



On Algebraic Properties of Inverse Fuzzy Languages

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Abstract

Aims/Objectives: In this paper, we investigate some of the algebraic properties of inverse fuzzy languages. We proved that a fuzzy automaton is inverse if and only if the transition monoid is an inverse monoid. A fuzzy language is an inverse fuzzy language if the minimal fuzzy automaton recognizing that fuzzy language is an inverse fuzzy automaton. We also discuss some more properties of an inverse fuzzy language based on the fact that an inverse monoid is one which is regular and idempotent commute.

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1 Introduction

Fuzzy sets were introduced by L. A. Zadeh as a generalization of the classical notion of sets [1] and since then it is applied in many fields of sciences. W. G. Wee introduced fuzzy automata theory. Zadeh and Lee [2] generalized the classical notion of languages to the concept of fuzzy languages. Algebraic automata theory deals with the study of the transition structure associated with fuzzy automata. Corresponds to every fuzzy automata there exists a finite monoid of fuzzy transition matrices and correspond to every finite monoid we can construct a fuzzy automata. This one-one correspondence allow us to study the structure of a fuzzy automaton through the study of the structure of the associated transition monoid.

Eilenberg-type variety theorem is proved for fuzzy languages by Tatjana Petkovic [3] and it says that there is a one to one correspondence between the variety of finite monoids, variety of languages and the variety of fuzzy languages. It is proved by Tatjana Petkovic [3] that every monoid is the syntactic monoid of some fuzzy language while this is not true in the case of crisp languages.

[4] discussed Fuzzy language recognizability via finite monoids (called m-recognizability). Fuzzy languages computed by (\max, \min) -automata, (\max, Δ_L) -automata and (\max, Δ_D) -automata are m-recognizable (Δ_L, Δ_D) are the Lucasiewicz intersection and the drastic intersection, respectively). The syntactic monoid associated to each m-recognizable language can be effectively constructed.

Mordeson J. N, Malik D. S independently and together with Nair P. S, Sen M. K proved many results on algebraic fuzzy automata theory and languages. These and many other references are available in [5].

[6], [7], they defined inverse fuzzy automata, inverse fuzzy languages and a characterization of inverse fuzzy languages. [7] defined regular and inverse fuzzy automata, its construction, and prove that the corresponding transition monoids are regular and inverse monoids respectively. The languages accepted by an inverse fuzzy automata is an inverse fuzzy language and gave a characterization of an inverse fuzzy language. As an application, converted a finite state automaton to a finite fuzzy state automaton. A classical automata determine whether a word is accepted by the automaton where as a finite fuzzy state automaton determine the degree of acceptance of the word by the automaton.

[8] studied l-fuzzy languages recognized by finite monoids and show that the class of monoid recognizable l-fuzzy languages is closed under scalar products, quotients, inverse homomorphic images and c-cuts. The notion of variety of monoid recognizable l-fuzzy languages were introduced and obtained an Eilenberg type variety theorem for l-fuzzy languages. [9] discussed l-fuzzy languages recognized by finite idempotent semirings. They proved that the class of semiring recognizable l-fuzzy languages is closed under quotients and inverse homomorphic images. They introduced the concept of conjunctive variety of l-fuzzy languages.

In this paper, we prove some results on inverse fuzzy languages depending on the fact that the syntactic monoid of an inverse fuzzy language is an inverse monoid.

2 Preliminaries

A fuzzy subset on a set X is a function $\mu : X \rightarrow [0, 1]$, [10]. A fuzzy language over an alphabet X is a fuzzy subset of X^* , free monoid generated by X . To each fuzzy language λ over X we

associate a congruence P_λ called syntactic congruence as follows. For $u, v \in X^*$, $uP_\lambda v$ if and only if $\lambda(xuy) = \lambda(xvy)$ for all $x, y \in X^*$. The quotient monoid $Syn(\lambda) = X^*/P_\lambda$ is called the syntactic monoid of λ [5].

Theorem 2.1. *A fuzzy language λ is regular if and only if $Im(\lambda)$ is finite and λ_c is regular for every $c \in [0, 1]$.*

Theorem 2.2. *(Myhill Nerode theorem) A fuzzy language λ is regular if and only if P_λ has finite index.*

Definition 2.1. A *fuzzy automaton* on an alphabet X is a five tuple $M = (Q, X, \mu, i, \tau)$ where Q is a finite set of states, X is a finite set of input symbols and μ is a fuzzy subset of $Q \times X \times Q$ representing the transition mapping, i is a fuzzy subset of Q called initial state, τ is a fuzzy subset of Q called final state.

