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Reformulating Special Relativity on a Two-World Background

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The sole author designed, analysed, interpreted and prepared the manuscript.

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ABSTRACT

A new spacetime is isolated and added to the existing spacetime, yielding a pair of co-existing spacetimes, which are four-dimensional inversions of each other. The separation of the spacetimes by the special-relativistic event horizon, compels an interpretation of a pair of symmetrical worlds (or universes) in nature. Furthermore, a two-dimensional intrinsic spacetime that underlies the four-dimensional spacetime in each universe is introduced. The four-dimensional spacetime is the outward manifestation of the two-dimensional intrinsic spacetime, just as the special theory of relativity (SR) on flat four-dimensional spacetime is the outward manifestation of the intrinsic special theory of relativity (∅SR) on flat two-dimensional intrinsic spacetime in each universe. A new set of spacetime/intrinsic spacetime diagrams in the two-world picture is developed, from which intrinsic Lorentz transformation in ∅SR and Lorentz transformation in SR are derived and intrinsic Lorentz invariance and Lorentz invariance are validated in each universe. The SR remains unchanged, but the exposition of its two-world background, the isolated parallel new theory ∅SR and other isolated new features in this article, allow a broader view of SR. This article includes a new addition to the

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conceptions of many worlds (or universes) in physics and it is effectively a review (in the two-world picture) of the existing geometrical representations of the Lorentz transformation (in a one-world picture).

Keywords: Two-world picture; our world; negative world; spacetime; intrinsic spacetime; special relativity in two-world; intrinsic special relativity; no light cones in two-world.

1 INTRODUCTION

The concept of other universe(s) (or world(s)) is not new in physics. In 1898, Schuster contemplated a universe containing negative mass [1]. The discovery in particle physics of the existence of an anti-particle to every particle afterward, led some physicists to suggest the existence of an anti-atom (composed of anti-particles) to every atom (composed of particles); an anti-molecule to every molecule and an anti-macroscopic-object to every macroscopic object. Then in order to explain the preponderance of particles and matter over anti-particles and anti-matter in this universe of ours, the existence of an anti-universe containing a preponderance of anti-matter was suggested, as discussed on page 695 of [2] for instance. However it has remained unknown whether the speculated universe containing negative mass of Schuster and an anti-universe containing a preponderance of anti-matter exist or not.

While the universe containing negative mass suggested by Schuster and an anti-universe containing anti-matter discussed above must be classified as speculations in physics, the theory of wormholes in the general theory of relativity makes a quantitative prediction of the existence of another universe. It is known in the general theory of relativity that the Schwarzschild geometry on the spacelike hypersurface, $t = \mathrm{const.}$, predicts (or consists) of wormhole (or Einstein-Rosen bridge) connecting two distinct points in spacetime within this universe of ours or two distinct asymptotically flat universes, inspired by the work of Einstein and Rosen [3], as discussed on pages 836-840 of [4] and also in [5], for example. The wormhole theory predicts the existence of two asymptotically flat universes. However the theory has not advanced the two-world concept in further detail. It can be said that the theory of wormhole provides a glimpse only of the existence of another universe and suggests independent existence of the other universe to our universe.

Although it involves the reformulation of the special theory of relativity on a pair of co-existing spacetimes (which are separated by the special-relativistic and gravitational event horizons), started in the second half of the 1980s, this paper is different in essential respects from several existing papers in more recent years, which formulate other theories of physics on a two-sheeted spacetime. They include the formulation of the scalar-tensor theory of gravitation on two-sheeted spacetime [6]; formulation of quantum dynamics on a non-commutative two-sheeted spacetime, leading to the doubling of fermionic states that are interpreted as matter and hidden matter states associated with matter and hidden matter universes [7]; model of left- and right-handed chiral fields on two different sheets of spacetime in the standard model [8], among many others.

A major difference between the existing papers on two-sheeted spacetime and this paper is that, although an existing paper may make implicit connection to the co-existence of a pair of worlds (or universes), such as in [7], none develops the associated two-world symmetry (or picture), which could lead to a characterization of the other universe in relation to our universe. Whereas, apart from explicit connection to two worlds (or universes), which are separated by event horizon, and the formulation of SR in two-world, the development of the associated two-world symmetry (or picture) and the characterization of the other universe in relation to our universe are the hallmarks of this paper.

Finally, another class of conceptions of other worlds (or universes) in physics is encountered in the many-world interpretation of quantum mechanics (MWI) and in quantum cosmology.

The various conceptions of other worlds in this class have been grouped into two categories Linde [9] namely, many different universes described by quantum cosmology, Everett [10]; Wheeler [11]; DeWitt [12, 13]; Page [14]; Kent [15] and others, and many different exponentially large parts of the same inflationary universe (or the entire ensemble of innumerable regions of disconnected spacetime), Linde [9]; Buosso and Susskind [16]; Deutsche [17]; Aguirre and Tegmark [18]; Linde and Vachurin [19] and others. These two categories are considered to be encompassed by the multiverse concept in Linde [9]. There is also the parallel branes conception of many worlds in the string theory, Maartens and Koyamme [20] and Abdel [21].

A distinguishing feature of the multiverse of inflationary cosmology and MWI and the parallel branes of the string theory in the preceding paragraph and the two universes to be derived in this article is that, the universes of the multiverse and parallel branes of string theory can have different number of dimensions of different extents and accommodate different natural laws, whereas the two universes to be isolated in this article have four dimensions of spacetime of equal extents and exhibit symmetry of the distribution of material particles and bodies, as well as symmetry of the special theory of relativity.

This article is a major revision of its earlier version [22]. Its central purpose is to show formally that the special theory of relativity (SR) rests on a background of a two-world picture, in which an identical partner universe in a different spacetime co-exists with this universe of ours in our spacetime, where the two spacetimes are separated by an interface of discontinuity, referred to as event horizon (the special-relativistic event horizon in this paper), and to commence the development of the two-world picture thus introduced. The placement of the spacetime of the other universe relative to the spacetime of our universe, as well as the distribution of matter in it shall be derived.

The definite intrinsic interaction between the two universes in SR (involving a flat two-dimensional intrinsic spacetime that underlies the flat four-dimensional spacetime in each universe to be introduced), shall also be shown. The symmetry of Lorentz transformation and Lorentz invariance between the two universes shall be established.

While offering a new addition to the conceptions of many worlds (or universes) in physics, this article is also effectively a review of the existing geometrical interpretations of the Lorentz transformation (the Lorentz boost) and its inverse, which involve the Minkowski's diagram [23], the Loedel diagram [24] and the Brehme diagram [25].

2 TWO SCHEMES TOWARD THE DERIVATION OF LORENTZ BOOST BY THE GRAPHICAL APPROACH

As can be easily demonstrated, the two schemes summarized in Table I both lead to the Lorentz boost (which shall also be referred to as the Lorentz transformation (LT)), and the Lorentz invariance (LI). Although the, $\gamma = \cosh\alpha$, parametrization of the Lorentz boost in scheme I is more familiar, the $\gamma = \sec\psi$ parametrization in scheme II is also known.

Now letting $v/c = 0$ in Table I we obtain the following

$$\cosh\alpha = 1\ ;\ \sinh\alpha = \tanh\alpha = 0\ \Rightarrow \alpha = 0$$
$$\sec\psi = 1\ ;\ \tan\psi = \sin\psi = 0\ \Rightarrow \psi = 0$$

Letting $v/c = 1$ we have

$$\cosh\alpha = \sinh\alpha = \infty\ ;\ \tanh\alpha = 1\ \Rightarrow \alpha = \infty$$
$$\sec\psi = \tan\psi = \infty\ ;\ \sin\psi = 1$$
$$\Rightarrow \psi = \pi/2,\ 5\pi/2,\ 9\pi/2\ \cdots$$

And letting $v/c = -1$ we have

$$\cosh\alpha = \infty \ ; \ \sinh\alpha = -\infty \ ; \ \tanh\alpha = -1$$
$$\Rightarrow \alpha = -\infty \ .$$
$$\sec\psi = \infty \ ; \ \tan\psi = -\infty \ ; \ \sin\psi = -1$$
$$\Rightarrow \psi = -\pi/2, \ 3\pi/2, \ 7\pi/2, \cdots$$

Table 1. Two schemes toward the derivation of the Lorentz boost graphically

Scheme I	Scheme II
$x' = x\cosh\alpha - ct\sinh\alpha$ $ct' = ct\cosh\alpha - x\sinh\alpha$ $y' = y \ ; \ z' = z$ $\cosh\alpha = \frac{1}{\sqrt{1 - v^2/c^2}} = \gamma$ $\sinh\alpha = \frac{v/c}{\sqrt{1 - v^2/c^2}} = \beta\gamma$ $\tanh\alpha = v/c = \beta$	$x' = xsec\psi - ct\tan\psi$ $ct' = ct\sec\psi - x\tan\psi$ $y' = y \ ; \ z' = z$ $\sec\psi = \frac{1}{\sqrt{1 - v^2/c^2}} = \gamma$ $\tan\psi = \frac{v/c}{\sqrt{1 - v^2/c^2}} = \beta\gamma$ $\sin\psi = v/c = \beta$

Thus there are the following equivalent ranges of values of the parameter α and the angle ψ between the two schemes:

$$0 \leq \alpha \leq \infty \text{ (Scheme I)} \equiv 0 \leq \psi \leq \pi/2 \text{ (Scheme II)}$$
$$-\infty \leq \alpha \leq \infty \text{ (Scheme I)} \equiv -\pi/2 \leq \psi \leq \pi/2 \text{ (Scheme II)}$$

The second range, $-\infty \leq \alpha \leq \infty$ (Scheme I) or $-\pi/2 \leq \psi \leq \pi/2$ (Scheme II), generates the positive half of the 4-dimensional spacetime hyperplane (to be referred to as positive half-hyperplane for brevity), shown shaded in Figs. 1a and 1b.

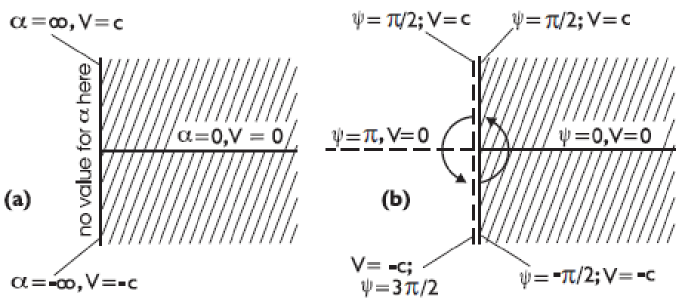

Fig. 1. (a) All values of the number α generate the positive spacetime half-hyperplane in scheme I, (b) all values of the angle ψ in the first cycle generate the positive and negative spacetime half-hyperplanes (or the entire hyperplane) in scheme II.

If we consider Scheme I, then clearly there is only the positive half-hyperplane as illustrated in Fig. 1a. This is so since the range, $-\infty \leq \alpha \leq \infty$, generates the positive half of the four-dimensional spacetime hyperplane only, and there are no other values of α outside this range. Thus going to the negative half-hyperplane is impossible in the context of SR in scheme I.

If we consider scheme II, on the other hand, then the range, $-\pi/2 \leq \psi \leq \pi/2$, which generates the positive half-hyperplane in Fig. 1b is not exhaustive of the values of angle ψ in the first cycle. There is also the range, $\pi/2 \leq \psi \leq 3\pi/2$, which generates the negative half-hyperplane. Thus going into the negative half-hyperplane is conceivable in SR in the context of scheme II. There is actually no gap between the solid line and the broken line along the vertical unlike as appears in Fig. 1b.

In translating Figs. 1a and 1b into spacetime diagrams, the positive horizontal lines along which, $v = 0, \alpha = 0$ and $\psi = 0$, in the figure, correspond to the 3-dimensional Euclidean space Σ with mutually orthogonal dimensions, x, y and z, in the Cartesian system of coordinates; the positive vertical lines along which, $v = c, \alpha = \infty$ and $\psi = \pi/2$, correspond to the positive time dimension ct, while the negative vertical lines along which $v = -c, \alpha = -\infty$ and $\psi = -\pi/2$, correspond to the negative time dimension (or the time reversal dimension) $-ct^*$. In addition, the horizontal line in the negative half-hyperplane in Fig. 1b, corresponds to a negative 3-dimensional Euclidean space (not known in physics until now), to be denoted by $-\Sigma^*$, with mutually orthogonal dimensions, $-x^*, -y^*$ and $-z^*$, in the Cartesian coordinate system. Thus Figs. 1a and 1b translate into the spacetime diagrams of Figs. 2a and 2b respectively. It shall also be noted that the speed v with extreme values c and $-c$ in Figs. 1a and 1b and Figs. 2a and 2b, is not the relative dynamical speed of SR, but shall be identified as static 'geodetic flow speed' (which is the same relative to all observers or frames of reference), and may be re-denoted by V_s.

Representation of the Euclidean 3-spaces by lines, which may be referred to as three-dimensional 'hyperlines', along the horizontal and the time dimensions by vertical pseudo-orthogonal lines to the "space axes", as done in Figs. 2a and 2b, is a well known practice in the graphical representation of four-dimensional spacetime in the modern Minkowski diagrams [23].

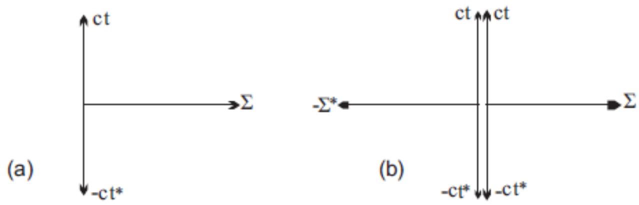

Fig. 2. The spacetimes generated by (a) all values of the number α in scheme I and (b) all values of the angle ψ in the first cycle in scheme II.

Fig. 2a pertains to scheme I in Table I. The four-dimensional spacetime (Σ, ct) with dimensions, x, y, z and ct, is the Minkowski space as known. In addition, there is the negative time dimension $-ct^*$ that serves the role of time reversal dimension (which is different from the *past time* axis in the past light cone). There are no second and third quadrants in Fig. 2a, since the negative half-hyperplane is inaccessible in scheme I.

Fig. 2b pertains to scheme II in Table 1. There are two 'anti-parallel' Minkowski spaces in Fig. 2b namely, the one with positive dimensions, $(\Sigma, ct) \equiv (x, y, z, ct)$, generated by the range of angles, $0 \leq \psi \leq \pi/2$, in the first quadrant in Fig. 1b, to be referred to as the positive Minkowski space, and the other with all negative dimensions, $(-\Sigma^*, -ct^*) \equiv (-x^*, -x^*, -y^*, -ct^*)$, generated by the range of angles, $\pi \leq \psi \leq 3\pi/2$, in the third quadrant, to be referred to as the negative Minkowski space.

There are, in addition, the negative time dimension $-ct^*$ that serves the role of the time reversal dimension to the positive Minkowski space, while the positive time dimension ct serves the role of time reversal dimension to the negative Minkowski space.

As can be observed, the dimensions of the negative Minkowski space constitute parity inversion and time reversal with respect to the spacetime dimensions of the positive Minkowski space and conversely. Figure 2b says that this situation exists naturally, quite apart from the fact that parity inversion (by coordinate reflection), $x \to -x; y \to y; z \to z$ or $x \to -x; y \to -y; z \to -z$ and time reversal $t \to -t$ are achievable within the positive half-hyperplane, that is, within the positive Minkowski space (first quadrant) plus the fourth quadrant in Figs. 2a and 2b.

Finally, it shall be noted that both schemes I and II in Table I have been restricted to the positive half-hyperplane in physics until now. There has not seemed to be any need to consider the second range, $\pi/2 \leq \psi \leq 3\pi/2$ (or the negative half-hyperplane), in scheme II (or in Fig. 1b and Fig. 2b) in physics until now, because the parity inversion and time reversal associated with it can be achieved by reflection of coordinates of 3-space in the ranges, $-\infty \leq \alpha \leq \infty$ and $-\pi/2 \leq \psi \leq \pi/2$ (or in the positive half-hyperplane), which also includes time reversal (in both Figs. 2a and 2b). However we consider it worthy of investigation whether the range, $\pi/2 \leq \psi \leq 3\pi/2$, and the parity inversion it represents in scheme II exist naturally, apart from the possibility of parity inversion by coordinate reflections in the positive half-hyperplane.

Reasoning that parity inversion and time reversal might not be the only physical significance of the second range, $\pi/2 \leq \psi \leq 3\pi/2$ (or the negative half-hyperplane), in Fig. 1b that generates Fig. 2b, should it exist in nature, we deem it judicious to carry both the ranges, $-\pi/2 \leq \psi \leq \pi/2$ and $\pi/2 \leq \psi \leq 3\pi/2$, in scheme II along in the present development, with hope that the theory shall ultimately justify the existence of the second range or otherwise. The investigation of the implications of the existence naturally of the negative half-hyperplane in parallel with the positive half-hyperplane in Figs. 1b and 2b shall be started in this paper.

3 ON THE MINKOWSKI DIAGRAMS AS GEOMETRICAL REPRESENTATION OF LORENTZ TRANSFORMATION IN SCHEME I

The inclusion of this section is necessary because of the critique of the Minkowski's diagrams to be done in this section and the review of the Minkowski's graphical approach to the derivation of the Lorentz boost (to also be referred to as Lorentz transformation (LT)) and its inverse, which is effectively being done in this article. The Minkowski spacetime diagrams from which the LT and its inverse in scheme I in Table I have sometimes been derived for two frames in relative motion along their collinear coordinates x' and x, are shown as Figs. 3a and 3b, where the future light cone is shown with broken lines.

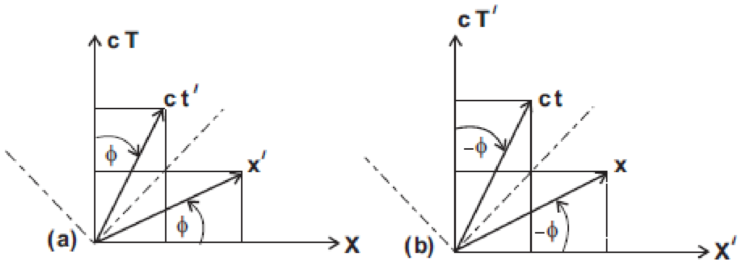

Fig. 3. The Minkowski diagrams sometimes used to derive the Lorentz transformation and its inverse in the existing one-world picture

The coordinates y' and z' of the particle's frame, as well as the coordinates y and z of the observer's frame, along which the relative notion of SR does not occur, remain not rotated from the Euclidean 3-space as a hyper-surface along horizontal, and have not been shown in Figs. 3a and 3b.

For the relative motion of two frames (which involves positive time dimension), the time reversal dimension $-ct^*$ is irrelevant, leaving only the first quadrant in Fig. 2a (in the context of scheme I). Thus relative rotations of the spacetime coordinates of the particle's (or primed) frame and the observer's (or unprimed) frame are restricted to the interior of the first quadrant in scheme I, for every pair of frames in relative motion. This corresponds to the first quadrant in Figs. 1a and 2a. As is clear from Fig. 2a, scheme I implies the existence of a singular spacetime (or a singular world), including a time reversal dimension, to be referred to as one-world picture.

Now the Lorentz transformation (LT) is usually derived analytically in the special theory of relativity (SR), following Albert Einstein's 1905 paper, see pages 37 – 48 of [26]. In this reference, Einstein inferred from two principles of relativity, the LT (and its inverse) for motion along the coordinate x' of the coordinate system (ct', x', y', z') attached to a particle moving at speed v relative to an observer's frame (ct, x, y, z) respectively as follows, where the coordinates x' and x are collinear,

$$t' = \gamma(t - \frac{v}{c^2}x) \; ; \; x' = \gamma(x - vt) \; ; \; y' = y \; ; \; z' = z \; , \tag{1}$$

and

$$t = \gamma(t' + \frac{v}{c^2}x') \; ; \; x = \gamma(x' + vt') \; ; \; y = y' \; ; \; z = z' \; , \tag{2}$$

where $\gamma = (1 - v^2/c^2)^{-1/2}$. As demonstrated in [26], system (1) (or (2)) satisfies the Lorentz invariance,

$$c^2t'^2 - x'^2 - y'^2 - z'^2 = c^2t^2 - x^2 - y^2 - z^2 \; . \tag{3}$$

Very soon afterwards (in 1906), Minkowski exposed the coordinate-geometrical (or graphical) implication of the LT and its inverse [27]. The modern forms of the spacetime diagrams he drew are depicted in Figs. 3a and 3b. The LT and its inverse (in terms of $\cosh\alpha$ and $\sinh\alpha$) usually associated with Figs. 3a and 3b, are contained in scheme I in Table I.

