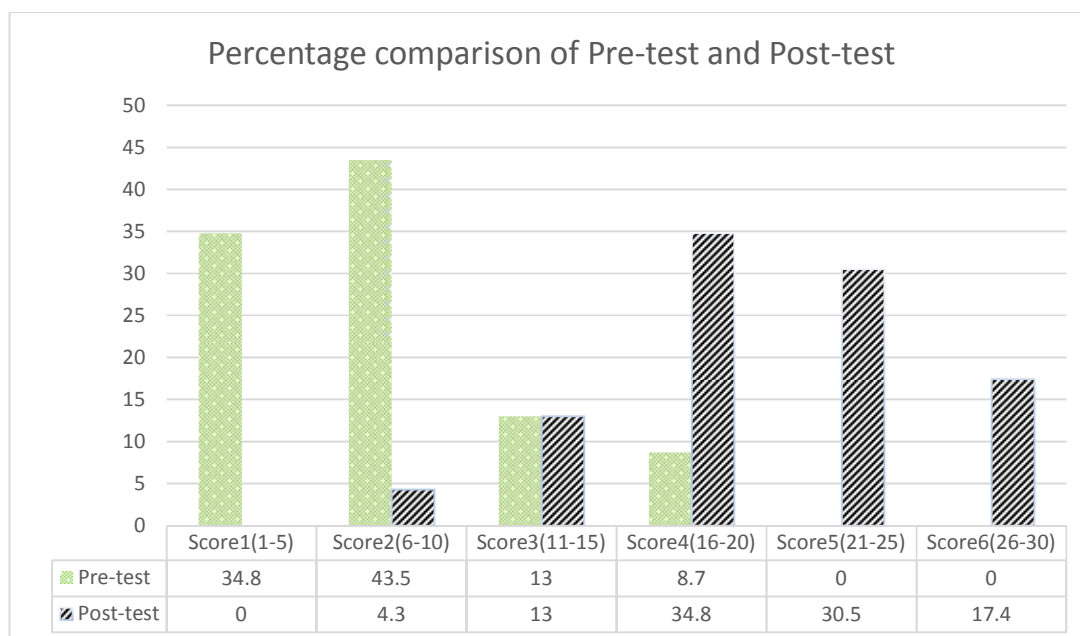
Percentage comparison of Pre-test and Post-test

	Score1(1-5)	Score2(6-10)	Score3(11-15)	Score4(16-20)	Score5(21-25)	Score6(26-30)
Pre-test	34.8	43.5	13	8.7	0	0
Post-test	0	4.3	13	34.8	30.5	17.4

Fig. 1. Percentages of students pre-test and post-test performance comparison

Table 3. Descriptive statistics of pre–test and post–test

Paired samples statistics					
		Mean	N	Std. deviation	Std. error mean
Pair 1	Pretest	7.8913	46	4.28552	.63187
	Posttest	20.2174	46	4.79775	.70739

The authors undertook inferential analysis of the pre–test and post–test, and the data used for this analysis were the scores obtained by the students in both tests. Statistical Package for Social Scientist (SPSS) was employed by the author to obtain the results of the analysis. Table 3 indicates the mean, standard deviation and standard error mean of the paired samples.

The results therefore indicated that there is a significant difference between the pre–test scores and that of the post–test which is in favor of the post–test. And this was attributed to the intervention processes the researcher took the students through.

7. DISCUSSION

Considering the scores obtained by the students in the pre – test and post – test, as shown in Tables 1 and Table 2 respectively, it can be confirmed that the performance of the students before the intervention was very low. The respective frequency distribution tables of the pre–test and post-test (Tables 1 and 2) clearly

showed the difference in the scores obtained by the students. For instance, in the pre–test (see Table 1), forty two (42) out of the forty six (46) students scored marks which are either less than half or half of the 30 marks for the test representing 91.3% of the total number of students. This poor performance by the students in the pre – test can be attributed to the kind of teaching and learning they may have experienced in their learning process as students which took the normal rote/lecture form of teaching. With this teaching process, the students did not have the opportunity to use their knowledge to build their own understanding; rather, they had already been given lessons in a structured manner. The students became acquainted to only memorizing and imitating teachers and this did not help the students to survive independently by applying the concept of algebraic word problems in solving real world situations. It was realized from the pre–test that (see Table 1), only four (4) out of the forty six (46) students scored marks more than half of the 30 marks for the test representing 8.7% of the total number of students. Therefore, it was also

identified that the students lacked cooperative learning abilities and hence the good students could not help the low performing colleagues. This support the argument made by Atteh et al. [22] that though the lecture method encourages the delivery of large amounts of information within a short time and the students tend to memorize the material but this form of learning does not encourage critical thinking and problem solving on the part of students. This suggests that students could not analyze simple mathematical problems involving algebraic word problems and come out with its solution pattern.