A fuzzy automaton can also be represented as a five tuple $(Q, X, \{T_u | u \in X\}, i, \tau)$ where $\{T_u | u \in X\}$ is the set of fuzzy transition matrices, $i = \{i_1, \dots, i_n\}$, $i_k \in [0, 1]$; $\tau = \{j_1, \dots, j_n\}^T$, $j_k \in [0, 1]$ for $k = 1, \dots, n$.

μ can be extended to the set $Q \times X^* \times Q$ by

$$\mu(q, \Lambda, p) = \begin{cases} 1 & q = p \\ 0 & q \neq p \end{cases}$$

$$\mu(q, u, p) = \bigvee_{q_i \in Q} \{ \mu(q, x_1, q_1) \wedge \mu(q_1, x_2, q_2) \wedge \dots \wedge \mu(q_{k-1}, x_k, p) | x_1 x_2 \dots x_k = u \}$$

The fuzzy language recognized by this fuzzy automaton is $f_M(u) = \bigvee_{q \in Q} \bigvee_{p \in Q} i(q) \wedge \mu(q, u, p) \wedge \tau(p)$ which can also written as $f_M(u) = i \circ T_u \circ \tau$, where the composition is the *max-min* composition of fuzzy matrices [11].

Definition 2.2. A *deterministic fuzzy automaton* is a fuzzy automaton $M = (Q, X, \mu, i, \tau)$ such that there exist a unique $s \in Q$ with $i(s) > 0$ and there exist a unique $q \in Q$ such that $\mu(s, x, q) > 0$ for every $x \in X^*$.

For each fuzzy automaton we can construct a deterministic fuzzy automaton such that the language recognized by them are the same.

Definition 2.3. If $M = (Q, X, \mu)$ be a fuzzy automaton, then $M = (Q, X, \mu)$ is said to be an *inverse fuzzy automaton* if for all $x \in X^*$, there exist a unique $y \in X^*$ such that and $\mu(q, xyx, p) = \mu(q, x, p)$; $\mu(q, yxy, p) = \mu(q, y, p)$ for all $p, q \in Q$ [6].

To assure the existence of such y , we take the free monoid \tilde{X}^* on $X \cup X^{-1}$ so that for every $x \in \tilde{X}^*$, $\mu(q, xx^{-1}x, p) = \mu(q, x, p)$ and $\mu(q, x^{-1}xx^{-1}, p) = \mu(q, x^{-1}, p)$ for every $p, q \in Q$. In the case of a deterministic inverse fuzzy automaton this can be redefined as, for every $x \in \tilde{X}^*$, $\mu(q, x, p) = \mu(p, x^{-1}, q)$ and $\mu(p, x, q) = \mu(r, x, q)$ which implies $p = r$ for all $p, q, r \in Q$. A deterministic inverse fuzzy automaton can be represented by transition matrices with each row and column contains atmost one non-zero entry.

Definition 2.4. A fuzzy language λ on an alphabet \tilde{X} is said to be an *inverse fuzzy language (IFL)* if the minimal fuzzy automaton recognizing that fuzzy language is an inverse fuzzy automaton.

Theorem 2.3. *(Characterization of an inverse fuzzy language) A fuzzy language λ on \tilde{X} is inverse if and only if for every $x \in \tilde{X}^*$, $\lambda(uxx^{-1}v) = \lambda(uxv)$ and $\lambda(ux^{-1}xx^{-1}v) = \lambda(ux^{-1}v)$ for all $u, v \in \tilde{X}^*$ [6].*

3 Properties of Inverse Fuzzy Languages

In [6], it is proved that IFL is closed under finite Boolean operations, homomorphic images.

Theorem 3.1. *IFL is closed under quotients.*

Proof. If $\lambda_1, \lambda_2 \in \text{IFL}$ and $x, u, v \in \tilde{X}^*$.