What has usually been done in the Minkowski graphical approach to the derivations of LT and its inverse is obtaining coordinate projections along the horizontal and vertical temporarily in the context of Euclidean geometry directly from Figs. 3a and 3b respectively as

$$ct = ct'\cos\phi + x'\sin\phi \; ; \; x = x'\cos\phi + ct'\sin\phi \; ;$$
$$y' = y \; \text{ and } z' = z. \tag{4a}$$

and

$$ct' = ct\cos\phi - x\sin\phi \; ; \; x' = x\cos\phi - ct\sin\phi \; ;$$
$$y = y' \; \text{ and } z = z' \; ; \tag{4b}$$

where the trivial transformations, $y' = y$ and $z' = z$, have been added in order to be consistent with the four-dimensionality of LT and its inverse.

Consideration of the motion of the origin of the coordinates of 3-space $(x' = y' = z' = 0)$ of the primed frame relative to the unprimed frame reduces system (4a) as

$$ct = ct\cos\phi \; \text{ and } \; x = ct'\sin\phi \; . \tag{4c}$$

Division of the second equation into the first equation of system (4c) gives

$$\frac{x}{ct} = \frac{v}{c} = \tan\phi \; . \tag{4d}$$

The relation (4d) says that the speed v has its maximum value c along the line, $\tan\phi = 1$, or $\phi = \pi/4$, on the surface of the future light cone, as known.

Now systems (4a) and (4b) along with relation (4d) derived from system (4a), do not yield the LT and its inverse, or they are incompatible with LT and its inverse. There was therefore the need to replace the temporary transformations (4a) and (4b) by others that will lead to LT and its inverse in the Minkowski's scheme. The following replacements are adopted.

$$ct = ct'\cosh\alpha + x'\sinh\alpha\,; x = x'\cosh\alpha + ct'\sinh\alpha\,;$$
$$y = y' \text{ and } z = z' \tag{5a}$$

and

$$ct' = ct\cosh\alpha - x\sinh\alpha\,; x' = x\cosh\alpha - ct\sinh\alpha\,;$$
$$y' = y \text{ and } z' = z\,. \tag{5b}$$

Again considering the motion of the spatial origin, $x' = y' = z' = 0$, of the primed frame relative to the unprimed frame, system (5a) reduces as

$$x = ct'\sinh\alpha \text{ and } ct = ct'\cosh\alpha\,. \tag{6a}$$

Division of the first into the second equation of system (6a) gives

$$\frac{x}{ct} = \frac{v}{c} = \tanh\alpha\,,$$

hence,

$$\cosh\alpha = (1 - \frac{v^2}{c^2})^{-1/2}\,;\ \sinh\alpha = \frac{v}{c}(1 - \frac{v^2}{c^2})^{-1/2}. \tag{6b}$$

System (6b) converts system (5a) to the inverse Lorentz transformation of system (1) and system (5b) to the LT of system (2), derived by Einstein [26].

System (5a) that replaces the temporary system (4a) derived from Fig. 3a, leads to the inverse Lorentz transformation, while system (5b) that replaces system (4b) derived from Fig. 3b, leads to the Lorentz transformation. Systems (5a) and (5b) are the natural options as inverse LT and LT in the Minkowski's scheme in the one-world picture.

The Lorentz boost (5b) in scheme I in Table I is the following in the matrix form

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cosh\alpha & -\sinh\alpha & 0 & 0 \\ -\sinh\alpha & \cosh\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}, \tag{7}$$

which is of the form, $\mathbf{x}' = L(\alpha)\,\mathbf{x}$.

There is a known mathematical significance of the Lorentz boost (or the LT) of system (5b) or (7) and its inverse system (5a), and the Minkowski diagrams of Figs. 3a and 3b. This is the fact that the 4×4 matrix $L(\alpha)$ that generates the Lorentz boost (7), which contains the parameter α in the unbounded range, $(-\infty,\infty)$, is a member of the pseudo-orthogonal Lorentz group SO(3,1), which is a non-compact Lie group [28]. Moreover the matrix $L(\alpha)$ is non-singular for any finite value of α in the range $(-\infty,\infty)$, as required for all group SO(3,1) matrices. This implies that non-physical discontinuities do not appear in the Minkowski space generated. Singularities appear in systems (5a) and (5b) for the extreme values of α namely, $\alpha = \infty$ and $\alpha = -\infty$ only, which are not included in the values of α.

The Lorentz boost is just a special Lorentz transformation. The general Lorentz transformation Λ is the following in the factorized form [28],

$$\Lambda = R(\gamma,\beta,0)L(\alpha)R(\gamma,\beta,0)^{-1}\,, \tag{8}$$

where $L(\alpha)$ is the Lorentz boost along the $z-$axis with speed, $v = c\tanh\alpha; 0 \leq \alpha < \infty$ (the time reversal dimension in Fig. 2a is not involved in SR), and the Euler angles for rotation in the Euclidean 3-space have their usual finite ranges.

Since the group SO(3) matrices R are closed and bounded and, are hence compact, the compactness or otherwise of Λ is determined by the Lorentz boost $L(\alpha)$. Thus since $L(\alpha)$ is non-compact, SO(3,1) is non-compact as known [28]. There is no way of making SO(3,1) compact in the Minkowski's one-world picture. This is so because the parameter α naturally lies within the unbounded exhaustive range $-\infty < \alpha < \infty$ that pertains to the positive half-hyperplane in Fig. 1a, implying the existence of one world in this picture. Thus the Minkowski's diagrams of Figs. 3a and 3b and the LT and its inverse of systems (5b) and (5a), or the implied transformation matrix $L(\alpha)$ in Eq. (7) derived from them, have been known as the physical significance of the Lorentz group in mathematics, or conversely.

Apart from the factorized form of the special orthogonal group SO(3,1) component of the generalized Lorentz group O(3,1) that contains the factors of Lorentz boost along the z-axis and the rotation group SO(3) in Eq. (8), the connected part of the Lorentz group that contains the identity matrix, which can be generated from the factors of Lorentz boost along x-, y- and z- directions and the rotation group SO(3), is also non-compact.

3.1 Critique of the Minkowski, the Loedel and the Brehme Diagrams

From the point of view of physics, on the other hand, one observes that the coordinates x' and ct' of the primed frame are not pseudo-orthogonal (or are skewed) in Fig. 3a, and the coordinates x and ct of the unprimed frame are skewed in Fig. 3b. These are pseudo-orthogonal coordinates in the absence of relative motion of the frames. Even in relative motion, an observer at rest relative to the primed frame cannot detect the uniform motion of his frame. The primed frame is stationary relative to an observer at rest relative to it, with or without the motion of the primed frame relative to the unprimed frame. Yet Fig. 3a shows that the coordinates of the primed frame are skewed with respect to an observer at rest relative to it when it is in uniform motion relative to the unprimed frame. The skewness of the spacetime coordinates of a frame is then an effect of the uniform motion of the frame, which an observer at rest relative to it can detect. This contradicts the fact that an observer cannot detect any effect of the uniform motion of his frame.

The skewness of rotated coordinates is unavoidable in the Minkowski's diagrams, because relative rotation of coordinates are restricted to the first quadrant in scheme I (or in the one-world picture). This is so, because the time-reversal dimension $-ct^*$ and, hence, the fourth quadrant of the spacetime hyperplane in Fig. 1a are not involved in SR, as deduced earlier.

The skewness of spacetime coordinates of frames of reference is not peculiar to the Minkowski diagrams. It is a general feature of all the existing spacetime diagrams in SR in the one-world picture. There are at least two other spacetime diagrams in special relativity, apart from the Minkowski diagram namely, the Loedel diagram [24] and the Brehme diagram [25]. The spacetime coordinates of two frames in relative motion are skewed in the Loedel and Brehme diagrams shown as Figs. 4a and 4b respectively, for two frames in relative motion along their collinear $x'-$ and $x-$ axes.

The skewness of the coordinates of a frame of reference in uniform relative motion is undesirable, because it is an effect of uniform motion of a frame that an observer at rest relative to the frame could observe, which negates the fundamental principle that no effect of uniform motion is detectable, as mentioned earlier. Moreover it gives apparent preference for one of two frames of reference in uniform relative motion, which, again, is a contradiction of a tenet of special relativity.

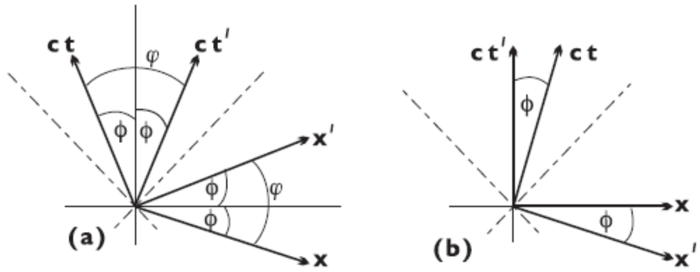

Fig. 4. (a) The Loedel diagram and (b) the Brehme diagram for two frames in relative motion

4 GEOMETRICAL REPRESENTATION OF LORENTZ TRANSFORMATION IN SCHEME II

Having discussed the existing geometrical representation of the Lorentz transformation (actually the Lorentz boost) and its inverse in special relativity in the context of scheme I in Table I (or in the one-world picture) in the preceding section, we shall develop a new set of spacetime diagrams that are compatible with Lorentz transformation and its inverse in the context of scheme II in Table I in the rest of this paper. We shall, in particular, watch out for the possibility of making the Lorentz group SO(3,1) compact and for removing the skewness of rotated spacetime coordinates of frames of reference in the existing spacetime diagrams of special relativity (in the one-world picture or in the context of scheme I).

4.1 Co-existence of Two Identical Universes in the Context of Scheme II

As shall be sufficiently justified with progress in this article, the co-existence of two anti-parallel Minkowski spaces in Fig. 2b implies the co-existence of two "anti-parallel" worlds (or universes) in nature. The dimensions x, y, z and ct of the positive Minkowski space, which are accessible to us by direct experience, are the dimensions of our universe (or world). The dimensions $-x^*, -y^*, -z^*$ and $-ct^*$ of the negative Minkowski space, which are inaccessible to us by direct experience, and hence, have remained unknown until now, are the dimensions of another universe (or world). Dummy star label is put on the dimensions of the other universe, which are non-observable to us in our universe, in order to distinguish them from the dimensions of our universe.

The negative spacetime dimensions $-x^*, -y^*, -z^*$ and $-ct^*$ are inversions in the origin (or four-dimensional inversion) of the positive spacetime dimensions x, y, z and ct. Thus the spacetime dimensions of the universe with negative dimensions, to be referred to as the negative universe for brevity, and the spacetime dimensions of our universe (also to be referred to as the positive universe), have an inversion-in-the-origin symmetry.

There is one-to-one mapping of points in spacetimes between the positive (or our) universe and the negative universe. This means that, to every point in spacetime in our universe, there corresponds a unique symmetry-partner point in spacetime in the negative universe.

In addition to the inversion in the origin relationship between the spacetime dimensions of the positive and negative universes, we shall prescribe reflection symmetry of spacetime geometry between the two universes. In other words, if we denote the spacetime manifold of the positive universe by **M** and that of the negative universe by $-$**M***, then spacetime geometry at a point in spacetime in the positive universe shall be prescribed by **M** and the metric tensor $g_{\mu\nu}$ at that point, that is, by (**M**, $g_{\mu\nu}$), while spacetime geometry shall be prescribed at the symmetry-partner point in the negative universe by $(-\mathbf{M}^*, g_{\mu\nu})$, where it must be remembered that the metric tensor is invariant with reflections of coordinates. Symmetry of spacetime geometry between the two universes can only be prescribed at this point of development of the two-world picture.

Now the Mach's principle is very fundamental. We shall make recourse to the principle here for the purpose of advancing our argument for the symmetry of state between the positive and negative universes, while knowing that the principle in itself has noting to do with special relativity. Essentially Mach's principle states that the geometry of a space is determined by the distribution of mass (and energy) in that space, see page 400 of [29]. It follows from the foregoing paragraph and Mach's principle that there is a reflection symmetry of the distribution of mass-energy in spacetimes between the two universes. Actually this is also a prescription at this point, since the symmetry of spacetime geometry is a prescription.

Reflection symmetry of geometry of spacetime and of the distribution of mass-energy in spacetime also imply reflection symmetry of motions of particles and objects, natural or caused by animate object, between the two universes. In other words, corresponding to an event, natural or man-made, taking place within a local region of spacetime in our universe, there is an identical event within the symmetry-partner local region of spacetime in the negative universe. This is the symmetry of state between the two universes. The two universes are perfectly identical in state at all times. The perfect symmetry of natural and man-made events (or perfect symmetry of state) between the two universes is a prescription at this point.

There is also a perfect symmetry of laws between the two universes, which implies that natural laws take on perfectly identical forms in the two universes. Symmetry of laws between the two universes is simply the extension of the invariance of laws found in our universe to the negative universe, which follows partly from the validity of Lorentz invariance in the negative universe to be demonstrated shortly. The two universes cannot possess symmetry of state if the laws that guide events and phenomena in them are different. The perfect symmetry of laws between the two universes shall be investigated elsewhere.

The negative spacetime dimensions of the negative universe implies that distance in space, which is a positive scalar quantity in our (or positive) universe, is a negative scalar quantity in the negative universe, and interval of time, which is a positive quantity in the positive universe is a negative quantity in the negative universe (it does not connote going to the past in our time dimension). These can be ascertained from the definition of distance, which is given in the Euclidean 3-space in the negative universe as, $d = ((-x^*)^2 + (-y^*)^2 + (-z^*)^2)^{1/2}$. If we consider motion along the dimension $-x^*$ solely, for instance, then we must let $-y^* = -z^* = 0$, to have $d = ((-x^*)^2)^{1/2} = -x^*$. Likewise the worldline element of special relativity in the negative universe is, $ds^* = ((-ct^*)^2 - (-x^*)^2 - (-y^*)^2 - (-z^*)^2)^{1/2}$. If we let $-x^* = -y^* = -z^* = 0$, for propagation in time only, then $ds^* = ((-ct^*)^2)^{1/2} = -ct^*$. Interestingly the negative worldline element ($ds^* < 0$) in the negative universe is the negative root $-ds$ of the quadratic line element ds^2, which is usually discarded, since it conveys nothing to us from the point of view of experience in the positive universe.

4.2 Non-separation of Symmetry-partner Points in Spacetimes in the Positive and Negative Universes

It shall be shown here that a point in spacetime in our (or positive) universe is effectively not separated in space or along the time dimension from its symmetry-partner point in spacetime in the negative universe, for every pair of symmetry-partner points in spacetimes in the two universes. Now let us consider the larger spacetime of combined positive and negative universes (Fig. 2b), which is re-illustrated as Fig. 5.

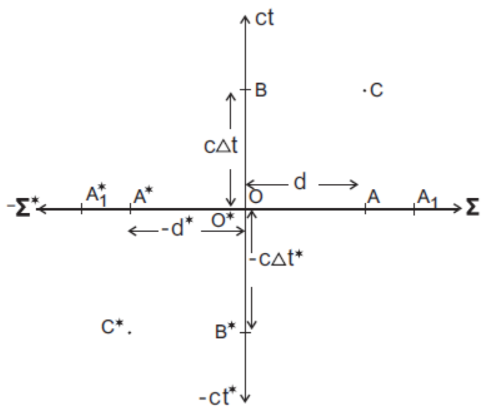

Fig. 5. Combined positive and negative Minkowski's spaces of the positive and negative universes

Point A* in the negative Euclidean 3-space $-\Sigma^*$ of the negative universe is the symmetry-partner to point A in the positive Euclidean 3-space Σ of the positive universe. Point B* in the negative time dimension $-ct^*$ of the negative universe is the symmetry-partner to point B in the positive time dimension ct of the positive universe. Hence points C* and C are symmetry-partner points on four-dimensional spacetimes in the two universes.

Now let points A and O in the positive 3-space Σ of the positive universe be separated by a positive distance d, since distances in space are positive scalar quantities in the positive universe. Then the symmetry-partner points A* and O* in the negative 3-space $-\Sigma^*$ of the negative universe are separated by negative distance $-d^*$, since distances in space are negative scalar quantities in the negative universe. Hence the distance in 3-space between point A in the positive universe and its symmetry-partner point A* in the negative universe is, $d-d^* = 0$, since d and $-d^*$ are equal in magnitude. This implies that the symmetry-partner points A and A* are effectively separated by zero distance in 3-space with respect to observers (or people) in the positive and negative universes.

Likewise, if the interval of the positive time dimension ct between point O and point B is the positive quantity $c\Delta t$, then the interval of the negative time dimension $-ct^*$ between point O* and point B* is the negative quantity $-c\Delta t^*$, since intervals of time are negative quantities in the negative universe. Hence the interval of time dimension between point B in ct in the positive universe and its symmetry-partner point B* in $-ct^*$ in the negative universe is, $c\Delta t - c\Delta t^* = 0$. This implies that the symmetry-partner points B and B* in the time dimensions are effectively separated by zero interval of time dimension with respect to observers (or people) in the positive and negative universes. It then follows that the time t of an event in the positive universe is effectively separated by zero time interval from the time $-t^*$ of the symmetry-partner event in the negative universe. Thus an event in the positive universe and its symmetry-partner in the negative universe occur simultaneously.

It follows from the foregoing two paragraphs that symmetry-partner points C and C* in spacetimes in the positive and negative universes are not separated in space or time, and this is true for every pair of symmetry-partner points in spacetimes in the two universes. Although symmetry-partner points in spacetimes in the positive and negative universes coincide at the same point, or are not separated, they do not touch, because they exist in different spacetimes.

Now let an object located at point A in the 3-space Σ of our (or positive) universe move to another point A_1 in Σ shown in Fig. 5. The symmetry-partner object in the negative universe will simultaneously move from point A* to point A_1^* in the 3-space $-\Sigma^*$ of the negative universe, where point A_1^* in $-\Sigma^*$ is the symmetry-partner to point A_1 in Σ. Again the symmetry-partner objects now located at points A_1 and A_1^* are effectively separated by zero distance in space. The non-separated symmetry-partner objects have effectively moved together in their respective 3-spaces from point A to point A_1 in Σ with respect to observers (or peoples) in our universe, while they have effectively moved together in their respective 3-spaces from point A^* to point A_1^* in $-\Sigma^*$ with respect to observers (or peoples) in the negative universe.

One consequence of the foregoing is that local spacetime coordinates $(\Sigma, ct) \equiv (x, y, z, ct)$, originating from a point O in the positive universe and the symmetry-partner local spacetime coordinates $(-\Sigma^*, -ct^*) \equiv (-x^*, -y^*, -z^*, -ct^*)$, originating from the symmetry-partner point O* in spacetime in the negative universe, can be drawn from the same point on paper, as done in Fig. 5, and geometrical construction whose predictions will conform with observation or experiment in each of the two universes, can be based on this in the two-world picture, as shall be done in the rest of this section.

4.3 Introducing a Flat Two-dimensional Intrinsic Spacetime Underlying the Flat Four-dimensional Spacetime

Since it is logically required for this paper to propagate beyond this point and since space limitation does not permit the presentation of its derivation, which shall be elsewhere, we shall present (as *ansatz*) at this point certain flat two-dimensional intrinsic spacetime, to be denoted by $(\varnothing\rho, \varnothing c\varnothing t)$, where $\varnothing\rho$ is intrinsic space dimension and $\varnothing c\varnothing t$ is intrinsic time dimension. The two-dimensional intrinsic spacetime $(\varnothing\rho, \varnothing c\varnothing t)$ underlies the flat four-dimensional spacetime (the Minkowski space) — the manifold — of the special theory of relativity, usually denoted by (x^0, x^1, x^2, x^3); $x^0 = ct$, but which shall be denoted by (Σ, ct) in this article for convenience, where Σ is the Euclidean 3-space with dimensions x^1, x^2 and x^3. The spacetime coordinates of all frames exist in the four-dimensional spacetime manifold (Σ, ct).