Results from the post–test scores by the students, as indicated in Table 2, clearly confirmed that the students performed much better as compared to the pre–test scores (see Table 1). This suggests that they have improved upon their ability to find solution to mathematical problems involving algebraic word problems through the use of constructivism in the classroom. From the frequency distribution of the post – test scores (Table 2), out of the forty six (46) students who took part in the test, thirty-eight (38) of them obtained more than half of the total mark of 30 for the test, which represents 82.7% of the total students number. The results from the post-test showed an upwards trend which suggests that the intervention activities (constructivist approach of learning) were effective in assisting the students to overcome their challenges and helping them in their learning process. In a study conducted by Andam et al. [6] and Eggen and Kauchak [21] showed that constructivist approach of learning do not just passively assists students to receive information but constantly create new knowledge based on previously acquired knowledge in conjunction with new experiences thereby improving their performance and understanding.

However, the improvement in the performance of the students, which became evident in the post–test scores they obtained, was not by chance, but through the use constructivist approach of learning that the authors adopted during the intervention activities in the classroom. With the use of constructivist approach of learning in teaching, the author designed a well–planned intervention activity in the lessons with the students. The adoption of constructivist approach of teaching and learning in the classroom enabled the students to participate actively in the lessons and also encouraged cooperative learning among the students. With the post-test

mean score of 20.2174 which is significantly higher than the pre-test mean score of 7.8913. This supports the argument by Good and Brophy [26] that in the constructivist classroom, learners construct their own understanding; new learning depends on current understanding; learning is facilitated by social interaction and meaningful learning occurs with the use of authentic learning tasks. And in effect, each student in the group is not only responsible for learning what was being taught alone, but also helped their colleagues who were still having problems and thus created a conducive learning atmosphere and a co-operative learning spirit amongst themselves.

8. CONCLUSION AND RECOMMENDA-TIONS

The results obtained from the post test support the research findings that learning is improved when students are exposed to constructivist approach of learning even if they have learning challenges. The findings from the research study were related to the research questions and assessed whether they had undoubtedly answered the research questions.

The answer to research question 1, "To what extent does teaching through constructivism help to improve student performance of the algebraic word problems?"

Comparing Table 1 and Table 2, showed the result of the intervention by comparing the pre-test scores of the individual students' with their respective post test scores. It revealed a significant improvement in students' performance after the introduction of the constructivist approach of teaching and learning by the author. Table 3 showed the result of the intervention by comparing the pre-test scores of the individual students' with their respective post test scores. The post-test mean score of 20.2174 (Standard deviation of 4.79775) is significantly higher than the pre-test mean score of 7.8913 (Standard deviation of 4.28552) revealing a significant improvement in students' performance after the introduction of the constructivist approach of teaching and learning by the author. This confirmed that the use of constructivist approach of teaching and learning algebraic word problems brought about a tremendous improvement in students' performance.

The answer to research question 2, "How does teaching and learning through constructivism

increase the ability and capability of students in solving algebraic word problems?"

The author observed that through the introduction of constructivist approach of learning, students who previously employed the chew and poor method of solving mathematical problems involving algebraic word problems were discouraged and refrained from such methods. This became evident in the post-test results. From the frequency distribution of the post – test scores (Table 2), out of the forty six (46) students who took part in the test, thirty-eight (38) of them obtained more than half of the total mark of 30 for the test, which represents 82.7% of the total students number as compared to the pre–test scores that (see Table 1), where only four (4) out of the forty six (46) students scored marks more than half of the 30 marks for the test representing 8.7% of the total number of students. This is clear that the use of constructivist approach of teaching and learning through the interventional processes has helped the students to now understand the concept of algebraic word problems in solving mathematical problems systematically and not relying on the memorized procedures. Therefore, the findings from the results of the study confirmed that the use of constructivist approach of teaching and learning will assist the students to have the opportunity of using their own experience to create their conceptual understanding (ability and capability) which will consequently improve their academic performance.

It is recommended that regular professional development should be organized for mathematics teachers at all levels to refresh them on practical approaches to teaching mathematics through constructivism. Also government and stakeholders must make conscious effort to provide the schools with adequate resources to facilitate the practical approach to teaching and learning of mathematics through constructivism.

CONSENT AND ETHICAL APPROVAL

As per international standard or university standard guideline participant consent and ethical approval has been collected and preserved by the authors.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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APPENDIX

Pre-Test/Post-Test Questions Administered to the Students

1. The sum of three times a certain number and 48 is 138. Find the number.
2. The difference between five times a number and twelve is 48. Find the number.
3. When 21 is taken from two – thirds of a certain number, the result is one – fifth of that number. Find the number.
4. Three times the sum of 8 and a certain number is equal to twice the sum of the number and 7. Find the number
5. The sum of three numbers is 81. The second number is twice the first, and the third number is six more than the second. Find the numbers
6. The sum of three consecutive even numbers is 42. Find the numbers.