Then $\lambda_1(uxx^{-1}xv) = \lambda_1(uxv)$ and $\lambda_1(ux^{-1}xx^{-1}v) = \lambda_1(ux^{-1}v)$. Now,

$$\begin{aligned} (\lambda_2^{-1}\lambda_1)(uxx^{-1}xv) &= \bigvee_{v_1 \in \tilde{X}^*} \{\lambda_1(v_1 u x x^{-1} x v) \wedge \lambda_2(v_1)\} \\ &= \bigvee_{v_1 \in \tilde{X}^*} \{\lambda_1(v_1 u) x x^{-1} x v \wedge \lambda_2(v_1)\} \\ &= \bigvee_{v_1 \in \tilde{X}^*} \{\lambda_1(v_1 u x v) \wedge \lambda_2(v_1)\} \\ &= \lambda_2^{-1}\lambda_1(uxv) \end{aligned}$$

and

$$\begin{aligned} (\lambda_2^{-1}\lambda_1)(ux^{-1}xx^{-1}v) &= \bigvee_{v_1 \in \tilde{X}^*} \{\lambda_1(v_1 u x^{-1} x x^{-1} v) \wedge \lambda_2(v_1)\} \\ &= \bigvee_{v_1 \in \tilde{X}^*} \{\lambda_1(v_1 u) x^{-1} x x^{-1} v \wedge \lambda_2(v_1)\} \\ &= \bigvee_{v_1 \in \tilde{X}^*} \{\lambda_1(v_1 u x^{-1} v) \wedge \lambda_2(v_1)\} \\ &= \lambda_2^{-1}\lambda_1(ux^{-1}v) \end{aligned}$$

In a similar way, we can prove that

$$\lambda_1 \lambda_2^{-1}(uxx^{-1}xv) = \lambda_1 \lambda_2^{-1}(uxv) \text{ and } \lambda_1 \lambda_2^{-1}(ux^{-1}xx^{-1}v) = \lambda_1 \lambda_2^{-1}(ux^{-1}v).$$

Thus $\lambda_1^{-1}\lambda_2$ and $\lambda_2\lambda_1^{-1} \in \text{IFL}$ □

Theorem 3.2. *IFL is closed under multiplication by constants.*

Proof. If λ be an inverse fuzzy language on \tilde{X} and $x \in \tilde{X}^*$. Then there exist a unique inverse $x^{-1} \in \tilde{X}^*$ such that $\lambda(uxx^{-1}xv) = \lambda(uxv)$ and $\lambda(ux^{-1}xx^{-1}v) = \lambda(ux^{-1}v)$ for every $u, v \in \tilde{X}^*$. If $c \in [0, 1]$, then $(c\lambda)(uxx^{-1}xv) = c.\lambda(uxx^{-1}xv) = c.\lambda(uxv) = (c\lambda)(uxv)$ and $(c\lambda)(ux^{-1}xx^{-1}v) = c.\lambda(ux^{-1}xx^{-1}v) = c.\lambda(ux^{-1}v) = (c\lambda)(ux^{-1}v)$.

So $c\lambda \in \text{IFL}$. □

Theorem 3.3. *If λ is an inverse fuzzy language on \tilde{X} , then for each $c \in [0, 1]$, λ_c is an inverse language on \tilde{X} .*

Proof. If $M = (Q, \tilde{X}, \mu, i, \tau)$ is an inverse fuzzy automaton recognizing λ . Then for every $x \in \tilde{X}^*$ there exist a unique $y \in \tilde{X}^*$ such that $\mu(p, x, q) = \mu(p, xyx, q)$ for all $p, q \in Q$.

If $c \in \lambda_c$ and $uxv \in \lambda_c$, then $\lambda(uxv) \geq c$.

$$\begin{aligned} \lambda(uxy xv) &= \bigvee_{p,q \in Q} i(p) \wedge \mu(p, uxy xv, q) \wedge \tau(q) \\ &= \bigvee_{p,q \in Q} i(p) \wedge \left(\bigvee_{r,r' \in Q} \mu(p, u, r) \wedge \mu(r, xyx, r') \wedge \mu(r', v, q) \right) \wedge \tau(q) \\ &= \bigvee_{p,q \in Q} i(p) \wedge \left(\bigvee_{r,r' \in Q} \mu(p, u, r) \wedge \mu(r, x, r') \wedge \mu(r', v, q) \right) \wedge \tau(q) \\ &= \bigvee_{p,q \in Q} i(p) \wedge \mu(p, uxv, q) \wedge \tau(q) \\ &= \lambda(uxv) \end{aligned}$$

Thus, $\lambda(uxv) \geq c$ if and only if $\lambda(uxy xv) \geq c$.