Every particle or object with inertial mass m in the Euclidean 3-space Σ has its line of intrinsic inertial mass, to be denoted by $\varnothing m$, underlying it in the one-dimensional intrinsic space $\varnothing\rho$. The one-dimensional intrinsic space $\varnothing\rho$ underlying the Euclidean 3-space Σ is an isotropic dimension with no unique orientation in Σ with respect to observers in spacetime (Σ, ct). This means that $\varnothing\rho$ can be considered to be orientated along a non-unique direction in Σ with respect to these observers. The straight line intrinsic time dimension $\varnothing c\varnothing t$ likewise lies parallel to the straight line time dimension ct along the vertical, in the graphical presentation of the flat spacetime of SR of Fig. 2 or Fig. 5.

If we temporarily consider the Euclidean 3-space Σ as a hyper-surface, $t = const.$, represented by a plane-surface along the horizontal (instead of a line along the horizontal in the previous diagrams), and the time dimension ct as a vertical pseudo-orthogonal line to the hyper-surface, then the graphical representation of the flat four-dimensional spacetime (Σ, ct) and its underlying flat two-dimensional intrinsic spacetime $(\varnothing\rho, \varnothing c\varnothing t)$ in the context of SR, described in the foregoing paragraph is depicted in Fig. 6a.

Figure 6a is valid with respect to observers on the flat physical four-dimensional spacetime (Σ, ct). The one-dimensional intrinsic masses of all particles and objects are aligned along the singular isotropic one-dimensional intrinsic space $\varnothing\rho$, whose inertial masses are scattered arbitrarily in the physical Euclidean 3-space Σ, with respect to these observers (in (Σ, ct)), as illustrated for three such particles and objects in Fig. 6a.

On the other hand, the intrinsic space is actually a flat three-dimensional manifold, to be denoted by $\varnothing\Sigma$, with mutually orthogonal intrinsic dimensions, $\varnothing x^1, \varnothing x^2$ and $\varnothing x^3$, with respect to intrinsic-mass-observers in $(\varnothing\Sigma, \varnothing c\varnothing t)$. The intrinsic masses $\varnothing m$ of particles and objects are likewise three-dimensional with respect to the intrinsic-mass-observers in $(\varnothing\Sigma, \varnothing c\varnothing t)$. The intrinsic mass $\varnothing m$ of a particle or object in the intrinsic space $\varnothing\Sigma$ lies directly underneath the inertial mass m of the particle or object in the physical Euclidean 3-space Σ, as illustrated for three such particles or objects in Fig. 6b.

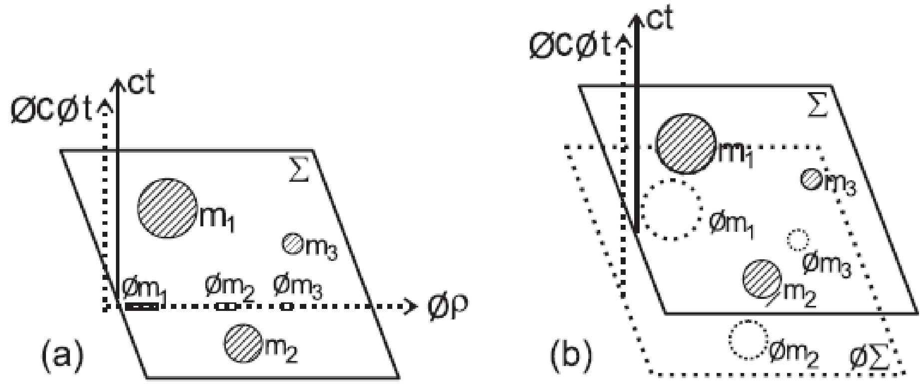

Fig. 6. (a) The flat 4-dimensional spacetime and its underlying flat 2-dimensional intrinsic spacetime with the inertial masses of three objects scattered in the Euclidean 3-space and their intrinsic inertial masses aligned along the one-dimensional isotropic intrinsic space with respect to observers in spacetime. (b) The flat 2-dimensional intrinsic spacetime with respect to observers in spacetime in (a) is a flat four-dimensional intrinsic spacetime containing intrinsic inertial masses of particles and objects in 3-dimensional intrinsic space with respect to hypothetical intrinsic-mass-observers in intrinsic spacetime

The flat four-dimensional physical spacetime (Σ, ct) containing the inertial masses m of particles and objects in the Euclidean 3-space Σ is the outward manifestation of the flat four-dimensional intrinsic spacetime $(\varnothing\Sigma, \varnothing c\varnothing t)$ containing three-dimensional intrinsic masses $\varnothing m$ of the particles and objects in $\varnothing\Sigma$ (with respect to intrinsic-mass-observers in $(\varnothing\Sigma, \varnothing c\varnothing t)$) in Fig. 6b. It is due to the fact that the flat three-dimensional intrinsic space $\varnothing\Sigma$ is an isotropic space, that is, all directions in $\varnothing\Sigma$ are the same with respect to observers in the physical spacetime (Σ, ct) that the dimensions $\varnothing x^1, \varnothing x^2$ and $\varnothing x^3$ of $\varnothing\Sigma$, which are mutually orthogonal with respect to the intrinsic-mass-observers in $(\varnothing\Sigma, \varnothing c\varnothing t)$, are effectively directed along the same non-unique direction in $\varnothing\Sigma$, thereby effectively constituting a singular one-dimensional intrinsic space (or an intrinsic space dimension) $\varnothing\rho$ with no unique orientation in $\varnothing\Sigma$ and, consequently, with no unique orientation in the physical Euclidean 3-space Σ overlying $\varnothing\Sigma$, with respect to observers on the flat physical spacetime (Σ, ct), as illustrated in Fig. 6a.

It is appropriate to expand the preceding paragraph by discussing the conceptual contraction procedure used to contract the intrinsic 3-space $\varnothing\Sigma$, with respect to observers in it in Fig. 6b, to the one-dimensional isotropic intrinsic space $\varnothing\rho$, with respect to observers in Σ in Fig. 6a.

Now the symbol $\varnothing$ in $\varnothing\Sigma$, $\varnothing c\varnothing t$, $\varnothing\rho$ and $\varnothing m$, $\varnothing M$ in Fig. 6b, is used to denote “empty” (as in empty set), “null”, or the suffix “no-” (as in nothing). Thus $\varnothing\Sigma$ is “empty space” or “null space” or “no-space”; $\varnothing m$ is “empty mass” or “null mass” or “no-mass”; $\varnothing c\varnothing t$ is “empty time dimension” or “null time dimension” or “no-time dimension”, etc. However “no-space”, “no-mass”, “no-time dimension”, etc, are preferred and are adulterated as nospace, nomass, notime dimension, etc.

Nospace $\varnothing\Sigma$ is also alternatively referred to as intrinsic space, nomass $\varnothing m$ as intrinsic mass, notime dimension $\varnothing c\varnothing t$ as inttinsic time dimension, nospeed $\varnothing v$ as intrinsic speed, and any other noparameter $\varnothing Q$ as intrinsic parameter. Hence the symbol $\varnothing$ denotes “intrinsic” or “no-”. Thus $\varnothing\beta$ must be pronounced as nobeta or intrinsic beta. Intrinsic spacetime and intrinsic parameters are used uniformly in this paper.

Any extent of nospace (or intrinsic space) $\varnothing\Sigma$ is equivalent to zero extent of space Σ, $(\varnothing\Sigma \equiv 0 \times \Sigma)$; any magnitude of nomass (or intrinsic mass) $\varnothing m$ is equivalent to zero magnitude of mass m, $(\varnothing m \equiv 0 \times m)$; any extent of notime (or intrinsic time) $\varnothing t$ is equivalent to zero extent of time t, $(\varnothing t \equiv 0 \times t)$; and any quantity of a noparameter (or an intrinsic parameter) $\varnothing Q$ is equivalent to zero quantity of the parameter Q, $(\varnothing Q \equiv 0 \times Q)$.

The preceding paragraph makes nospace (or intrinsic space), nomass (or intrinsic mass), notime (or intrinsic time), nospeed (or intrinsic speed), or any noparameter (or intrinsic parameter) in nospace-notime (or intrinsic spacetime) non-observable and non-detectable to observers in spacetime. Whereas no-observers (or intrinsic observers) in intrinsic spacetime can observe intrinsic spacetime and intrinsic parameters in intrinsic spacetime.

Now a given pair of distinct directions in 3-dimensional nospace (or intrinsic space) $\varnothing\Sigma$, which are separated by non-zero noangles (or intrinsic angles) $\varnothing\alpha$, $\varnothing\beta$ and $\varnothing\gamma$, with respect to no-observer (or intrinsic observers) in $\varnothing\Sigma$, are separated by zero angles, $\varnothing\alpha = 0\times\alpha$, $\varnothing\beta = 0\times\beta$ and $\varnothing\gamma = 0 \times \gamma$, with respect to observers in 3-space Σ. Consequently the pair of directions in $\varnothing\Sigma$ with respect to intrinsic observers in $\varnothing\Sigma$, are perfectly the same relative to all observers in the 3-space Σ. And this is true for every pair of distinct directions in $\varnothing\Sigma$ with respect to intrinsic observers in $\varnothing\Sigma$.

It follows from the preceding paragraph that the 3-dimensional nospace (or intrinsic space) $\varnothing\Sigma$, with respect to intrinsic observers in it, is perfectly isotropic (i.e. all directions in $\varnothing\Sigma$ are perfectly the same) relative to all observers in 3-space Σ. Consequently the 3-dimensional intrinsic space $\varnothing\Sigma$ with respect to intrinsic observers in it, is naturally contracted to a one-dimensional intrinsic space, denoted by $\varnothing\rho$, relative to all observers in 3-space Σ.

The one-dimensional nospace (or intrinsic space) $\varnothing\rho$ has no unique orientation in $\varnothing\Sigma$ that contracts to it, relative to observers in Σ. Or the singular intrinsic dimension $\varnothing\rho$ can be considered to be effectively orientated along every direction in $\varnothing\Sigma$ that contracts to it, relative to observers in Σ. Since $\varnothing\Sigma$ is embedded in the 3-space Σ, then $\varnothing\rho$ is embedded in Σ, but has no unique orientation in Σ, relative to observers in Σ. Or $\varnothing\rho$ is a singular isotropic intrinsic space dimension embedded in Σ, which is effectively orientated along every direction in Σ, relative to observers in Σ. This is the basis for the transformation of Fig. 6b that is valid with respect to intrinsic observers in $\varnothing\Sigma$ and observers in Σ, to Fig. 6a that is valid with respect to observers in Σ solely.

As follows from the above, Fig. 6a is the correct diagram with respect to observers in spacetime (Σ, ct). It is still valid to say that the flat four-dimensional spacetime (Σ, ct) is the outward (or physical) manifestation of the flat two-dimensional intrinsic spacetime $(\varnothing\rho, \varnothing c\varnothing t)$ and that the inertial mass m of a particle or object in Σ is the outward (or physical) manifestation of the line of intrinsic mass $\varnothing m$ of the particle or object in $\varnothing\rho$, with respect to observers in (Σ, ct) in Fig. 6a. Observers on the flat four-dimensional spacetime (Σ, ct) must formulate intrinsic physics in intrinsic spacetime as two-dimensional intrinsic physics on flat intrinsic spacetime $(\varnothing\rho, \varnothing c\varnothing t)$.

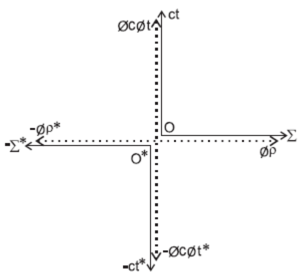

Fig. 7. Combined flat four-dimensional spacetimes and underlying combined flat two-dimensional intrinsic spacetimes of the positive and negative universes

It is for convenience that the three-dimensional Euclidean space Σ shall be represented by a line along the horizontal as done in Figs. 2a and 2b and Fig. 5 and as shall be done in the rest of this article, instead of a plane surface along the horizontal in Figs. 6a and 6b. Thus the flat four-dimensional spacetime and its underlying flat two-dimensional intrinsic spacetime shall be presented graphically in the two-world picture as Fig. 7. The origins O and O* are not actually separated (although they do not touch), contrary to their separation in Fig. 7.

Figure 7 is Fig. 5 modified by incorporating the flat two-dimensional intrinsic spacetimes underlying the flat four-dimensional spacetimes of the positive and negative universes into Fig. 5. Figure 7 is a fuller diagram than Fig. 5. As mentioned earlier, the intrinsic spacetime and intrinsic parameters in it along with their properties and notations shall be derived elsewhere.

The intrinsic spacetime dimensions $\varnothing\rho$ and $\varnothing c\varnothing t$ and one-dimensional intrinsic masses $\varnothing m$ of particles and objects in the intrinsic space $\varnothing\rho$ are hidden (or non-observable) to observers on the flat spacetime (Σ, ct). The symbol $\varnothing$ attached to the intrinsic dimensions, intrinsic coordinates and intrinsic masses is used to indicate their intrinsic (or hidden) natures with respect to observers in spacetime. By removing the symbol $\varnothing$ from the flat two-dimensional intrinsic spacetime $(\varnothing\rho, \varnothing c\varnothing t)$, one obtains the observed flat four-dimensional spacetime (Σ, ct) and by removing $\varnothing$ from the one-dimensional intrinsic mass $\varnothing m$ in $\varnothing\rho$, one obtains the observed three-dimensional mass m in the Euclidean 3-space Σ.

As the mass m moves at velocity $\vec{v}$ in the Euclidean 3-space Σ of the flat four-dimensional spacetime (Σ, ct), relative to an observer in (Σ, ct), the intrinsic mass $\varnothing m$ performs intrinsic motion at intrinsic speed $\varnothing v$ in the one-dimensional intrinsic space $\varnothing\rho$ of the flat two-dimensional intrinsic spacetime $(\varnothing\rho, \varnothing c\varnothing t)$, relative to the observer in (Σ, ct), where $|\varnothing v| = |\vec{v}|$. The mass m of a particle in Σ and its intrinsic mass $\varnothing m$ in $\varnothing\rho$ are together always in their respective spaces, irrespective of whether m is in motion or at rest relative to the observer.

Finally on the *ansatz* being presented in this subsection: the intrinsic motion of the intrinsic rest mass $\varnothing m_0$ of a particle at intrinsic speed $\varnothing v$ in a primed intrinsic frame, to be denonated by $(\varnothing\tilde{x}', \varnothing c\varnothing\tilde{t}')$, relative to the unprimed intrinsic frame, to be denoted by, $(\varnothing\tilde{x}, \varnothing c\varnothing\tilde{t})$, on the flat two-dimensional intrinsic spacetime $(\varnothing\rho, \varnothing c\varnothing t)$, pertains to two-dimensional intrinsic special theory of relativity, to be denoted by $\varnothing$SR, while the corresponding motion of the rest mass m_0 of the particle at velocity $\vec{v}$ in the primed frame, to be denoted by $(\tilde{x}', \tilde{y}', \tilde{z}', c\tilde{t}')$, relative to the unprimed frame, to be denoted by, $(\tilde{x}, \tilde{y}, \tilde{z}, c\tilde{t})$, on the flat four-dimensional spacetime (Σ, ct), pertains to the special theory of relativity (SR) as usual. The SR on flat four-dimensional spacetime (Σ, ct) is mere outward manifestation of $\varnothing$SR on the underlying flat two-dimensional intrinsic spacetime $(\varnothing\rho, \varnothing c\varnothing t)$.

The intrinsic motion at intrinsic speed $\varnothing v$ of the intrinsic rest mass $\varnothing m_0$ of a particle in the primed intrinsic frame $(\varnothing\tilde{x}', \varnothing c\varnothing\tilde{t}')$, relative to the unprimed intrinsic frame $(\varnothing\tilde{x}, \varnothing c\varnothing\tilde{t})$, gives rise to rotation of the primed intrinsic coordinates, $\varnothing\tilde{x}'$ and $\varnothing c\varnothing\tilde{t}'$, relative to the unprimed intrinsic coordinates, $\varnothing\tilde{x}$ and $\varnothing c\varnothing\tilde{t}$, on the vertical two-dimensional intrinsic spacetime hyperplane (i. e. on the vertical $(\varnothing\rho, \varnothing c\varnothing t)$-hyperplane) in Fig. 7. It is to be observed that the rotations of the intrinsic coordinates $\varnothing\tilde{x}'$ and $\varnothing c\varnothing\tilde{t}'$ can take place on the vertical intrinsic spacetime hyperplane only in Fig. 6a or Fig. 7.

Two-dimensional intrinsic spacetime diagram and its inverse must be drawn on the vertical $(\varnothing\rho, \varnothing c\varnothing t)$-plane in Fig. 7 in the two-world picture and intrinsic Lorentz transformation ($\varnothing$LT) and its inverse derived from them in the context of $\varnothing$SR. The intrinsic Lorentz invariance ($\varnothing$LI) on the flat two-dimensional intrinsic spacetime must be validated and every result in the context of the two-dimensional intrinsic special theory of relativity, each of which has its counterpart in SR, must be derived from the $\varnothing$LT and its inverse in the manner that the results of SR are derived from the LT and its inverse.

Once $\varnothing$SR has been formulated as described above then SR, being mere outward (or physical) manifestation on the flat four-dimensional spacetime (Σ, ct) of $\varnothing$SR on the flat two-dimensional intrinsic spacetime $(\varnothing\rho, \varnothing c\varnothing t)$, the results of SR namely, the LT and its inverse, the Lorentz invariance (LI) on the flat four-dimensional spacetime and every other results of SR, can be written directly from the corresponding results of ∅SR, without having to draw spacetime diagrams involving the rotation of the coordinates, $\tilde{x}'$, $\tilde{y}'$, $\tilde{z}'$ and $c\tilde{t}'$, of the primed frame relative to the coordinates, $\tilde{x}$, $\tilde{y}$, $\tilde{z}$ and $c\tilde{t}$, of the unprimed frame on the flat four-dimensional spacetime (Σ, ct), in the context of SR. This procedure shall be demonstrated in the next sub-section.

4.4 New Spacetime/intrinsic Spacetime Geometrical Representation of Lorentz Transformation/intrinsic Lorentz Transformation in the Two-world Picture

The classical (or Newtonian) gravitational field exists on the flat four-dimensional proper metric spacetime (with constant Lorentzian metric tensor), to be denoted by (Σ', ct'). The rest masses M_0 of classical gravitational field sources and m_0 of particles and objects, exist in the proper Euclidean 3-space Σ'. It is postulated in the general theory of relativity (GR) that the flat proper spacetime (Σ', ct'), usually denoted by (x'^0, x'^1, x'^2, x'^3) in GR, evolves into a curved ('relativistic') spacetime (Σ, ct), usually denoted by (x^0, x^1, x^2, x^3) in GR, with pseudo-Riemannian metric tensor $g_{\mu\nu}$ in a relativistic gravitational field, see pages 111 – 120 of [26].

The rest masses m_0 and M_0 of particles and bodies on the flat proper metric spacetime (Σ', ct') in the classical gravitational field, evolve into inertial masses m and M on the prescribed curved spacetime (Σ, ct), but the principle of equivalence provides that, $M = M_0$ and $m = m_0$, in the context of GR [1]. The states of affairs with the evolution of the flat proper spacetime (Σ', ct') into the 'relativistic' spacetime (Σ, ct) and evolutions of rest masses m_0 and M_0 on the flat (Σ', ct'), in the classical gravitational field, into inertial masses m and M in (Σ, ct) in a relativistic gravitational field, in the context of the present theory on a two-world background, are issues for investigation elsewhere.