That is, $uxv \in \lambda_c$ if and only if $uxy xv \in \lambda_c$.

In a similar way, we can prove that $uyv \in \lambda_c$ if and only if $uyxyv \in \lambda_c$. □

Theorem 3.4. *IFL is not closed under inverse homomorphic images.*

Proof. Take $\tilde{X}_1 = \{a, a^{-1}\}$, $\tilde{X}_2 = \{b, b^{-1}\}$. Now define $\beta : \tilde{X}_1 \rightarrow \tilde{X}_2$ as $\beta(a) = \beta(a^{-1}) = b$. Then β can be extended to a homomorphism $\beta^* : \tilde{X}_1^* \rightarrow \tilde{X}_2^*$. If λ be an inverse fuzzy language on \tilde{X}_1 . Then $\lambda(u'bv') = \lambda(u'bb^{-1}bv')$ for all $u', v' \in \tilde{X}_2^*$. If $\beta^{*-1}\lambda$ is an inverse fuzzy language. Then $\beta^{*-1}\lambda(uav) = \beta^{*-1}\lambda(uaa^{-1}av)$; for all $u, v \in \tilde{X}_1^*$. That is, $\lambda(\beta^*(uav)) = \lambda(\beta^*(uaa^{-1}av))$. Thus, $\lambda(\beta^*(u)b\beta^*(v)) = \lambda(\beta^*(u)bbb\beta^*(v))$ and this says the inverse is not unique which is a contradiction. So $\beta^{*-1}\lambda$ is not an inverse fuzzy language. □

Theorem 3.5. *IFL is not a variety of fuzzy languages.*

Proof. A collection of fuzzy languages is a variety if it is closed under finite Boolean operations, homomorphic and inverse homomorphic images, quotients, multiplication by constants and c-cuts. □

Thus an inverse fuzzy language can be considered as a regular fuzzy language with its syntactic monoid is an inverse monoid. A characterization for an inverse monoid by Wagner in (1952) is that a monoid is an inverse monoid if and only if it is regular and any two idempotents commute each other [12]. It is also proved that a monoid is regular if and only if every \mathcal{L} -class (\mathcal{R} -class) contains an idempotent. Thus a fuzzy language is an inverse fuzzy language then idempotents in the syntactic monoid commute each other and every \mathcal{L} -class (\mathcal{R} -class) contains an idempotent.

Proposition 3.1. *If λ be an inverse fuzzy language on \tilde{X} and $[e]$ be an idempotent in $M(\lambda)$, then $\mu(p, xe, q) \leq \mu(p, x, q)$ and $\mu(p, ex, q) \leq \mu(p, x, q)$ for all $p, q \in Q$; $x \in \tilde{X}^*$.*

Proof. Since λ is an inverse fuzzy language every element of $M(\lambda)$ acts as one-one partial fuzzy transformations on Q and idempotents in $M(\lambda)$ can be considered as fuzzy matrices with non-zero entries only in the diagonal and so T_e acts as a sub-identity on Q .

Thus, $\mu(p, e, q) \neq 0$ if $p = q$ and $\mu(p, e, q) = 0$ if $p \neq q$.

$$\begin{aligned} \mu(p, xe, q) &= \bigvee_{q' \in Q} \mu(p, x, q') \wedge \mu(q', e, q) \\ &= \mu(p, x, q) \wedge \mu(q, e, q) \\ &\leq \mu(p, x, q) \end{aligned}$$

Similarly, $\mu(p, ex, q) \leq \mu(p, x, q)$ □

Theorem 3.6. *If a regular fuzzy language λ , is an inverse fuzzy language, then (1) Idempotents of $M(\lambda)$ commute (2) there exist $n \in \mathbb{N}$ such that $\lambda(xu^n y) \leq \lambda(xy)$ for all $x, u, y \in \tilde{X}^*$.*

Proof. If λ is an inverse fuzzy language, then (1) follows, since $M(\lambda)$ is an inverse monoid and idempotents in an inverse monoid commute.

To prove (2), if $x, u, y \in \tilde{X}^*$. Then $[x], [u], [y] \in M(\lambda)$. Since $M(\lambda)$ is a finite inverse monoid, there exist $n > 0$ such that $[u]^n$ is an idempotent in $M(\lambda)$.

$$\begin{aligned} \lambda(xu^n y) &= \bigvee_{p, q \in Q} i(p) \wedge \mu(p, xu^n y, q) \wedge \tau(q) \\ &= \bigvee_{p, q \in Q} i(p) \wedge \left(\bigvee_{q' \in Q} \mu(p, xu^n, q') \wedge \mu(q', y, q) \right) \wedge \tau(q) \\ &\leq \bigvee_{p, q \in Q} i(p) \wedge \left(\bigvee_{q' \in Q} \mu(p, x, q') \wedge \mu(q', y, q) \right) \wedge \tau(q) \\ &= \bigvee_{p, q \in Q} i(p) \wedge \mu(p, xy, q) \wedge \tau(q) \\ &= \lambda(xy) \end{aligned}$$

□

Take λ , is a regular fuzzy language of \tilde{X}^* ; $\pi : \tilde{X}^* \rightarrow M(\lambda)$, the syntactic morphism; $\lambda^+ = \{x \in \tilde{X}^* : \lambda(x) > 0\}$, the support of λ and $\pi(\lambda^+)$, the syntactic image of λ .

Theorem 3.7. *For every regular fuzzy language the following conditions are equivalent: (1) for every $x, u, y \in \tilde{X}^*$, there exist $n \in \mathbb{N}$ such that $\lambda(xy) \geq \lambda(xu^n y)$ (2) for every $[x], [y] \in M(\lambda)$ and for every idempotent $[e] \in M(\lambda)$, $[xey] \in \pi(\lambda^+)$ implies $[xy] \in \pi(\lambda^+)$.*

Proof. Suppose condition(1) holds. If $[x], [e], [y] \in M(\lambda)$ such that $[xey] \in \pi(\lambda^+)$. Since π is onto, there exist $x, u, y \in \tilde{X}^*$ such that $\pi(x) = [x]$, $\pi(y) = [y]$, $\pi(u) = [e]$. Also, there exist $n \in \mathbb{N}$ such that $\lambda(xy) \geq \lambda(xu^n y)$. $\pi(xu^n y) = \pi(x)\pi(u^n)\pi(y) = [x][e]^n[y] = [x][e][y] \in \pi(\lambda^+)$ by assumption. It follows that $xu^n y \in \lambda^+$ and $xy \in \lambda^+$. Hence $[x][y] = [xy] \in \pi(\lambda^+)$ Thus condition (1) implies condition (2).

Conversely, suppose that for every $[x], [y] \in M(\lambda)$ and for every idempotent $[e] \in M(\lambda)$, $[xey] \in \pi(\lambda^+)$ implies $[xy] \in \pi(\lambda^+)$ Then condition (1) holds when $\lambda(xu^n y) = 0$ for every n . Suppose $x, u, y \in \tilde{X}^*$ such that $\lambda(xu^n y) > 0$ for all $n \in \mathbb{N}$. That is, $xu^n y \in \lambda^+$ for all $n \in \mathbb{N}$. Then $[x], [y], [u] \in M(\lambda)$ and since $M(\lambda)$ is finite there exist some $k \in \mathbb{N}$ such that $[u]^k$ is an idempotent say $[e]$. Now, $[xey] = [x][e][y] = \pi(x)\pi(u^k)\pi(y) = \pi(xu^k y) \in \pi(\lambda^+)$. So $[xy] \in \pi(\lambda^+)$ by the assumption. That is, $\pi(xy) \in \pi(\lambda^+)$ and this implies $xy \in \lambda^+$. Thus $\lambda(xy) > 0$ which says that there exist some $k \in \mathbb{N}$ such that $\lambda(xy) \geq \lambda(xu^k y)$. □

Theorem 3.8. *If λ is an inverse fuzzy language. Then (1) idempotents of $M(\lambda)$ commute. (2) For every $x, y \in \tilde{X}^*$, there exist an idempotent $[e] \in M(\lambda)$ such that $[xey] \in \pi(\lambda^+)$ if and only if $[xy] \in \pi(\lambda^+)$.*

Proof. Since the syntactic monoid of an inverse fuzzy language is an inverse monoid, condition (1) follows. To prove condition (2), take $x \in \tilde{X}^*$. Since $M(\lambda)$ is an inverse monoid, $\mathcal{L}_{[x]}$ contains a unique idempotent say $[e]$ which is a right identity for elements of $\mathcal{L}_{[x]}$. Then $T_x \circ T_e = T_x$. Suppose $(Q, \tilde{X}, \mu, i, \tau)$ be the minimal fuzzy automata recognizing λ and $y \in \tilde{X}^*$ such that $[xey] \in \pi(\lambda^+)$.