The rest mass m_0 of a particle or object can be in low velocity relative motion $(v/c << 1)$ on the flat proper metric spacetime (Σ', ct') in the context of classical mechanics (CM). Or it can be in large velocity relative motion $(v/c \approx 1)$ on (Σ', ct') in the context of SR, in the classical gravitational field. We shall restrict SR to the flat four-dimensional proper metric spacetime (Σ', ct'), with the assumption that only classical gravitational field exists, in this article. The intrinsic special theory of relativity (∅SR) shall consequently be restricted to the flat two-dimensional proper intrinsic metric spacetime $(\varnothing\rho', \varnothing c\varnothing t')$ underlying the flat (Σ', ct') in this article.

Thus let us prescribe a frame of reference with extended straight line primed affine coordinates, $\tilde{x}', \tilde{y}', \tilde{z}'$ and $c\tilde{t}'$. respectively, which is embedded in the flat four-dimensional proper metric spacetime (Σ', ct') of our (or positive) universe. Let a three-dimensional observer (or a 3-observer), Peter, say, be located in the metric proper Euclidean 3-space Σ'. Corresponding to the 3-dimensional observer Peter in the proper metric Euclidean 3-space Σ', we shall prescribe a one-dimensional observer (or 1-observer) in the straight line proper metric time dimension ct', to be denoted by Peter^0. Thus there is the 4-observer (Peter, Peter^0) on the flat four-dimensional proper metric spacetime (Σ', ct') of our universe.

The existence of 1-observers and 1-objects in the time is consistent with the known four-dimensionality of objects in the four-dimensional spacetime in SR. Possible more formal justification for it in the two-world picture is worth investigation elsewhere.

Corresponding to the primed affine frame with extended straight line affine coordinates, $\tilde{x}', \tilde{y}', \tilde{z}'$, and $c\tilde{t}'$, prescribed on the flat four-dimensional proper metric spacetime (Σ', ct') above, there is the primed intrinsic affine frame with extended straight line intrinsic affine coordinates, $\varnothing\tilde{x}'$ and $\varnothing c\varnothing\tilde{t}'$, on the flat two-dimensional proper intrinsic metric spacetime $(\varnothing\rho', \varnothing c\varnothing t')$ underlying (Σ', ct'), in the first quadrant in Fig. 7. And corresponding to the 4-observer (Peter, Peter^0) on the flat four-dimensional proper metric spacetime (Σ, ct), there is the intrinsic 2-observer (∅Peter, ∅Peter^0) on the flat two-dimensional proper intrinsic metric spacetime $(\varnothing\rho', \varnothing c\varnothing t')$.

Before proceeding further, let us shine some light on the concepts of metric spacetime and affine spacetime, which have been introduced in the preceding two paragraphs. From its literal definition, a metric spacetime is a ponderable, that is, observable and measurable spacetime, while an affine spacetime is a non-ponderable, that is, non-observable and non-measurable spacetime (i.e. without metric quality).

As known, the proper physical four-dimensional spacetime, usually denoted by $(x^{0'}, x^{1'}, x^{2'}, x^{3'})$, where, $x^{0'} = ct'$, is the proper time dimension, but which is being denoted by (Σ', ct') for convenience in this article, where Σ' is the flat proper 3-space with dimensions, x', y' and z', in the Cartesian coordinate system, is the proper metric 3-space. The (Σ', ct') is flat with the Lorentzian metric tensor in the classical gravitational field. The rest masses, m_0 or M_0, of particles and bodies are contained in the proper metric 3-space Σ', hence they exist and move on the flat four-dimensional proper metric spacetime (Σ', ct'), with the assumed absence of relativistic gravitational field. The coordinates of the proper metric spacetime shall be denoted by x', y', z' and ct' (in the Cartesian system of coordinates of 3-space) in this article.

The flat four-dimensional intrinsic spacetime $(\varnothing\Sigma', \varnothing c\varnothing t')$ with respect to intrinsic observers in it in Fig. 6b, is ponderable, that is, it is observable and measurable to intrinsic observers in $(\varnothing\Sigma', \varnothing c\varnothing t')$. The $(\varnothing\Sigma', \varnothing c\varnothing t')$ is consequently a metric spacetime with respect to the intrinsic observers in $(\varnothing\Sigma', \varnothing c\varnothing t')$, while it is an intrinsic metric spacetime with respect to observers in the proper metric spacetime (Σ', ct').

The flat four-dimensional proper intrinsic metric spacetime $(\varnothing\Sigma', \varnothing c\varnothing t')$, with respect to observers in the metric spacetime (Σ', ct'), naturally contracts to flat two-dimensional proper intrinsic metric spacetime $(\varnothing\rho', \varnothing c\varnothing t')$, with respect observers in (Σ', ct') in Fig. 6b. The one-dimensional intrinsic rest masses, $\varnothing m_0$ or $\varnothing M_0$, of particles and bodies are contained in the one-dimensional proper intrinsic metric space $\varnothing\rho'$. Hence they exist and undergo intrinsic motion on the flat $(\varnothing\rho', \varnothing c\varnothing t')$ in the classical gravitational field. The intrinsic metric coordinates of $(\varnothing\rho', \varnothing c\varnothing t')$ shall be denoted by $\varnothing x'$ and $\varnothing c\varnothing t'$.

On the other hand, the proper affine space coordinates, $\tilde{x}', \tilde{y}'$ and $\tilde{z}'$, in Cartesian coordinates of affine 3-space, of the proper (or primed) affine frame attached to the rest mass m_0 of a particle in motion, are loci of points traced by m_0 on the flat proper metric 3-space Σ', such that, $\tilde{x}'$, $\tilde{y}'$ and $\tilde{z}'$, lie along the metric coordinates, x', y' and z', of the proper metric 3-space Σ' respectively. The proper affine time coordinate $c\tilde{t}'$ is a locus of point that is simultaneously traced along the proper metric time dimension ct' as the m_0 moves.

Thus the affine spacetime coordinates, $\tilde{x}'$, $\tilde{y}'$, $\tilde{z}'$ and $c\tilde{t}'$, which constitute the proper (or primed) affine frame $(\tilde{x}', \tilde{y}', \tilde{z}', c\tilde{t}')$, of the rest mass m_0 of the particle in motion, are loci of points traced along the respective metric spacetime coordinates, x', y', z' and ct' of the flat four-dimensional proper metric spacetime (Σ', ct'), as m_0 moves.

The proper (or primed) intrinsic affine spacetime coordinates, $\varnothing\tilde{x}'$ and $\varnothing c\varnothing\tilde{t}'$, are likewise loci of points traced along the proper intrinsic metric spacetime coordinates, $\varnothing x'$ and $\varnothing c\varnothing t'$, respectively of the flat proper intrinsic metric spacetime $(\varnothing\rho', \varnothing c\varnothing t')$, by the intrinsic rest mass $\varnothing m_0$ of the particle, as it undergoes intrinsic motion.

The coordinates of a primed affine spacetime shall be differentiated from those of a metric spacetime by an additional over-head tilde label as, $\tilde{x}', \tilde{y}', \tilde{z}'$ and $c\tilde{t}'$. These are mere mathematical entities without physical (or metrical) quality, used to identify the positions and to track the motions of the rest masses of material particles and bodies (as mass-points), relative to a specified origin on the flat proper metric spacetime (Σ', ct'). The primed affine coordinates, $\tilde{x}', \tilde{y}', \tilde{z}'$ and $c\tilde{t}'$, are straight line coordinates that can be of any extensions on the flat proper metric spacetime. The loci of points (or affine coordinates) of a material point through a metric spacetime are without metrical quality. The extended three-dimensional affine space cannot hold matter (or mass) of a particle or object. However the mass of a particle or body contained in a volume of the metric space can propagate along affine spacetime coordinates (or in an affine frame) through metric spacetime.

The perfect symmetry of state between the positive and negative universes prescribed earlier in this article, requires that an identical symmetry-partner primed affine frames with extended negative straight line affine coordinates, $-\tilde{x}'^{*}, -\tilde{y}'^{*}, -\tilde{z}'^{*}$ and $-c\tilde{t}'^{*}$, be prescribed on the flat four-dimensional proper metric spacetime $(-\Sigma'^{*}, -ct'^{*})$ and an underlying symmetry-partner primed intrinsic affine frame with extended negative straight line intrinsic affine coordinates, $-\varnothing\tilde{x}'^{*}$ and $-\varnothing c\varnothing\tilde{t}'^{*}$, be prescribed on flat two-dimensional proper intrinsic metric spacetime $(-\varnothing\rho'^{*}, -\varnothing c\varnothing t'^{*})$ that underlies $(-\Sigma'^{*}, -ct'^{*})$ in the third quadrant (or in negative universe) in Fig. 7. There is likewise the symmetry-partner 4-observer* (Peter*, Peter^{0}*) on the flat proper metric spacetime $(-\Sigma'^{*}, -ct'^{*})$ and the symmetry-partner intrinsic 2-observer (∅Peter*, ∅Peter^{0}*) on the flat proper intrinsic metric spacetime $(-\varnothing\rho'^{*}, -\varnothing c\varnothing t'^{*})$ in the negative universe.

Initially the primed affine frames, $(\tilde{x}', \tilde{y}', \tilde{z}', c\tilde{t}')$ and $(-\tilde{x}'^{*}, -\tilde{y}'^{*}, -\tilde{z}'^{*}, -c\tilde{t}'^{*})$, are at rest relative to the 'stationary' 4-observer (Peter, Peter^{0}) in the flat four-dimensional proper metric spacetime (Σ', ct') and 'stationary' 4-observer* (Peter*, Peter^{0}*) in the flat proper metric spacetime $(-\Sigma'^{*}, -ct'^{*})$ respectively. Consequently the primed intrinsic affine frames $(\varnothing\tilde{x}', \varnothing c\varnothing\tilde{t}')$ embedded in the flat two-dimensional proper intrinsic metric spacetime $(\varnothing\rho', \varnothing c\varnothing t')$ and $(-\varnothing\tilde{x}'^{*}, -\varnothing c\varnothing\tilde{t}'^{*})$ embedded in the flat two-dimensional proper intrinsic metric spacetime $(-\varnothing\rho'^{*}, -\varnothing c\varnothing t'^{*})$, are at rest relative to the 'stationary' intrinsic 2-observer (∅Peter, ∅Peter^{0}) in $(\varnothing\rho', \varnothing c\varnothing t')$ in the our universe and (∅Peter*, ∅Peter^{0}*) in $(-\varnothing\rho'^{*}, -\varnothing c\varnothing t'^{*})$ in the negative universe respectively initially. The primed intrinsic affine frames, $(\varnothing\tilde{x}', \varnothing c\varnothing\tilde{t}')$ and $(-\varnothing\tilde{x}'^{*}, -\varnothing c\varnothing\tilde{t}'^{*})$, are consequently at rest relative to the 'stationary' 4-observer (Peter, Peter^{0}) in (Σ', ct') overlying proper intrinsic metric spacetime $(\varnothing\rho', \varnothing c\varnothing t')$ and the 'stationary' 4-observer* (Peter*, Peter^{0}*) in $(-\Sigma'^{*}, -ct'^{*})$ overlying $(-\varnothing\rho'^{*}, -\varnothing c\varnothing t'^{*})$ respectively initially.

Implied by the preceding paragraph is the fact that the primed affine frame $(\tilde{x}', \tilde{y}', \tilde{z}', c\tilde{t}')$ lies in (or is embedded in) the flat proper metric spacetime (Σ', ct'), such that the extended straight line affine coordinates, $\tilde{x}', \tilde{y}', \tilde{z}'$ and $c\tilde{t}'$, lie along the extended straight line metric coordinates, x', y', z' and ct', respectively, of the flat proper metric spacetime (Σ', ct') in our universe initially, and the primed affine frame $(-\tilde{x}'^{*}, -\tilde{y}'^{*}, -\tilde{z}'^{*}, -c\tilde{t}'^{*})$ lies (or is embedded) in the flat proper metric spacetime $(-\Sigma'^{*}, -ct'^{*})$, such that the extended affine coordinates, $-\tilde{x}'^{*}, -\tilde{y}'^{*}, -\tilde{z}'^{*}$ and $-c\tilde{t}'^{*}$, lie along the extended straight line metric coordinates, $-x'^{*}, -y'^{*}, -z'^{*}$ and $-ct'^{*}$, respectively, of the flat proper metric spacetime $(-\Sigma'^{*}, -ct'^{*})$ in the negative universe initially, when the symmetry-partner particles are yet at rest relative to the symmetry-partner 'stationary' observers on the flat four-dimensional proper metric spacetimes in our universe and the negative universe.

The extended straight line primed intrinsic affine coordinates, $\varnothing\tilde{x}'$ and $\varnothing c\varnothing\tilde{t}'$, of the primed intrinsic affine frame $(\varnothing\tilde{x}', \varnothing c\varnothing\tilde{t}')$, lie along the extended straight line intrinsic metric coordinates, $\varnothing x'$ and $\varnothing c\varnothing t'$, respectively of the flat proper intrinsic metric spacetime $(\varnothing\rho', \varnothing c\varnothing t')$ in our universe initially. The extended intrinsic affine coordinates, $-\varnothing\tilde{x}'^{*}$ and $-\varnothing c\varnothing\tilde{t}'^{*}$, of the primed intrinsic affine frame $(-\varnothing\tilde{x}'^{*}, -\varnothing c\varnothing\tilde{t}'^{*})$, likewise lie along the extended intrinsic metric coordinates, $-\varnothing x'^{*}$ and $-\varnothing c\varnothing t'^{*}$, respectively of the flat proper intrinsic metric spacetime $(-\varnothing\rho'^{*}, -\varnothing c\varnothing t'^{*})$ in the negative universe initially, when the symmetry-partner particles or objects are yet at rest relative to the symmetry-partner observers in the flat four-dimensional proper metric spacetimes in our universe and the negative universe.

The initial state of rest of the symmetry-partner particles relative to the symmetry-partner 'stationary' observers in our universe and the negative universe, described in the preceding three paragraphs, will persist for as long as no forces act on the particles. However we shall allow identical (symmetry-partner) impressed forces to act on the initially 'stationary' symmetry-partner particles, which accelerate them to identical velocities $\vec{v}$, at which point the forces are removed. The symmetry-partner particles thereby continue to move at the identical velocity $\vec{v}$ relative to the symmetry-partner 'stationary' observers in the two universes.

Let us now consider the propagation at a constant speed v of the rest mass m_0 of a particle along the coordinate $\tilde{x}'$ of the primed affine frame $(\tilde{x}', \tilde{y}', \tilde{z}', c\tilde{t}')$ relative to the 'stationary' 3-observer Peter in the proper metric 3-space Σ' in the positive universe (or our universe). Correspondingly, the intrinsic rest mass $\varnothing m_0$ of the particle is in intrinsic motion at intrinsic speed $\varnothing v$ along the intrinsic affine space coordinate $\varnothing\tilde{x}'$ of the primed intrinsic affine frame $(\varnothing\tilde{x}', \varnothing c\varnothing\tilde{t}')$ relative to the intrinsic 1-observer $\varnothing$Peter in the one-dimensional proper intrinsic metric space $\varnothing\rho'$ and, consequently, relative to the 3-observer Peter in Σ' overlying $\varnothing\rho'$.

The intrinsic motion at intrinsic speed $\varnothing v$ of the intrinsic rest mass $\varnothing m_0$ of the particle along the intrinsic affine space coordinate $\varnothing\tilde{x}'$ of the primed intrinsic affine frame $(\varnothing\tilde{x}', \varnothing c\varnothing\tilde{t}')$, relative to the 'stationary' intrinsic 1-observer $\varnothing$Peter in $\varnothing\rho'$ and, consequently, relative to the 'stationary' 3-observer Peter in Σ' overlying $\varnothing\rho'$, will cause the simultaneous anti-clockwise rotation of the extended straight line primed intrinsic affine spaetime coordinates, $\varnothing\tilde{x}'$ and $\varnothing c\varnothing\tilde{t}'$, of the primed intrinsic affine frame by equal intrinsic angle $\varnothing\psi$ relative to the proper intrinsic metric space dimension $\varnothing\rho'$ along the horizontal and proper intrinsic metric time dimension $\varnothing c\varnothing t'$ along the vertical respectively, such that the inclined $\varnothing\tilde{x}'$ lies in the first and the inclined $\varnothing c\varnothing\tilde{t}'$ lies in the second quadrant in Fig. 7. The rotated (or inclined) primed intrinsic affine spacetime coordinates, $\varnothing\tilde{x}'$ and $\varnothing c\varnothing\tilde{t}'$, will then project unprimed intrinsic affine coordinates, $\varnothing\tilde{x}$ and $\varnothing c\varnothing\tilde{t}$, into $\varnothing\rho'$, along the horizontal and $\varnothing c\varnothing t'$ along the vertical respectively in the first quadrant in Fig. 7.

The perfect symmetry of state between the positive and the negative universes discussed earlier, implies that the rest mass $-m_0^*$ (its negative sign is discussed below) of the symmetry-partner particle is in simultaneous motion at equal constant speed v along the primed affine coordinate $-\tilde{x}'^*$ of the primed affine frame $(-\tilde{x}'^*, -\tilde{y}'^*, -\tilde{z}'^*, -c\tilde{t}'^*)$, relative to the symmetry-partner 'stationary' 3-observer* Peter* in the proper Euclidean metric 3-space $-\Sigma'^*$ in the negative universe.

Correspondingly, the intrinsic rest mass $-\varnothing m_0^*$ of the symmetry-partner particle, is in intrinsic motion at equal constant intrinsic speed $\varnothing v$ along the intrinsic affine coordinate $-\varnothing\tilde{x}'^*$ of the primed intrinsic affine frame $(-\varnothing\tilde{x}'^*, -\varnothing c\varnothing\tilde{t}'^*)$, relative to the 'stationary' intrinsic 1-observer* $\varnothing$Peter* in the proper intrinsic metric space $-\varnothing\rho'^*$ and, consequently, relative to the 'stationary' 3-observer* Peter* in the proper metric 3-space $-\Sigma'^*$ in the negative universe.

It is a cannon in the two-world picture that mass assumes the sign of the distance in space in which it exists. This makes mass and intrinsic mass negative in the negative universe. Apart from this, the negativity of mass and intrinsic mass in the negative universe shall be shown to be implied by the Lorentz transformation in the two-world picture (or in Scheme II in Table I), as well as from the requirement of symmetry of the natural laws between our (or positive) universe and the negative universe, elsewhere with further development of the two-world picture.

Again the intrinsic motion at intrinsic speed $\varnothing v$ of the intrinsic rest mass $-\varnothing m_0^*$ of the particle along the intrinsic affine space coordinate $-\varnothing\tilde{x}'^*$ of the primed intrinsic affine frame $(-\varnothing\tilde{x}'^*, -\varnothing c\varnothing\tilde{t}'^*)$, relative to the 'stationary' intrinsic 1-observer* $\varnothing$Peter* in $-\varnothing\rho'^*$ and, consequently, relative to the 'stationary' 3-observer* Peter* in $-\Sigma'^*$ overlying $-\varnothing\rho'^*$, will cause the simultaneous anti-clockwise rotations of the extended straight line intrinsic affine spacetime coordinates, $-\varnothing\tilde{x}'^*$ and $-\varnothing c\varnothing\tilde{t}'^*$, of the primed intrinsic affine frame by equal intrinsic angle $\varnothing\psi$ relative to the proper intrinsic metric space dimension $-\varnothing\rho'^*$ along the horizontal and the proper intrinsic metric time dimension $-\varnothing c\varnothing t'^*$ along the vertical respectively, in the third quadrant in Fig. 7. The rotated (or inclined) intrinsic affine coordinates, $-\varnothing\tilde{x}'^*$ will lie in the third quadrant and the rotated)pr inclined) $-\varnothing c\varnothing\tilde{t}'^*$ will lie in the fourth quadrant consequently. They will project unprimed intrinsic affine coordinates, $-\varnothing\tilde{x}^*$ and $-\varnothing c\varnothing\tilde{t}^*$, into $-\varnothing\rho'^*$ along the horizontal and $-\varnothing c\varnothing t'^*$ along the vertical respectively in the third quadrant in Fig. 7.