$$\begin{aligned} \lambda(xey) &= i \circ T_x \circ T_e \circ T_y \circ \tau \\ &= i \circ T_x \circ T_y \circ \tau \\ &= \lambda(xy) \end{aligned}$$

So $\lambda(xey) > 0$ if and only if $\lambda(xy) > 0$. That is, $[xey] \in \pi(\lambda^+)$ if and only if $[xy] \in \pi(\lambda^+)$. \square

Theorem 3.9. *If $M = (Q, \tilde{X}, \mu)$ is a fuzzy automaton. Then for every $x, y \in \tilde{X}^*$ and $m, n \in N$ with $[x]^m, [y]^n$ are idempotents in \tilde{X}^*/θ_M , $\mu(p, x^m y^n, q) = \mu(p, y^n x^m, q)$ for every $p, q \in Q$ if and only if \tilde{X}^*/θ_M has commuting idempotents.*

Proof. Since \tilde{X}^*/θ_M is a finite semigroup, for every $[x]$ in \tilde{X}^*/θ_M , there exists $n \in N$ such that $[x]^n$ is an idempotent. Suppose $[x]^m, [y]^n$ are two idempotents in \tilde{X}^*/θ_M . Now for every $p, q \in Q$, $\mu(p, x^m y^n, q) = \mu(p, y^n x^m, q)$ if and only if $[x^m y^n] = [y^n x^m]$

That is, $\mu(p, x^m y^n, q) = \mu(p, y^n x^m, q)$ if and only if \tilde{X}^*/θ_M has commuting idempotents using the fact that $[x^m][y^n] = [x]^m[y]^n$. \square

Theorem 3.10. *If λ is a fuzzy language. Then for every $x, y \in \tilde{X}^*$ and $m, n \in N$; $[x]^m, [y]^n$ are idempotents, $\lambda(ux^m y^n v) = \lambda(uy^n x^m v)$ for all $u, v \in \tilde{X}^*$ if and only if the syntactic monoid of λ has commuting idempotents.*

Proof. Since for every $x, y \in \tilde{X}^*$ and $m, n \in N$ such that $[x]^m, [y]^n$ are idempotents, $\lambda(ux^m y^n v) = \lambda(uy^n x^m v)$, it follows that $x^m y^n P_\lambda y^n x^m$ if and only if $[x]_{P_\lambda}^m [y]_{P_\lambda}^n = [y]_{P_\lambda}^n [x]_{P_\lambda}^m$. That is, if and only if syntactic monoid of λ has commuting idempotents. \square

Thus we have proved the following theorem.

Theorem 3.11. *If λ is an inverse fuzzy language, then (1) for every $x, y \in \tilde{X}^*$ there exist $m, n \in N$ such that $\lambda(ux^m y^n v) = \lambda(uy^n x^m v)$ for all $u, v \in \tilde{X}^*$. (2) for all $x, u, y \in \tilde{X}^*$, there exist $n \in N$ such that $\lambda(xu^n y) \leq \lambda(xy)$.*

By Eilegnberg-type variety theorem, the collection of all fuzzy languages such that for every $x, y \in \tilde{X}^*$ and $m, n \in N$ such that $[x]^m, [y]^n$ are idempotents, $\lambda(ux^m y^n v) = \lambda(uy^n x^m v)$ for all $u, v \in \tilde{X}^*$, form a variety of fuzzy languages and the associated psuedo variety is the variety generated by inverse semi groups.

4 Conclusion

We analysed some of the algebraic properties of inverse fuzzy languages. We proved that inverse fuzzy languages is not closed under inverse homomorphic images. We proved that a fuzzy automaton is inverse if and only if the transition monoid is an inverse monoid. A fuzzy language is an inverse fuzzy language if the minimal fuzzy automaton recognizing that fuzzy language is an inverse fuzzy automaton. We derived an equivalent condition for a regular fuzzy language. We also discussed some more properties of an inverse fuzzy language based on the fact that an inverse monoid is one which is regular and idempotents commute.

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Competing Interests

Authors have declared that no competing interests exist.

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