As a summary of the foregoing, the extended intrinsic affine coordinates, $-\varnothing\tilde{x}'^*$ and $-\varnothing c\varnothing\tilde{t}'^*$, of the primed intrinsic affine frame will be simultaneously rotated anti-clockwise by equal intrinsic angle $\varnothing\psi$ relative to their projective extended straight line unprimed intrinsic affine coordinates, $-\varnothing\tilde{x}^*$ and $-\varnothing c\varnothing\tilde{t}^*$, along the horizontal and vertical respectively in the negative universe. This will happen simultaneously with the anti-clockwise rotation of the extended straight line primed intrinsic affine coordinates, $\varnothing\tilde{x}'$ and $\varnothing c\varnothing\tilde{t}'$, of the primed intrinsic affine frame, at equal intrinsic angle $\varnothing\psi$ relative to their projective extended straight line unprimed intrinsic affine coordinates, $\varnothing\tilde{x}$ and $\varnothing c\varnothing\tilde{t}$, along the horizontal and vertical respectively in the positive universe.

Now the extended straight line primed intrinsic affine time coordinate $\varnothing c\varnothing\tilde{t}'$ of the primed intrinsic affine frame in the first quadrant, can rotate into the second quadrant with respect to the 3-observer Peter in the proper metric 3-space Σ' (as a 'hyper-line') along the horizontal in the first quadrant, on the larger spacetime/intrinsic spacetime of combined positive universe and negative universe depicted in Fig. 7. This is so because the intrinsic angle $\varnothing\psi$ can take on values in the negative half of the four-dimensional spacetime hyperplane (negative half-hyperplane) in Fig. 1b, which corresponds to the second and third quadrants in Fig. 7.

Similarly the extended straight line primed intrinsic affine time coordinate $-\varnothing c\varnothing\tilde{t}'^*$ of the primed intrinsic affine frame in the third quadrant, can rotate into the fourth quadrant with respect to 3-observer* Peter* in the proper metric 3-space $-\Sigma'^*$ (as a 'hyper-line') along the horizontal in the third quadrant, since $\varnothing\psi$ can take on values in the positive half-hyperplane with respect to 3-observers* in $-\Sigma'^*$ in Fig. 1b, which corresponds to the fourth and first quadrants in Fig. 7. Thus the rotations of the intrinsic affine coordinates, $\varnothing\tilde{x}'$ and $\varnothing c\varnothing\tilde{t}'$, relative to their projections, $\varnothing\tilde{x}$ and $\varnothing c\varnothing\tilde{t}$, respectively and of, $-\varnothing\tilde{x}'^*$ and $-\varnothing c\varnothing\tilde{t}'^*$, relative to their projections, $-\varnothing\tilde{x}^*$ and $-\varnothing c\varnothing\tilde{t}^*$, respectively, shown in Fig. 8a, are possible (or will ensue) in the two-world picture.

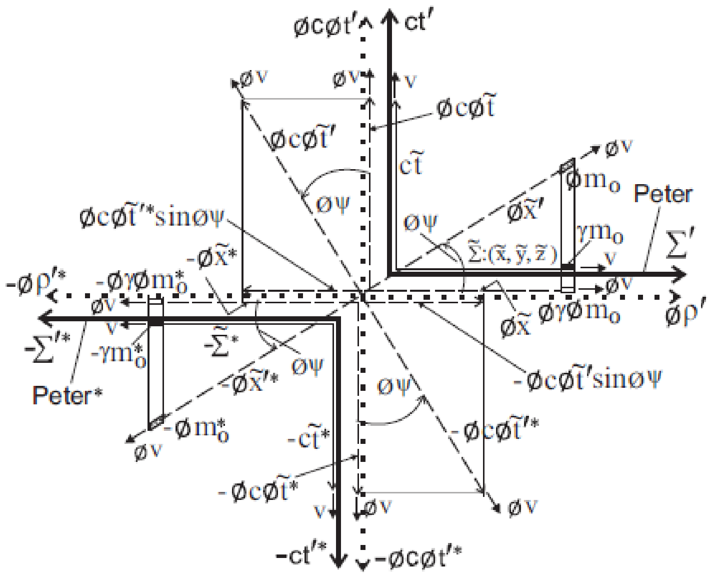

Fig. 8. (a) The diagram used to derive partial intrinsic Lorentz transformations and partial Lorentz transformations with respect to 3-observers in the Euclidean 3-spaces in the positive and negative universes

The projective extended straight line unprimed intrinsic affine spacetime coordinates, $\varnothing\tilde{x}$ and $\varnothing c\varnothing\tilde{t}$, are embedded in the extended straight line proper intrinsic metric spacetime dimensions, $\varnothing\rho'$ and $\varnothing c\varnothing t'$, respectively of the flat two-dimensional proper intrinsic metric spacetime $(\varnothing\rho', \varnothing c\varnothing t')$ in the first quadrant (or in our universe) in Fig. 8a. They constitute an unprimed intrinsic affine frame $(\varnothing\tilde{x}, \varnothing c\varnothing\tilde{t}\,)$ embedded in $(\varnothing\rho', \varnothing c\varnothing t')$, which is in intrinsic motion at intrinsic speed $\varnothing v$ in $(\varnothing\rho', \varnothing c\varnothing t')$ relative to the 'stationary' intrinsic 1-observer $\varnothing$Peter in the proper intrinsic metric space $\varnothing\rho'$ and, consequently, relative to the 'stationary' 3-observer Peter in the proper metric 3-space Σ', in the first quadrant (or our universe) in Fig. 8a.

The projective unprimed intrinsic affine spacetime coordinates, $\varnothing\tilde{x}$ and $\varnothing c\varnothing\tilde{t}$, embedded in the proper intrinsic metric spacetime dimensions, $\varnothing\rho'$ and $\varnothing c\varnothing t'$, respectively are then made manifested outwardly in the unprimed affine spacetime coordinates, $\tilde{x}, \tilde{y}, \tilde{z}$ and $c\tilde{t}$, that constitute an unprimed affine frame $(\tilde{x}, \tilde{y}, \tilde{z}, c\tilde{t}\,)$ (or $(\tilde{\Sigma}, c\tilde{t}\,)$) in the first quadrant in Fig. 8a. The unprimed affine frame $(\tilde{x}, \tilde{y}, \tilde{z}, c\tilde{t}\,)$ (or $(\tilde{\Sigma}, c\tilde{t}\,)$) is embedded in the flat four-dimensional proper metric spacetime (Σ', ct'), as illustrated in Fig. 8a, and it is in motion at speed v along its coordinate $\tilde{x}$ on the flat proper metric sacetime (Σ', ct'), relative to the 'stationary' 3-observer Peter in the proper metric 3-space Σ' in the first quadrant in Fig. 8a.

In symmetry, the projective straight line unprimed intrinsic affine coordinates, $-\varnothing\tilde{x}^{\,*}$ and $-\varnothing c\varnothing\tilde{t}^{\,*}$, are embedded in the straight line proper intrinsic metric spacetime dimensions, $-\varnothing\rho'^{*}$ and $-\varnothing c\varnothing t'^{*}$, respectively of the flat two-dimensional proper intrinsic metric spacetime $(-\varnothing\rho'^{*}, -\varnothing c\varnothing t'^{*})$ in the third quadrant (or in the negative universe) in Fig. 8a. They constitute an unprimed intrinsic affine frame $(-\varnothing\tilde{x}^{\,*}, -\varnothing c\varnothing\tilde{t}^{\,*})$ embedded in the flat $(-\varnothing\rho'^{*}, -\varnothing c\varnothing t'^{*})$, which is in intrinsic motion at intrinsic speed $\varnothing v$ in $(-\varnothing\rho'^{*}, -\varnothing c\varnothing t'^{*})$ relative to the 'stationary' symmetry-partner intrinsic 1-observer* $\varnothing$Peter* in the proper intrinsic metric space $-\varnothing\rho'^{*}$ and, consequently, relative to the 'stationary' 3-observer* Peter* in the proper metric 3-space $-\Sigma'^{*}$ in the third quadrant (or in the negative universe) in Fig. 8a.

The projective unprimed intrinsic affine coordinates, $-\varnothing\tilde{x}^{\,*}$ and $-\varnothing c\varnothing\tilde{t}^{\,*}$, embedded in the proper intrinsic metric spacetime dimensions, $-\varnothing\rho'^{*}$ and $-\varnothing c\varnothing t'^{*}$, respectively, are then made manifested outwardly in the unprimed affine coordinates, $-\tilde{x}^{\,*}, -\tilde{y}^{\,*}, -\tilde{z}^{\,*}$ and $-c\tilde{t}^{\,*}$, that constitute an unprimed affine frame $(-\tilde{x}^{\,*}, -\tilde{y}^{\,*}, -\tilde{z}^{\,*}, -c\tilde{t}^{\,*})$ (or $(-\tilde{\Sigma}^{*}, -c\tilde{t}^{\,*})$) in the third quadrant in Fig. 8a, which is embedded in the flat four-dimensional proper metric spacetime $(-\Sigma'^{*}, -ct'^{*})$, as illustrated. It is in motion at speed v along its coordinate $-\tilde{x}^{\,*}$ on the flat $(-\Sigma'^{*}, -ct'^{*})$, relative to the 'stationary' 3-observer* Peter* in the proper metric 3-space $-\Sigma'^{*}$ in the third quadrant in Fig. 8a.

The special-relativistic mass, $m = \gamma m_0$, of the particle is moving at speed v in the unprimed affine frame $(\tilde{x}, \tilde{y}, \tilde{z}, c\tilde{t}\,)$ (or $(\tilde{\Sigma}, c\tilde{t}\,)$) embedded in the flat proper metric spacetime (Σ', ct'), relative to the 'stationary' 3-observer Peter in the proper metric 3-space Σ' in our universe in Figs. 8a. The special-relativistic mass, $-m^{*} = -\gamma m_0^{*}$, of the symmetry-partner particle is likewise moving at equal speed v in the unprimed affine frame $(-\tilde{x}^{\,*}, -\tilde{y}^{\,*}, -\tilde{z}^{\,*}, -c\tilde{t}^{\,*})$ (or $(-\tilde{\Sigma}^{*}, -c\tilde{t}^{\,*})$) embedded in the flat proper metric spacetime $(-\Sigma'^{*}, -ct'^{*})$, relative to the 'stationary' 3-observer* Peter* in the proper metric 3-space $-\Sigma'^{*}$ in the negative universe in Fig 8a.

It is also important to note that the primed affine frames, $(\tilde{x}\,', \tilde{y}\,', \tilde{z}\,', c\tilde{t}\,')$ (or $(\tilde{\Sigma}', c\tilde{t}\,')$) and $(-\tilde{x}\,'^{*}, -\tilde{y}\,'^{*}, -\tilde{z}\,'^{*}, -c\tilde{t}\,'^{*})$ (or $(-\tilde{\Sigma}'^{*}, -c\tilde{t}\,'^{*})$), in which the rest masses, m_0 and $-m_0^{*}$, are at rest relative to 3-observers Peter in Σ' and Peter* in $-\Sigma'^{*}$ respectively initially, no longer exist in the geometry of Figs. 8a. If $(\tilde{x}\,', \tilde{y}\,', \tilde{z}\,', c\tilde{t}\,')$ and $(-\tilde{x}\,'^{*}, -\tilde{y}\,'^{*}, -\tilde{z}\,'^{*}, -c\tilde{t}\,'^{*})$ were to appear in Fig. 8a, they would be the outward manifestations of the inclined primed intrinsic affine frames, $(\varnothing\tilde{x}\,', \varnothing c\varnothing\tilde{t}\,')$ and $(-\varnothing\tilde{x}\,'^{*}, -\varnothing c\varnothing\tilde{t}\,'^{*})$, respectively.

However the inclined primed intrinsic affine frames, $(\varnothing\tilde{x}\,', \varnothing c\varnothing\tilde{t}\,')$ and $(-\varnothing\tilde{x}\,'^{*}, -\varnothing c\varnothing\tilde{t}\,'^{*})$, have no outward manifestations; only the non-inclined (or flat) unprimed intrinsic affine frames, $(\varnothing\tilde{x}, \varnothing c\varnothing\tilde{t}\,)$ and $(-\varnothing\tilde{x}^{\,*}, -\varnothing c\varnothing\tilde{t}^{\,*})$ have. The primed affine frames, $(\tilde{x}\,', \tilde{y}\,', \tilde{z}\,', c\tilde{t}\,')$ (or $(\tilde{\Sigma}', c\tilde{t}\,')$) and $(-\tilde{x}\,'^{*}, -\tilde{y}\,'^{*}, -\tilde{z}\,'^{*}, -c\tilde{t}\,'^{*})$ (or $(-\tilde{\Sigma}'^{*}, -c\tilde{t}\,'^{*})$), and the rest masses, m_0 and $-m_0^{*}$, in them no longer exist in Fig. 8a of partial geometrical representations of intrinsic Lorentz transformation/Lorentz transformation (∅LT/LT) in the two-world picture, with respect to 'stationary' 3-observers in the metric proper Euclidean 3-spaces, Σ' and $-\Sigma'^{*}$.

The unprimed affine frames, $(\tilde{x}, \tilde{y}, \tilde{z}, c\tilde{t}\,)$ (or $(\tilde{\Sigma}, c\tilde{t}\,)$) and $(-\tilde{x}^{\,*}, -\tilde{y}^{\,*}, -\tilde{z}^{\,*}, -c\tilde{t}^{\,*})$ (or $(-\tilde{\Sigma}^{*}, -c\tilde{t}^{\,*})$), that support the motions of the special-relativistic masses, $m = \gamma m_0$ and $-m^{*} = -\gamma m_0^{*}$, are not the projective frames of inclined primed affine frames, $(\tilde{x}\,', \tilde{y}\,', \tilde{z}\,', c\tilde{t}\,')$ and $(-\tilde{x}\,'^{*}, -\tilde{y}\,'^{*}, -\tilde{z}\,'^{*}, -c\tilde{t}\,'^{*})$, respectively, which do not exist in Fig. 8a. Rather they are the outward manifestations on the flat metric spacetimes, (Σ', ct') and $(-\Sigma'^{*}, -ct'^{*})$, of the projective unprimed intrinsic affine frames, $(\varnothing\tilde{x}, \varnothing c\varnothing\tilde{t}\,)$ and $(-\varnothing\tilde{x}^{\,*}, -\varnothing c\varnothing\tilde{t}^{\,*})$, which support the intrinsic motion of the relativistic intrinsic masses, $\varnothing m = \varnothing\gamma\varnothing m_0$ and $-\varnothing m^{*} = -\varnothing\gamma\varnothing m_0^{*}$, on the flat proper intrinsic metric spacetimes, $(\varnothing\rho', \varnothing c\varnothing t')$ and $(-\varnothing\rho'^{*}, -\varnothing c\varnothing t'^{*})$.

The usual practice of rotating the primed coordinates, x' and ct' (of the particle's frame), relative to the unprimed coordinates, x and ct (of the 'stationary' observer's frame), of the four-dimensional spacetime in the Minkowski's diagrams (Figs. 3a and 3b), as well as in the Loedel diagram (Fig. 4a) and Brehme diagram (Fig. 4b), in the existing one-world picture, does not arise in the present context in the two-world picture.

The primed intrinsic affine frames, $(\varnothing\tilde{x}\,', \varnothing c\varnothing\tilde{t}\,')$ and $(-\varnothing\tilde{x}\,'^{*}, -\varnothing c\varnothing\tilde{t}\,'^{*})$, and their projective unprimed intrinsic affine frames, $(\varnothing\tilde{x}, \varnothing c\varnothing\tilde{t}\,)$ and $(-\varnothing\tilde{x}^{\,*}, -\varnothing c\varnothing\tilde{t}^{\,*})$, are all in intrinsic motion at intrinsic speed $\varnothing v$ relative to the 'stationary' 3-observers in the metric 3-spaces, Σ' and $-\Sigma'^{*}$, in Fig. 8a. The unprimed affine frames, $(\tilde{x}\,', \tilde{y}\,', \tilde{z}\,', c\tilde{t}\,')$ and $(\tilde{x}\,, \tilde{y}\,, \tilde{z}\,, c\tilde{t})$, are likewise in motion at speed v relative to the 'stationary' 3-observers in Σ' and $-\Sigma'^{*}$ in Fig. 8a.

Figure 8a is a partial diagram that is valid with respect to the 'stationary' 3-observers, Peter and Peter*, in the proper metric Euclidean 3-spaces, Σ' and $-\Sigma'^{*}$, respectively. There is partial intrinsic Lorentz transformation and partial Lorentz transformation to be derived with respect to each of these 3-observers, as shall be done shortly.

There is a second partial diagram that is valid with respect to 1-observers Peter^0 and Peter^{0*} in the proper metric time dimensions, ct' and $-ct'^{*}$, respectively from which partial intrinsic Lorentz transformations and Lorentz transformations shall also be derived with respect to each of these 1-observers shortly. The partial intrinsic Lorentz transformations and partial Lorentz transformations derived from the resulting two partial diagrams shall then be combined to obtain the full transformations. The second partial diagram with respect to 1-observers shall be described as complementary diagram to Fig. 8a. It is depicted in Fig. 8b.

As known, every object (including observers) have three-dimensional mass in the metric 3-space and one-dimensional mass in the metric time dimension, thereby being a four-dimensional object in spacetime. The one-dimensional masses of particles in the metric time dimensions have not been incorporated into Figs. 8a and 8b in this article, because they are not required for the development of this article.

The complementary diagram of Fig. 8b (with respect to 1-observers in the time-dimensions) must be drawn along with Fig. 8a (with respect to 3-observers in the 3-spaces) in order to derive the full intrinsic Lorentz transformation and full Lorentz transformation in each of the two universes of the two-world picture, as shall be done in the rest of this sub-section.

The essential difference between Fig. 8a with respect to the 'stationary' 3-observers Peter and Peter* in the metric proper Euclidean 3-spaces, Σ' and $-\Sigma'^{*}$, respectively and Fig. 8b with respect to 1-observers Peter^0 and Peter^{0*} in the proper metric time dimensions, ct' and $-ct'^{*}$, respectively is that, the primed intrinsic affine frames, $(\varnothing\tilde{x}\,', \varnothing c\varnothing\tilde{x}\,')$ and $(-\varnothing\tilde{x}\,'^{*}, -\varnothing c\varnothing\tilde{x}\,'^{*})$, are inclined anti-clockwise relative to their projective unprimed intrinsic affine frames, $(\varnothing\tilde{x}, \varnothing c\varnothing\tilde{x}\,)$ and $(-\varnothing\tilde{x}^{\,*}, -\varnothing c\varnothing\tilde{x}^{\,*})$, in Fig. 8a, whereas $(\varnothing\tilde{x}\,', \varnothing c\varnothing\tilde{x}\,')$ and $(-\varnothing\tilde{x}\,'^{*}, -\varnothing c\varnothing\tilde{x}\,'^{*})$ are inclined clockwise relative to their projective, $(\varnothing\tilde{x}, \varnothing c\varnothing\tilde{x}\,)$ and $(-\varnothing\tilde{x}^{\,*}, -\varnothing c\varnothing\tilde{x}^{\,*})$, in Fig. 8b. These features of the geometries of Figs. 8a and 8b require further development of the two-world picture than in this article to explain.

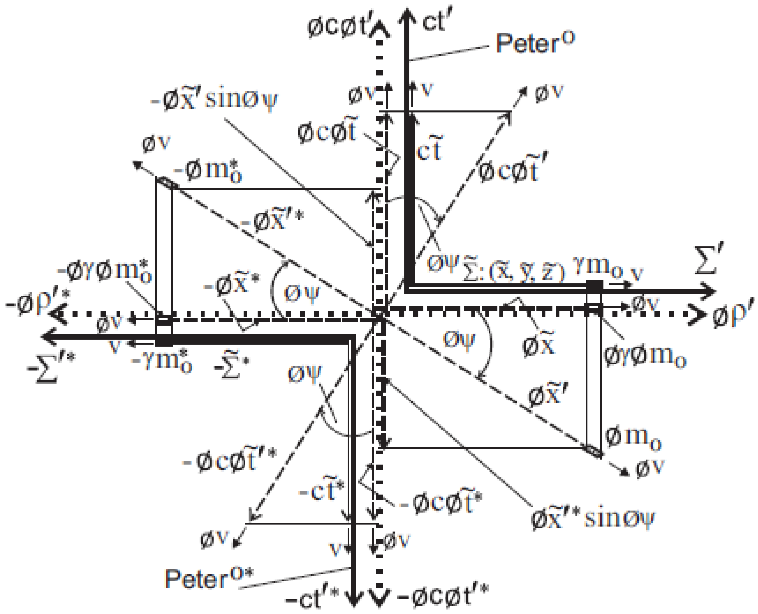

Fig. 8. (b) The complementary diagram to Fig. 8a used to derive partial intrinsic Lorentz transformations and partial Lorentz transformations with respect to 1-observers in the proper metric time dimensions in the positive and negative universes

Let us formally identify and appropriately entitle the intrinsic affine frames and affine frames encompassed by Figs.8a and 8b. There are the inclined primed intrinsic affine frames, $(\varnothing\tilde{x}\,', \varnothing c\varnothing\tilde{x}\,')$ and $(-\varnothing\tilde{x}\,'^{*}, -\varnothing c\varnothing\tilde{x}\,'^{*})$, which support the intrinsic motions of the intrinsic rest masses, $\varnothing m_0$ and $-\varnothing m_0^*$, of the symmetry-partner particles at intrinsic speed $\varnothing v$ relative to the symmetry-partner 'stationary' 3-observers in the proper metric 3-spaces, Σ' and $-\Sigma'^{*}$, in Fig. 8a and 'stationary' 1-observers in the proper metric time dimensions, ct' and $-ct'^{*}$, in Fig. 8b. They shall be referred to as the particle's primed intrinsic affine frames.

There is a pair of projective unprimed (or relativistic) intrinsic affine frames, $(\varnothing\tilde{x}, \varnothing c\varnothing\tilde{x})$ and $(-\varnothing\tilde{x}^{\,*}, -\varnothing c\varnothing\tilde{x}^{\,*})$, which support the intrinsic motions of the relativistic intrinsic masses, $\varnothing m = \varnothing\gamma\varnothing m_0$ and $-\varnothing m^* = -\varnothing\gamma\varnothing m_0^*$, of the symmetry-partner particles at intrinsic speed $\varnothing v$ on the flat proper intrinsic metric spacetimes, $(\varnothing\rho', \varnothing c\varnothing t')$ and $(-\varnothing\rho'^{*}, -\varnothing c\varnothing t'^{*})$, relative to the symmetry-partner 'stationary' 3-observers in the proper Euclidean 3-spaces, Σ' and $-\Sigma'^{*}$, in Fig. 8a and 'stationary' 1-observers in the proper metric time dimensions, ct' and $-ct'^{*}$, in Fig. 8b. They shall be referred to as the particle's relativistic (or unprimed) intrinsic affine frames.

Finally there are the relativistic affine frames, $(\tilde{x},\, \tilde{y}, \tilde{z},\, c\tilde{t})$ and $(-\tilde{x}^{\,*}, -\tilde{y}^{\,*}, -\tilde{z}^{\,*}, -c\tilde{t}^{\,*})$, which support the motions of the observed special-relativistic masses, $m = \gamma m_0$ and $-m^* = -\gamma m_0^*$, of the symmetry-partner particles at speed v on the flat four-dimensional proper metric spacetimes, (Σ', ct') and $(-\Sigma'^{*}, -ct'^{*})$, relative to the symmetry-partner 'stationary' 3-observers in the proper metric 3-spaces, Σ' and $-\Sigma'^{*}$, in Fig. 8a and 'stationary' 1-observers in the proper metric time dimensions, ct' and $-ct'^{*}$, in Fig. 8b. They shall be referred to as the particle's relativistic (or unprimed) affine frames.

What could have been inclined particle's proper (or primed) affine frames, $(\tilde{x}\,', \tilde{y}\,', \tilde{z}\,', c\tilde{t}\,')$ and $(-\tilde{x}\,'^{*}, -\tilde{y}\,'^{*}, -\tilde{z}\,'^{*}, -c\tilde{t}\,'^{*})$, that support the motions of the rest masses, m_0 and $-m_0^*$, of the symmetry-partner particles relative to the symmetry-partner 'stationary' 3-observers in the proper metric 3-spaces, Σ' and $-\Sigma'^*$, in Fig. 8a and 'stationary' 1-observers in the proper metric time dimensions, ct' and $-ct'^*$, in Fig. 8b, do not exist in Figs. 8a and 8b, as mentioned earlier. The primed affine frames, $(\tilde{x}', \tilde{y}', \tilde{z}', c\tilde{t}')$ and $(-\tilde{x}'^*, -\tilde{y}'^*, -\tilde{z}'^*, -c\tilde{t}'^*)$, were embedded in the proper metric spacetimes, (Σ', ct') and $(-\Sigma'^*, -ct'^*)$, when the particles were at rest relative to the observers initially. They become transformed (without rotations of their coordinates) into the relativistic affine frames, $(\tilde{x}, \tilde{y}, \tilde{z}, c\tilde{t})$ and $(-\tilde{x}^*, -\tilde{y}^*, -\tilde{z}^*, -c\tilde{t}^*)$, as the particles begin to move relative to the observers in Figs. 8a and 8b.

The 'stationary' 4-observers (Peter, Peter^0) and (Peter*,Peter^{0*}) are located 'at rest' (or are 'stationary') on the flat four-dimensional proper metric spacetimes, (Σ', ct') and $(-\Sigma'^*, -ct'^*)$, and are without (affine) frames in Figs. 8a and 8b. Apart from their two-world background, the new diagrams of Fig. 8a and Fig. 8b, represent a major break from the existing Minkowski diagrams (Figs. 3a and 3b), as well as the Loedel and Brehme diagrams (Figs. 4a and 4b). This is so because the metric coordinates, x' and ct', of the particle's frame (x', y', z', ct') are rotated relative to the metric coordinates, x and ct, of the 'stationary' observer's frame (x, y, z, ct), for relative motion along their collinear $x'-$ and $x-$ axes of the flat four-dimensional proper metric spacetime (Σ', ct') of our universe in these existing diagrams in the one-world picture.

In the twin paradox, the 'stationary' 3-observer Peter in the proper Euclidean 3-space Σ' in Fig. 8a, remained at home. His twin-brother (James), inside the observed particle with relativistic mass γm_0—a craft—in the particle's relativistic (or unprimed) affine frame $(\tilde{x}, \tilde{y}, \tilde{z}, c\tilde{t})$ (or $(\tilde{\Sigma}, c\tilde{t})$) in that figure, embarked on a return trip to a distant star at constant speed v relative to Peter. He aged faster because of time dilation in his frame relative to Peter.

The space devoted to the development of the geometries of Figs. 8a and 8b and the elucidations of some associated salient issues above is unavoidable in this foundation article. There is yet one other feature of the diagrams that must be pointed out. This is the fact that, as indicated, the intrinsic affine spacetime coordinates of the particle's primed and unprimed intrinsic affine frames, $(\varnothing\tilde{x}', \varnothing c\varnothing\tilde{t}')$ and $(\varnothing\tilde{x}, \varnothing c\varnothing\tilde{t})$, are in natural intrinsic 'flow' (or elongation) at the intrinsic speed of the particle relative to the observers. The particle's intrinsic masses, $\varnothing m_0$ and $\varnothing\gamma\varnothing m_0$, at rest with respect to $\varnothing\tilde{x}'$ and $\varnothing\tilde{x}$, at the edges of which they lie respectively, are naturally being translated at the intrinsic speed $\varnothing v$ relative to the observers by these naturally 'flowing' (or elongating) intrinsic affine coordinates.

The affine spacetime coordinates, $\tilde{x}$ and $c\tilde{t}$, of the particle's unprimed affine frame, $(\tilde{x}, \tilde{y}, \tilde{z}, c\tilde{t})$; the $\tilde{x}$ that lies along x' in Σ' being the affine coordinates along which motion takes place, are likewise in natural 'flow' (or elongation) at the speed v of the particle relative to the observer. The particle's relativistic mass γm_0 at rest with respect to $\tilde{x}$, at the edge of which it lies, is naturally being translated at the speed v relative to the observer by these naturally 'flowing' (or elongating) affine coordinate. The possible relationship to the concept of ether of SR (the Lorentzian ether [30]) of these naturally 'flowing' intrinsic affine spacetime coordinates and affine spacetime coordinates is striking and worthy of investigation elsewhere.

Now the unprimed intrinsic affine space coordinate $\varnothing\tilde{x}$ is the projection along the horizontal of the inclined primed intrinsic affine space coordinate $\varnothing\tilde{x}'$ in the first quadrant in Fig. 8a. That is, $\varnothing\tilde{x} = \varnothing\tilde{x}'\cos\varnothing\psi$ (recall that we are now operating in the context of scheme II in Table I). We must express the rotated $\varnothing\tilde{x}'$ in terms of its projection along the horizontal and write

$$\varnothing\tilde{x}' = \varnothing\tilde{x}\sec\varnothing\psi. \tag{9a}$$

The transformation of intrinsic affine space coordinates (9a) with respect to the 3-observer (Peter) in Σ' can be ascertained by 'measurement' with intrinsic laboratory rod; the $\varnothing\tilde{x}'$ is the 'measured' length from a specified origin along the intrinsic metric space $\varnothing\rho'$ before the commencement of motion of the particle relative to the observer and $\varnothing\tilde{x}$ is the 'measured' length along $\varnothing\rho'$ in relative motion. Equation (9a) shall be referred to as the transformation of the 'measurable' intrinsic affine space coordinates (with respect to the 3-observer in Σ'). The quotation marks of 'measured' and 'measurable' is used to indicate that these intrinsic affine coordinates cannot themselves be measured in reality.

Apart from the transformation of the 'measurable' intrinsic affine space coordinates (9a), the inclined negative primed intrinsic affine time coordinate $-\varnothing c\varnothing\tilde{t}'^{*}$ of the negative universe, which is rotated into the fourth quadrant in Fig. 8a, projects component $-\varnothing c\varnothing\tilde{t}'\sin\varnothing\psi$ into the proper intrinsic metric space $\varnothing\rho'$ along the horizontal, thereby making the net projective intrinsic affine coordinate along $\varnothing\rho'$ to be, $\varnothing\tilde{x} - \varnothing c\varnothing\tilde{t}\sin\varnothing\psi$, along the horizontal in the first quadrant in Fig. 8a, when the the particle is in motion relative to the observer.

Apart from the net projection, $\varnothing\tilde{x}\sec\varnothing\psi - \varnothing c\varnothing\tilde{t}\,'\sin\varnothing\psi$, into $\varnothing\rho'$ along the horizontal, derived between the first and fourth quadrants in Fig. 8a, there is also the projection $-\varnothing c\varnothing\tilde{t}\,^{*}$ along the vertical by the inclined $\varnothing c\varnothing\tilde{t}\,'^{*}$ in the fourth quadrant, for which the relation, $-\varnothing c\varnothing\tilde{t}\,^{*} = -\varnothing c\varnothing\tilde{t}\,'^{*}\cos\varnothing\psi$, implying $\varnothing c\varnothing\tilde{t}\,' = \varnothing c\varnothing\tilde{t}\sec\varnothing\psi$, obtains. This relation must complement the net projection into $\varnothing\rho'$ along the horizontal. It must therefore be used to replace $\varnothing c\varnothing\tilde{t}\,'$ by $\varnothing c\varnothing\tilde{t}\sec\varnothing\psi$ in the net projection into $\varnothing\rho'$ along the horizontal, giving $\varnothing\tilde{x}\sec\varnothing\psi - \varnothing c\varnothing\tilde{t}\tan\varnothing\psi$.

Now the projective intrinsic affine time coordinate along $\varnothing\rho'$ namely, $-\varnothing c\varnothing\tilde{t}\tan\varnothing\psi$, is non-measurable with intrinsic laboratory rod, unlike $\varnothing\tilde{x}$. It shall be referred to as 'non-measurable' intrinsic affine space coordinate. It exists along with the the 'measurable' intrinsic affine space coordinate $\varnothing\tilde{x}$ in the particle's relativistic (or unprimed) intrinsic affine frame $(\varnothing\tilde{x}, \varnothing c\varnothing\tilde{t})$. Whereas only the 'measurable' primed intrinsic affine space coordinate $\varnothing\tilde{x}'$ exists in the inclined particle's proper (or primed) intrinsic affine frame $(\varnothing\tilde{x}', \varnothing c\varnothing\tilde{t}')$.

While it is the transformation of the 'measurable' intrinsic affine space coordinates in Eq. (9a) (without a 'non-measurable' component) that man can establish by 'measurement' with intrinsic laboratory rod (as intrinsic affine length contraction formula to be derived later in this article), nature makes use of both the 'measurable' and 'non-measurable' intrinsic affine coordinates, $\varnothing\tilde{x}\sec\varnothing\psi$ and $-\varnothing c\varnothing\tilde{t}\tan\varnothing\psi$, along $\varnothing\rho'$ in the particle's relativistic (or unprimed) intrinsic affine frame, along with the only 'measurable' intrinsic affine space coordinate $\varnothing\tilde{x}'$ in the inclined particle's proper (or primed) intrinsic affine frame, to establish partial intrinsic Lorentz transformation with respect to the 3-observer (Peter) in Σ' in the first quadrant (or our universe) in Fig. 8a as follows

$$\varnothing\tilde{x}\,' = \varnothing\tilde{x}\sec\varnothing\psi - \varnothing c\varnothing\tilde{t}\tan\varnothing\psi\ ;$$
$$(\text{w.r.t } 3-\text{observer Peter in } \Sigma')\ .(9\text{b})$$

Equation (9b) states that the intrinsic affine coordinate $\varnothing\tilde{x}\,'$ along $\varnothing\rho'$ prior to relative motion of the particle, is equal to the net intrinsic affine coordinate $\varnothing\tilde{x} - \varnothing c\varnothing\tilde{t}'\tan\varnothing\psi$, projected along $\varnothing\rho'$ as the particle moves relative to the observer.

The dummy star label used to differentiate the the coordinates and parameters of the negative universe from those of the positive universe has been removed from the component, $-\varnothing c\varnothing\tilde{t}'^{*}\sin\varnothing\psi\,(= -\varnothing c\varnothing\tilde{t}'^{*}\tan\varnothing\psi)$, projected into $\varnothing\rho'$ along the horizontal by the inclined $-\varnothing c\varnothing\tilde{t}'^{*}$ of the negative universe rotated into the fourth quadrant. This is done because the projected component is now an intrinsic affine coordinate in the positive universe.

The transformation (9b) reduces as the pure 'measurable' intrinsic affine space transformation (9a) upon removing the 'non-measurable' intrinsic affine time coordinate in Eq. (9b). It is to be noted that the intrinsic affine coordinate transformation (9b) cannot be derived by direct intrinsic affine coordinate projections in Fig. 8a, thereby warranting the rather long explanation in the paragraphs that lead to that equation. Having derived the partial intrinsic Lorentz transformation (9b) with respect to the 3-observer in Σ' from Fig. 8a, let us now proceed to Fig. 8b to derive the corresponding partial intrinsic Lorentz transformation with respect to the 1-observer in ct'.

The unprimed intrinsic affine time coordinate $\varnothing c\varnothing\tilde{t}$ is the projection along the vertical of the inclined primed intrinsic affine time coordinate $\varnothing c\varnothing\tilde{t}\,'$ in the second quadrant in Fig. 8a. That is, $\varnothing c\varnothing\tilde{t} = \varnothing c\varnothing\tilde{t}\,'\cos\varnothing\psi$. We must express the rotated $\varnothing c\varnothing\tilde{t}\,'$ in terms of its projection along the vertical and write

$$\varnothing c\varnothing\tilde{t}\,' = \varnothing c\varnothing\tilde{t}\sec\varnothing\psi. \qquad (10a)$$

The transformation of intrinsic affine time coordinates (10a) with respect to the 1-observer (Peter^0) in ct' can be ascertained by 'measurement' with intrinsic laboratory clock. The $\varnothing\tilde{t}'$ is the 'measured' initial duration of intrinsic affine time along $\varnothing c\varnothing t'$ by Peter^0 before the commencement of the motion of the particle relative to the observer and $\varnothing\tilde{t}$ is the 'measured' duration of affine intrinsic time along $\varnothing c\varnothing t'$ by Peter^0 during relative motion. Equation (10a) shall be referred to as transformation of 'measurable' intrinsic affine time coordinates (with respect to the 1-observer in ct').

Apart from the transformation of the 'measurable' intrinsic affine time coordinates (10a), the inclined negative primed intrinsic affine space coordinate $-\varnothing\tilde{x}'^*$ of the negative universe, which is rotated into the second quadrant in Fig. 8b, projects a component, $-\varnothing\tilde{x}'\sin\varnothing\psi$ into the proper intrinsic metric time dimension $\varnothing c\varnothing t'$ along the vertical, thereby making the net projective intrinsic affine coordinate along $\varnothing c\varnothing t'$ along the vertical to be $\varnothing c\varnothing\tilde{t}\sec\varnothing\psi - \varnothing\tilde{x}'^*\sin\varnothing\psi$ in the first quadrant in Fig. 8b, when the the particle is in motion relative to the observer.

Apart from the net intrinsic affine coordinate projection $\varnothing c\varnothing\tilde{t}\sec\varnothing\psi - \varnothing\tilde{x}\,'\sin\varnothing\psi$ into $\varnothing c\varnothing t'$ along the vertical, derived between the first and second quadrants in Fig. 8b, there is also the projection $-\varnothing\tilde{x}\,^*$ along the horizontal by the inclined $-\varnothing\tilde{x}\,'^*$ in the second quadrant, for which the relation, $-\varnothing\tilde{x}\,^* = \varnothing\tilde{x}\,'^*\cos\varnothing\psi$, implying, $\varnothing\tilde{x}\,' = \varnothing\tilde{x}\sec\varnothing\psi$, obtains. This relation must complement the net intrinsic affine coordinate projection into $\varnothing c\varnothing t'$ along the vertical in the first quadrant. It must be used to replace $\varnothing\tilde{x}\,'$ by $\varnothing\tilde{x}\sec\varnothing\psi$ in the net projection into $\varnothing c\varnothing t'$, giving $\varnothing c\varnothing\tilde{t}\sec\varnothing\psi - \varnothing\tilde{x}\tan\varnothing\psi$.

Now the projective intrinsic affine space coordinate $-\varnothing\tilde{x}\tan\varnothing\psi$ along $\varnothing c\varnothing t'$ is 'non-measurable' with intrinsic laboratory clock, unlike $\varnothing c\varnothing\tilde{t}\sec\varnothing\psi$, by Peter^0 in ct'. It shall be referred to as 'non-measurable' intrinsic affine 'time' coordinate. It exists alongside the 'measurable' intrinsic affine time coordinate $\varnothing c\varnothing\tilde{t}\sec\varnothing\psi$ in the particle's relativistic (or unprimed) intrinsic affine frame $(\varnothing\tilde{x}, \varnothing c\varnothing\tilde{t})$. Whereas only the primed 'measurable' intrinsic affine time coordinate $\varnothing c\varnothing\tilde{t}\,'$ exists in the inclined particle's proper (or primed) intrinsic affine frame $(\varnothing\tilde{x}', \varnothing c\varnothing\tilde{t}')$.

While it is the transformation of the 'measurable' intrinsic affine time coordinates (10a) (without a 'non-measurable' component) that the 1-observer in ct' can establish by 'measurement' with intrinsic laboratory clock, nature makes use of both the 'measurable' and 'non-measurable' intrinsic affine coordinates, $\varnothing c\varnothing\tilde{t}\sec\varnothing\psi$ and $-\varnothing\tilde{x}\tan\varnothing\psi$, along $\varnothing c\varnothing t'$ in the particle's relativistic (or unprimed) intrinsic affine frame, along with the only 'measurable' intrinsic affine time coordinate $\varnothing c\varnothing\tilde{t}'$ in the inclined particle's proper (or primed) intrinsic affine frame, to establish partial intrinsic Lorentz transformation with respect to the 1-observer (Peter^0) in ct' in the first quadrant (or our universe) in Fig. 8b as

$$\varnothing c\varnothing\tilde{t}\,' = \varnothing c\varnothing\tilde{t}\sec\varnothing\psi - \varnothing\tilde{x}\tan\varnothing\psi\ , \qquad (10b)$$

$$(\text{w.r.t } 1-\text{observer Peter}^0 \text{ in } ct')\ .$$

Equation (10b) states that the intrinsic affine coordinate $\varnothing c\varnothing\tilde{t}\,'$ along $\varnothing c\varnothing t'$ prior to relative motion of the particle, is equal to the net intrinsic affine coordinate $\varnothing c\varnothing\tilde{t}\sec\varnothing\psi - \varnothing\tilde{x}\tan\varnothing\psi$, projected along $\varnothing c\varnothing t'$ as the particle moves relative to the observer.

The dummy star label used to differentiate the coordinates and parameters of the negative universe from those of the positive universe has been removed from the component, $-\varnothing\tilde{x}'^*\sin\varnothing\psi = -\varnothing\tilde{x}^*\tan\varnothing\psi$, projected into $\varnothing c\varnothing t'$ along the vertical by the inclined $-\varnothing\tilde{x}'^*$ of the negative universe rotated into the second quadrant. This is done because the projected component is now an intrinsic affine coordinate in the positive universe.

The partial intrinsic Lorentz transformation of intrinsic affine spacetime coordinates (9b) with respect to the 'stationary' 3-observer Peter in the metric 3-space Σ' and the partial intrinsic Lorentz transformation of intrinsic affine spacetime coordinates (10b) with respect to the 'stationary' 1-observer Peter^0 in the metric time dimension ct', must be collected to obtain the full intrinsic Lorentz transformation (∅LT) of extended straight line intrinsic affine spacetime coordinates with respect to the 'stationary' 4-observers (Peter,

Peter^0) on the flat four-dimensional proper metric spacetime (Σ', ct') as

$$\begin{aligned}
\varnothing c\varnothing\tilde{t}\,' &= \varnothing c\varnothing\tilde{t}\sec\varnothing\psi - \varnothing\tilde{x}\tan\varnothing\psi\ ; \\
&\quad (\text{w.r.t. } 1-\text{observer Peter}^{\,0} \text{ in } ct')\ ; \\
\varnothing\tilde{x}\,' &= \varnothing\tilde{x}\sec\varnothing\psi - \varnothing c\varnothing\tilde{t}\tan\varnothing\psi\ ; \\
&\quad (\text{w.r.t. } 3-\text{observer Peter in } \Sigma')\ ,
\end{aligned} \tag{11}$$

for $-\varnothing\pi/2 < \varnothing\psi < \varnothing\pi/2$ (temporarily). This temporary range of $\varnothing\psi$ shall be modified later in this section.

The fact that the intrinsic angle $\varnothing\psi$ can take on values in the range $[0, \varnothing\pi/2)$ in the first quadrant in Figs. 8a and 8b in the two-world picture, instead of the range $[0, \pi/4)$ of the angle ϕ in the Minkowski's diagrams, (Figs. 3a and 3b) in the one-world picture, is due to the non-existence of the light-cone concept in the two-world picture, as shall be shown later in this article.

There is an inverse to the full intrinsic Lorentz transformation (11) to be derived from the inverses of Figs. 8a and 8b. In obtaining the inverses to Figs. 8a and 8b, the projective non-inclined relativistic (or unprimed) particle's intrinsic affine frames, $(\varnothing\tilde{x}, \varnothing c\varnothing\tilde{t})$ and $(-\varnothing\tilde{x}^*, -\varnothing c\varnothing\tilde{t}^*)$, embedded in the flat proper intrinsic metric spacetimes, $(\varnothing\rho', \varnothing c\varnothing t')$ and $(-\varnothing\rho'^*, -\varnothing c\varnothing t'^*)$, respectively in those figures, must be considered to be in intrinsic motion at a negative intrinsic speed $-\varnothing v$ (of the same magnitude as $\varnothing v$ in Figs. 8a and 8b), relative to the inclined proper (or primed) particle's intrinsic affine frames, $(\varnothing\tilde{x}\,', \varnothing c\varnothing\tilde{t}\,')$ and $(-\varnothing\tilde{x}\,'^*, -\varnothing c\varnothing\tilde{t}\,'^*)$, in Figs. 8a and 8b. This implies that the non-inclined $(\varnothing\tilde{x}, \varnothing c\varnothing\tilde{t})$ and $(-\varnothing\tilde{x}^*, -\varnothing c\varnothing\tilde{t}^*)$ (without changing their positions in Figs. 8a and 8b), must be considered to be inclined at a negative intrinsic angle $-\varnothing\psi$ (of the same magnitude as $\varnothing\psi$ in Figs. 8a and 8b), relative to the inclined proper (or primed) particle's intrinsic affine frames, $(\varnothing\tilde{x}\,', \varnothing c\varnothing\tilde{t}\,')$ and $(-\varnothing\tilde{x}\,'^*, -\varnothing c\varnothing\tilde{t}\,'^*)$, respectively in those figures.

The inverse diagram to Fig. 8a that follows from the preceding paragraph is depicted in Figs. 9a. While Fig. 8a is valid with respect to the 3-observers Peter in Σ' and Peter* in $-\Sigma'^*$, its inverse Fig. 9a is valid with respect to the 1-observers Peter^0 in ct' and Peter^{0*} in $-ct'^*$, as indicated. This is so because, the clockwise rotation of the unprimed relative to the primed intrinsic affine coordinates by a negative intrinsic angle $-\varnothing\psi$ in Fig. 9a is equivalent to the clockwise rotation of the primed relative to the unprimed intrinsic affine coordinates by a positive intrinsic angle $\varnothing\psi$ in Fig. 8b. Hence Figs. 8b and 9a are both valid relative to the 'stationary' 1-observers in the proper metric time dimensions ct' and $-ct'^*$, with respect to whom clockwise rotation is positive.

Following the derivations of the transformations (9a) and (9b) with respect to the 3-observer in Σ' from Fig. 8a, the inclined primed intrinsic affine space coordinate $\varnothing\tilde{x}\,'$ is the projection of the non-inclined unprimed intrinsic affine space coordinate $\varnothing\tilde{x}$ embedded in $\varnothing\rho'$ along the horizontal in the first quadrant in Fig. 9a. That is, $\varnothing\tilde{x}\,' = \varnothing\tilde{x}\cos(-\varnothing\psi)$. Hence we must write the inverse transformation,

$$\varnothing\tilde{x} = \varnothing\tilde{x}\,'\sec\varnothing\psi\ . \tag{12a}$$

The 'measurable' intrinsic affine space coordinate transformation (12a) (with intrinsic laboratory rod by Peter^0 in ct'), is all that should have been possible with respect to Peter^0 in ct' in Fig. 9a, but for the fact that the unprimed intrinsic affine time coordinate $\varnothing c\varnothing\tilde{t}$ embedded in the proper intrinsic metric time dimension $\varnothing c\varnothing t'$ along the vertical in the first quadrant also projects component $\varnothing c\varnothing\tilde{t}\cos\varnothing\eta$ along the inclined $\varnothing\tilde{x}\,'$ in that quadrant.

Now $\varnothing\eta + \varnothing\psi = \varnothing\pi/2$, hence, $\varnothing\eta = \varnothing\pi/2 - \varnothing\psi$, in Fig. 9a. Therefore the component projected along the inclined $\varnothing\tilde{x}\,'$ by $\varnothing c\varnothing\tilde{t}$ along the vertical in the first quadrant is

$$\varnothing c\varnothing\tilde{t}\cos\varnothing\eta = \varnothing c\varnothing\tilde{t}\cos(\varnothing\pi/2 - \varnothing\psi) = \varnothing c\varnothing\tilde{t}\sin\varnothing\psi\ .$$

This 'non-measurable' projective intrinsic affine time coordinate $\varnothing c\varnothing\tilde{t}\,'\sin\varnothing\psi$ along the inclined $\varnothing\tilde{x}\,'$, must be added to $\varnothing\tilde{x}\,'\sec\varnothing\psi$ at the right-hand side of Eq. (12a) to obtain the net intrinsic affine coordinate projection along the inclined $\varnothing\tilde{x}\,'$ in the first quadrant as, $\varnothing\tilde{x}\,'\sec\varnothing\psi + \varnothing c\varnothing\tilde{t}\sin\varnothing\psi$.

Apart from the net intrinsic affine coordinate projection $\varnothing\tilde{x}\,'\sec\varnothing\psi + \varnothing c\varnothing\tilde{t}\sin\varnothing\psi$. along the inclined $\varnothing\tilde{x}\,'$ in the first quadrant, the inclined $\varnothing c\varnothing\tilde{t}\,'$ in the second quadrant is the projection of $\varnothing c\varnothing\tilde{t}$ along the vertical in the first quadrant in Fig. 9a, for which the relation, $\varnothing c\varnothing\tilde{t}\,' = \varnothing c\varnothing\tilde{t}\cos\varnothing\psi$, hence, $\varnothing c\varnothing\tilde{t} = \varnothing c\varnothing\tilde{t}\,'\sec\varnothing\psi$, obtains. This relation must complement the net intrinsic affine coordinate projection along the inclined $\varnothing\tilde{x}\,'$ in the first quadrant. It must be used to replace $\varnothing c\varnothing\tilde{t}$ by $\varnothing c\varnothing\tilde{t}\,'\sec\varnothing\psi$ in the net intrinsic affine coordinate projection along the inclined $\varnothing\tilde{x}\,'$ giving, $\varnothing\tilde{x}\,'\sec\varnothing\psi + \varnothing c\varnothing\tilde{t}\,'\tan\varnothing\psi$. Equation (12a) for pure 'measurable' intrinsic affine space coordinates, must be replaced by the following

$$\varnothing\tilde{x} = \varnothing\tilde{x}\,'\sec\varnothing\psi + \varnothing c\varnothing\tilde{t}\,'\tan\varnothing\psi\ , \qquad (12b)$$

w.r.t. 1-observer in ct'. It is again to be noted that this partial inverse intrinsic Lorentz transformation cannot be derived by direct intrinsic affine coordinate projections in Fig. 9a.

The inverse diagram to Fig. 8b, which follows from the discussion that leads to Fig. 9a as the inverse of Fig. 8a, is depicted in Fig. 9b. Figure 9b is valid with respect to the 3-observer Peter in Σ' and his symmetry-partner Peter* in $-\Sigma'^{*}$ as indicated. This is so because, the anti-clockwise rotation of the unprimed relative to the primed intrinsic affine coordinates by negative intrinsic angle $-\varnothing\psi$ in Fig. 9b is equivalent to the anti-clockwise rotation of the primed relative to the unprimed intrinsic affine coordinates by positive intrinsic angle $\varnothing\psi$ in Fig. 8a. Hence Figs. 8a and 9b are both valid relative to the 'stationary' 3-observers in the proper metric Euclidean 3-spaces Σ' and $-\Sigma'^{*}$, with respect to whom anti-clockwise rotation is positive.

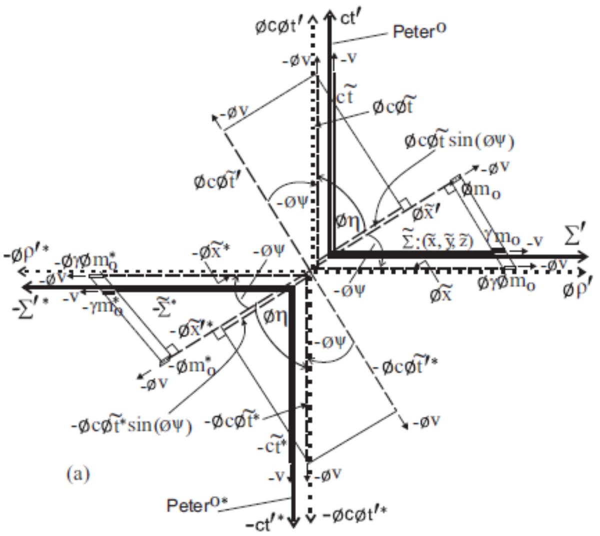

Fig. 9. (a) The inverse diagrams to Fig. 8a used to derive partial inverse intrinsic Lorentz transformations and partial inverse Lorentz transformations with respect to the 'stationary' 1-observers in the proper metric time dimensions in the positive and negative universes

The inclined primed intrinsic affine time coordinate $\varnothing c\varnothing\tilde{t}\,'$ is the projection of the unprimed intrinsic affine time coordinate $\varnothing c\varnothing\tilde{t}$ embedded in $\varnothing c\varnothing t'$ along the vertical in the first quadrant in Fig. 9b. That is, $\varnothing c\varnothing\tilde{t}\,' = \varnothing c\varnothing\tilde{t}\,\cos(-\varnothing\psi)$. Hence we must express $\varnothing c\varnothing\tilde{t}$ in terms of its projection $\varnothing c\varnothing\tilde{t}\,'$ and write

$$\varnothing c\varnothing\tilde{t} = \varnothing c\varnothing\tilde{t}\,'\sec\varnothing\psi\ . \qquad (13a)$$

The 'measurable' intrinsic affine time coordinate transformation (13a) (with intrinsic laboratory clock by Peter in Σ'), is all that should have been possible with respect to the 3-observer Peter in Σ' in Fig. 9b, but for the fact that the unprimed intrinsic affine space coordinate $\varnothing\tilde{x}$ embedded in the proper intrinsic metric space $\varnothing\rho'$ along the horizontal in the first quadrant, also projects the following component along the inclined $\varnothing c\varnothing\tilde{t}\,'$ in that quadrant,

$$\varnothing\tilde{x}\cos(\varnothing\pi/2 - \varnothing\psi) = \varnothing\tilde{x}\sin\varnothing\psi = \varnothing\tilde{x}\,'\tan\varnothing\psi \ ,$$

since, $\varnothing\tilde{x}\,' = \varnothing\tilde{x}\cos(-\varnothing\psi)$, which follows from the inclination of $\varnothing\tilde{x}$ along $\varnothing\rho'$ to the inclined $\varnothing\tilde{x}\,'$ in the fourth quadrant.

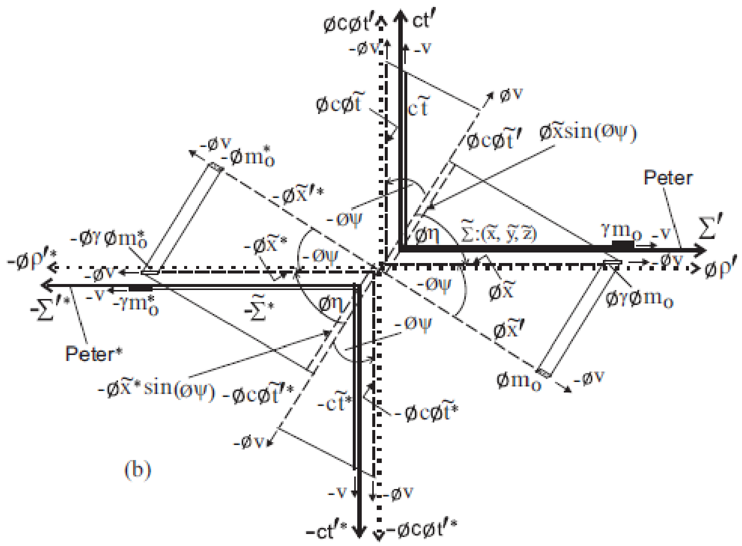

Fig. 9. (b) The inverse diagrams to Fig. 8b used to derive partial inverse intrinsic Lorentz transformations and partial inverse Lorentz transformations with respect to the 'stationary' 3-observers in the proper Euclidean 3-spaces in the positive and negative universes

The 'non-measurable' (with intrinsic laboratory clock) projective intrinsic affine space coordinate $\varnothing\tilde{x}\,'\times\tan\varnothing\psi$ along the inclined $\varnothing c\varnothing\tilde{t}\,'$, must be added to the right-hand side of the pure 'measurable' intrinsic affine time coordinate transformation (13a) to have

$$\varnothing c\varnothing\tilde{t} = \varnothing c\varnothing\tilde{t}\,'\sec\varnothing\psi + \varnothing\tilde{x}\,'\tan\varnothing\psi \ ; \qquad (\text{w.r.t. } 3-\text{observer Peter in } ct') \ . \tag{13b}$$

Collecting the partial intrinsic affine coordinate transformations (12b) and (13b) gives the inverse intrinsic Lorentz transformation (inverse $\varnothing$LT) with respect to 4-observer (Peter, Peter^0) on the flat proper metric spacetime (Σ', ct') as

$$\begin{aligned}\varnothing c\varnothing\tilde{t} &= \varnothing c\varnothing\tilde{t}\,'\sec\varnothing\psi + \varnothing\tilde{x}\,'\tan\varnothing\psi \ ; \\ &\quad (\text{w.r.t. } 3-\text{observer Peter in } \Sigma') \\ \varnothing\tilde{x} &= \varnothing\tilde{x}\,'\sec\varnothing\psi + \varnothing c\varnothing\tilde{t}\,'\tan\varnothing\psi \ ; \\ &\quad (\text{w.r.t. } 1-\text{observer Peter}^{\,0} \text{ in } ct') \ ,\end{aligned} \tag{14}$$

for $-\varnothing\pi/2 < \varnothing\psi < \varnothing\pi/2$ (temporarily). This temporary range of the intrinsic angles $\varnothing\psi$ in the positive universe shall be modified shortly in this section, as mentioned earlier.

Now considering the intrinsic motion of the origin, $\varnothing\tilde{x}' = 0$, of the intrinsic affine space coordinate $\varnothing\tilde{x}'$ of the particle's primed intrinsic affine frame, system (14) simplifies as

$$\varnothing\tilde{x} = \varnothing c\varnothing\tilde{t}' \tan\varnothing\psi \ \text{ and } \ \varnothing c\varnothing\tilde{t} = \varnothing c\varnothing\tilde{t}' \sec\varnothing\psi\ . \tag{15}$$

Division of the first into the second equation of system (15) gives

$$\varnothing\tilde{x}/\varnothing c\varnothing\tilde{t} = \varnothing v/\varnothing c = \sin\varnothing\psi\ ,$$

where the positive intrinsic speed, $\varnothing\tilde{x}/\varnothing\tilde{t} = \varnothing v$, is the intrinsic speed of the particle's primed intrinsic affine frame relative to the 'stationary' 3-observer Peter in Σ'. Hence,

$$\sin\varnothing\psi = \varnothing v/\varnothing c = \varnothing\beta\ ; \tag{16}$$

$$\sec\varnothing\psi = (1 - \varnothing v^2/\varnothing c^2)^{-1/2} = \varnothing\gamma\ . \tag{17}$$

Using relations (16) and (17) in systems (11) gives

$$\begin{aligned}
\varnothing c\varnothing\tilde{t}' &= (1 - \frac{\varnothing v^2}{\varnothing c^2})^{-1/2}(\varnothing c\varnothing\tilde{t} - \frac{\varnothing v}{\varnothing c}\varnothing\tilde{x})\ ;\\
&\quad (\text{w.r.t. } 1-\text{observer Peter}^{\,0} \text{ in } ct')\ ;\\
\varnothing\tilde{x}' &= (1 - \frac{\varnothing v^2}{\varnothing c^2})^{-1/2}(\varnothing\tilde{x} - \frac{\varnothing v}{\varnothing c}\varnothing c\varnothing\tilde{t})\ ;\\
&\quad (\text{w.r.t. } 3-\text{observer Peter in } \Sigma')
\end{aligned}$$

or

$$\begin{aligned}
\varnothing\tilde{t}' &= \varnothing\gamma(\varnothing\tilde{t} - \frac{\varnothing v}{\varnothing c^2}\varnothing\tilde{x})\ ;\\
&\quad (\text{w.r.t. } 1-\text{observer Peter}^{\,0} \text{ in } ct')\ ;\\
\varnothing\tilde{x}' &= \varnothing\gamma(\varnothing\tilde{x} - \varnothing v\varnothing\tilde{t})\ ;\\
&\quad (\text{w.r.t. } 3-\text{observer Peter in } \Sigma')\ .
\end{aligned} \tag{18}$$

And using equations (16) and (17) in system (14) give

$$\begin{aligned}
\varnothing c\varnothing\tilde{t} &= (1 - \frac{\varnothing v^2}{\varnothing c^2})^{-1/2}(\varnothing c\varnothing\tilde{t}' + \frac{\varnothing v}{\varnothing c}\varnothing\tilde{x}')\ ;\\
&\quad (\text{w.r.t. } 3-\text{observer Peter in } \Sigma')\ ;\\
\varnothing\tilde{x} &= (1 - \frac{\varnothing v^2}{\varnothing c^2})^{-1/2}(\varnothing\tilde{x}' + \frac{\varnothing v}{\varnothing c}\varnothing c\varnothing\tilde{t}')\ ;\\
&\quad (\text{w.r.t. } 1-\text{observer Peter}^{\,0} \text{ in } ct')
\end{aligned}$$

or

$$\begin{aligned}
\varnothing\tilde{t} &= \varnothing\gamma(\varnothing\tilde{t}' + \frac{\varnothing v}{\varnothing c^2}\varnothing\tilde{x}')\ ;\\
&\quad (\text{w.r.t. } 3-\text{observer Peter in } \Sigma')\ ;\\
\varnothing\tilde{x} &= \varnothing\gamma(\varnothing\tilde{x}' + \varnothing v\varnothing\tilde{t}')\ ;\\
&\quad (\text{w.r.t. } 1-\text{observer Peter}^{\,0} \text{ in } ct')\ .
\end{aligned} \tag{19}$$

Systems (18) and (19) are the explicit forms in terms of intrinsic speed $\varnothing v$ of the intrinsic Lorentz transformation ($\varnothing$LT) of extended intrinsic affine coordinates and its inverse respectively, on the flat two-dimensional proper intrinsic metric spacetime $(\varnothing\rho', \varnothing c\varnothing t')$ that underlies the flat four-dimensional proper metric spacetime (Σ', ct') in the positive universe. As can be easily verified, either system (11) or (14), or its explicit form (18) or (19), implies intrinsic Lorentz invariance ($\varnothing$LI) in terms of extended intrinsic affine spacetime coordinates in the positive universe namely,

$$\varnothing c^2\varnothing\tilde{t}^{\,2} - \varnothing\tilde{x}^{\,2} = \varnothing c^2\varnothing\tilde{t}'^{2} - \varnothing\tilde{x}'^{2}\ . \tag{20}$$

Just as the 4-observers (Peter, Peter^0) in the metric spacetime (Σ', ct') derives system (11), given explicitly as system (18), from Figs. 8a and 8b and system (14), given explicitly as system (19), from Figs. 9a and 9b in the positive universe, the symmetry-partner 4-observer* (Peter*, Peter^{0}*) in the flat proper metric spacetime $(-\Sigma'^{*}, -ct'^{*})$ in the negative universe, derives the ∅LT from Figs. 8a and 8b and its inverse from Figs. 9a and 9b and write respectively as follows

$$\begin{aligned} -\varnothing c\varnothing\tilde{t}'^{*} &= -\varnothing c\varnothing\tilde{t}^{*}\sec\varnothing\psi - (-\varnothing\tilde{x}^{*})\tan\varnothing\psi \;; \\ &\quad (\text{w.r.t. Peter}^{0*}\text{ in } -ct'^{*})\;; \\ -\varnothing\tilde{x}'^{*} &= -\varnothing\tilde{x}^{*}\sec\varnothing\psi - (-\varnothing c\varnothing\tilde{t}^{*})\tan\varnothing\psi \;; \\ &\quad (\text{w.r.t. Peter}^{*}\text{ in } -\Sigma'^{*}) \end{aligned} \tag{21}$$

and

$$\begin{aligned} -\varnothing c\varnothing\tilde{t}^{*} &= -\varnothing c\varnothing\tilde{t}'^{*}\sec\varnothing\psi + (-\varnothing\tilde{x}'^{*})\tan\varnothing\psi \;; \\ &\quad (\text{w.r.t. Peter}^{*}\text{ in } -\Sigma'^{*})\;; \\ -\varnothing\tilde{x}^{*} &= -\varnothing\tilde{x}'^{*}\sec\varnothing\psi + (-\varnothing c\varnothing\tilde{t}'^{*})\tan\varnothing\psi \;; \\ &\quad (\text{w.r.t. Peter}^{0*}\text{ in } -ct'^{*})\;, \end{aligned} \tag{22}$$

for $-\varnothing\pi/2 < \varnothing\psi < \varnothing\pi/2,\;(\text{temporarily})$. This temporary range of the intrinsic angles $\varnothing\psi$ in systems (21) and (22) in the negative universe shall be modified shortly in this section.

Systems (21) and (22) can also be put in their explicit forms in terms of the intrinsic speed $\varnothing v$ respectively as follows by virtue of Eqs. (16) and (17)

$$\begin{aligned} -\varnothing\tilde{t}'^{*} &= \varnothing\gamma(-\varnothing\tilde{t}^{*} - \frac{\varnothing v}{\varnothing c^2}(-\varnothing\tilde{x}^{*}))\;; \\ &\quad (\text{w.r.t. Peter}^{0*}\text{ in } -ct'^{*})\;; \\ -\varnothing\tilde{x}'^{*} &= \varnothing\gamma(-\varnothing\tilde{x}^{*} - \varnothing v(-\varnothing\tilde{t}^{*}))\;; \\ &\quad (\text{w.r.t. Peter}^{*}\text{ in } -\Sigma'^{*}) \end{aligned} \tag{23}$$

and

$$\begin{aligned} -\varnothing\tilde{t}^{*} &= \varnothing\gamma(-\varnothing\tilde{t}'^{*} + \frac{\varnothing v}{\varnothing c^2}(-\varnothing\tilde{x}'^{*}))\;; \\ &\quad (\text{w.r.t. Peter}^{*}\text{ in } -\Sigma'^{*})\;; \\ -\varnothing\tilde{x}^{*} &= \varnothing\gamma(-\varnothing\tilde{x}'^{*} + \varnothing v(-\varnothing\tilde{t}'^{*}))\;; \\ &\quad (\text{w.r.t. Peter}^{0*}\text{ in } -ct'^{*})\;. \end{aligned} \tag{24}$$

Again system (21) or (22), or the explicit form (23) or (24), implies intrinsic Lorentz invariance (∅LI) in terms of extended intrinsic affine spacetime coordinates in the negative universe namely,

$$(-\varnothing c^2\varnothing\tilde{t}^{*})^2 - (-\varnothing\tilde{x}^{*})^2 = (-\varnothing c^2\varnothing\tilde{t}'^{*})^2 - (-\varnothing\tilde{x}'^{*})^2 \tag{25}$$

The intrinsic Lorentz transformation (∅LT) of system (11) and its inverse of system (14), or their explicit forms of systems (18) and (19), and the intrinsic Lorentz invariance (20) they imply, pertain to two-dimensional intrinsic special theory of relativity (∅SR) on the flat two-dimensional proper intrinsic metric spacetime $(\varnothing\rho', \varnothing c\varnothing t')$ underlying the flat four-dimensional proper metric spacetime (Σ', ct') of the positive universe. In symmetry, the ∅LT and its inverse of systems (21) and (22), or their explicit forms (23) and (24), and the intrinsic Lorentz invariance (25) they imply, pertain to the intrinsic special theory of relativity (∅SR) on the flat two-dimensional proper intrinsic metric spacetime $(-\varnothing\rho'^{*}, -\varnothing c\varnothing t'^{*})$ underlying the flat four-dimensional proper metric spacetime $(-\Sigma'^{*}, -ct'^{*})$ of the negative universe.

Now, the projective particle's unprimed intrinsic affine frame $(\varnothing\tilde{x},\, \varnothing c\varnothing\tilde{t}\,)$ is embedded in the flat proper intrinsic metric spacetime $(\varnothing\rho', \varnothing c\varnothing t')$

in our universe and $(-\varnothing\tilde{x}^*, -\varnothing c\varnothing\tilde{t}^*)$ is embedded in the flat proper intrinsic metric spacetime $(-\varnothing\rho'^*, -\varnothing c\varnothing t'^*)$ in the negative universe in Figs. 8a and 8b and Figs. 9a and 9b. It then follows that the unprimed intrinsic affine coordinates, $\varnothing\tilde{x}$, $\varnothing c\tilde{t}, -\varnothing\tilde{x}^*$ and $-\varnothing c\tilde{t}^*$, correspond to the unprimed intrinsic metric coordinates, $\varnothing x, \varnothing c\varnothing t, -\varnothing x^*$ and $-\varnothing c\varnothing t^*$ respectively, in which they are embedded, where $\varnothing\tilde{x}$ and $\varnothing x$ are equal in length, $\varnothing c\varnothing\tilde{t}$ and $\varnothing c\varnothing t$ are equal in length, etc.

The inclined primed intrinsic affine coordinates, $\varnothing\tilde{x}\,'$, $\varnothing c\varnothing\tilde{t}\,'$, $-\varnothing\tilde{x}\,'^*$ and $-\varnothing c\varnothing\tilde{t}\,'^*$, likewise correspond to the primed intrinsic metric coordinates, $\varnothing x\,', \varnothing c\varnothing t'$, $-\varnothing x\,'^*$ and $-\varnothing c\varnothing t'^*$ respectively, in which they were embedded when the particles was at rest relative to the observers initially, where $\varnothing\tilde{x}\,'$ and $\varnothing x'$ are equal in length, $\varnothing c\varnothing\tilde{t}\,'$ and $\varnothing c\varnothing t'$ are equal in length, etc. The unprimed intrinsic metric coordinates, $\varnothing x, \varnothing c\varnothing t, -\varnothing x^*$ and $-\varnothing c\varnothing t^*$, and the primed intrinsic metric coordinates, $\varnothing x\,'$, $\varnothing c\varnothing t\,', -\varnothing x\,'^*$ and $-\varnothing c\varnothing t\,'^*$, are coordinates of the proper intrinsic metric spacetimes, $(\varnothing\rho', \varnothing c\varnothing t')$ and $(-\varnothing\rho'^*, -\varnothing c\varnothing t\,'^*)$, in the context of $\varnothing$SR in these notations.

Although the primed intrinsic metric coordinates $\varnothing x'$ and $\varnothing c\varnothing t\,'$ are not rotated relative to unprimed intrinsic metric coordinates $\varnothing x$ and $\varnothing c\varnothing t$ in our universe and $-\varnothing x\,'^*$ and $-\varnothing c\varnothing t\,'^*$ are not rotated relative to $-\varnothing x^*$ and $-\varnothing c\varnothing t^*$ in the negative universe in Figs. 8a and 8b and Figs. 9a and 9b, it follows from the preceding paragraph that the derivations from Eqs. (9a) and (9b) through Eq. (25) can be written in terms of the corresponding unprimed intrinsic metric coordinates, $\varnothing x$, $\varnothing c\varnothing t$, $-\varnothing x^*$ and $-\varnothing c\varnothing t^*$, and the corresponding primed intrinsic metric coordinates, $\varnothing x'$, $\varnothing c\varnothing t'$, $-\varnothing x\,'^*$ and $-\varnothing c\varnothing t\,'^*$, for fictitiously rotated $(\varnothing x', \varnothing c\varnothing t')$ relative to $(\varnothing x, \varnothing c\varnothing t)$ in our universe and fictitiously rotated $(-\varnothing x'^*, -\varnothing c\varnothing t'^*)$ relative to $(-\varnothing x^*, -\varnothing c\varnothing t^*)$ in the negative universe. We must simply remove the tilde label on the intrinsic affine coordinates in those equations. The resulting equations shall not be written in this article in order to conserve space. It shall be noted however that when the intrinsic Lorentz invariance ($\varnothing$LI) (20) and (25) are written in terms of the corresponding intrinsic metric coordinates, the resulting equations express $\varnothing$LI on the proper intrinsic metric spacetimes $(\varnothing\rho', \varnothing c\varnothing t')$ and $(-\varnothing\rho'^*, -\varnothing c\varnothing t'^*)$ in the context of $\varnothing$SR in our universe and the negative universe.

Having derived the $\varnothing$LT of system (11) on page 71 and its inverse of system (14) on page 73, and their explicit forms of systems (18) and (19), in the context of intrinsic 2-geometry $\varnothing$SR in the positive universe, we shall now obtain their outward (or physical) manifestations on the flat four-dimensional spacetime in the context of 4-geometry special theory of relativity (SR). We do not have to draw a new set of diagrams in the two-world picture in which extended straight line affine spacetime coordinates $\tilde{x}\,'$ and $c\tilde{t}\,'$ of the particle's primed affine frame are inclined relative to their projective extended affine coordinates $\tilde{x}$ and $c\tilde{t}$ respectively, of the particle's unprimed affine frame on the vertical (x, ct)-hyperplane, while the affine coordinates, $\tilde{y}\,'$ and $\tilde{z}\,'$, of the particle's primed frame along which relative motion of SR does not occur are not rotated on the vertical spacetime hyperplane. Indeed such diagram does exist, since, as noted earlier, the particle's primed affine frames, $(\tilde{x}\,', \tilde{y}\,', \tilde{z}\,', c\tilde{t}\,')$ and $(-\tilde{x}\,'^*, -\tilde{y}\,'^*, -\tilde{z}\,'^*, -c\tilde{t}\,'^*)$, no longer exist in the geometries of Figs. 8a and 8b and Figs. 9a and 9b. Figures 8a and 8b and their inverses, Figs. 9a and 9b, in which the affine intrinsic spacetime coordinates are actually rotated are the only diagrams from which the $\varnothing$LT and LT and their inverses must be derived in the two-world picture.

As discussed earlier, the flat four dimensional proper metric spacetime, $(\Sigma', ct') \equiv (x', y', z', ct')$, is the outward (or physical) manifestation of the flat two-dimensional proper intrinsic metric spacetime $(\varnothing\rho', \varnothing c\varnothing t')$ in Fig. 7. Likewise the extended mutually orthogonal straight line unprimed affine coordinates, $\tilde{x}, \tilde{y}$ and $\tilde{z}$, is considered to constitute a flat affine 3-space, shown as a straight line and denoted by $\tilde{\Sigma}$ along the horizontal in the first quadrant. The affine 3-space $\tilde{\Sigma}$, overlying (or embedded in) Σ', is the outward manifestation of the extended straight line unprimed intrinsic affine coordinate $\varnothing\tilde{x}$ overlying (or embedded in) $\varnothing\rho'$ in Figs. 8a and 8b. And the extended straight line unprimed affine time coordinate $c\tilde{t}$ is the outward manifestation of the extended straight line unprimed intrinsic affine time coordinate $\varnothing c\varnothing\tilde{t}$ along the vertical in Figs. 8a and 8b. The extended straight line primed affine spacetime coordinates, $\tilde{x}', \tilde{y}', \tilde{z}'$ and $\tilde{ct}'$, are likewise the non-existing outward manifestations of the inclined extended primed intrinsic affine spacetime coordinates, $\varnothing\tilde{x}'$ and $\varnothing c\varnothing\tilde{t}'$, in Figs. 9a and 9b.

It follows by virtue of the preceding paragraph that the LT and its inverse in the context of SR are the outward manifestations of the intrinsic Lorentz transformation ($\varnothing$LT) of system (11) or (19) and its inverse of system (14) or (19). We must simply remove the symbol $\varnothing$ in systems (11) and (14) to have the LT and its inverse in SR in their usual forms (but now in terms of affine spacetime coordinates) respectively as

$$\begin{aligned}
\tilde{ct}' &= \tilde{ct}\sec\psi - \tilde{x}\tan\psi \; ; \\
&\quad (\text{w.r.t. to } 1-\text{observer Peter}^0 \text{ in } ct') \; ; \\
\tilde{x}' &= \tilde{x}\sec\psi - \tilde{ct}\tan\psi \; ; \; \tilde{y}' = \tilde{y} \text{ and } \tilde{z}' = \tilde{z} \; ; \\
&\quad (\text{w.r.t. } 3-\text{observer Peter in } \Sigma')
\end{aligned} \tag{26}$$

and

$$\begin{aligned}
\tilde{ct} &= \tilde{ct}'\sec\psi + \tilde{x}'\tan\psi \; ; \\
&\quad (\text{w.r.t. } 3-\text{observer Peter in } \Sigma') \; ; \\
\tilde{x} &= \tilde{x}'\sec\psi + \tilde{ct}'\tan\psi \; ; \; \tilde{y} = \tilde{y}' \text{ and } \tilde{z} = \tilde{z}' \; ; \\
&\quad (\text{w.r.t. } 1-\text{observer Peter}^0 \text{ in } ct') \; ,
\end{aligned} \tag{27}$$

for $-\pi/2 < \psi < \pi/2$; (temporarily).

The trivial transformations, $\tilde{y} = \tilde{y}'$ and $\tilde{z} = \tilde{z}'$, of the affine coordinates along which relative motion of SR does not occur have been added to the second equation of systems (26) obtained by simply dropping the symbol $\varnothing$ in system (11) on page 71 and to the second equation of system (27) obtained by simply dropping the symbol $\varnothing$ in system (14) on page 73, thereby making the resulting LT of system (26) and its inverse of system (27) consistent with the 4-geometry of SR. The angle ψ being the outward manifestation in spacetime of the intrinsic angle $\varnothing\psi$ in intrinsic spacetime, has the same temporary range in systems (26) and (27) as $\varnothing\psi$ in systems (11) and (14). This temporary range of ψ shall also be modified shortly in this section.

System (26) indicates that the primed affine spacetime coordinates, $\tilde{x}'$ and $\tilde{ct}'$, are rotated by equal angle ψ relative to the unprimed affine spacetime coordinates, $\tilde{x}$ and $\tilde{ct}$, respectively, while $\tilde{y}'$ is not rotated relative $\tilde{y}$ and $\tilde{z}'$ is not rotated relative to $\tilde{z}$ in the context of SR, and system (27) indicates that $\tilde{x}$ and $\tilde{ct}$ are rotated by equal negative angle $-\psi$ relative to $\tilde{x}'$ and $\tilde{ct}'$ respectively. However the relative rotations of the affine coordinates of the four-dimensional affine spacetime do not exist in reality, as mentioned earlier. The implied rotations of affine spacetime coordinates by systems (26) and (27) may be referred to as intrinsic (i.e. non-observable or hypothetical) relative rotations of affine spacetime coordinates. This is what the actual relative rotations of intrinsic affine spacetime coordinates in Figs. 8a and 8b and Figs. 9a and 9b represent.

Considering the motion of the spatial origin, $\tilde{x}' = \tilde{y}' = \tilde{z}' = 0$, of the particle's primed affine frame, system (27) reduces as

$$\tilde{ct} = \tilde{ct}'\sec\psi \;\text{ and }\; \tilde{x} = \tilde{x}'\tan\psi \; . \tag{28}$$

And dividing the second equation into the first equation of system (28) gives

$$\tilde{x}/\tilde{ct} = v/c = \sin\psi \; ,$$

where, $\tilde{x}/\tilde{t} = v$, is the speed of the particle's unprimed affine frame $(\tilde{x}, \tilde{y}, \tilde{z}, c\tilde{t})$ relative to the 'stationary' 3-observer Peter in the metric 3-space Σ'. Hence

$$\begin{aligned} \sin\psi &= v/c = \beta\,; \quad (29)\\ \sec\psi &= (1 - v^2/c^2)^{-1/2} = \gamma\,. \quad (30) \end{aligned}$$

Relations (29) and (30) on flat 4-dimensional spacetime corresponds to relations (16) and (17) respectively on flat 2-dimensional intrinsic spacetime. By using Eqs. (29) and (30) in systems (26) and (27) we obtain the LT and its inverse in their usual explicit forms respectively as

$$\begin{aligned} \tilde{t}\,' &= \gamma\,(\tilde{t} - \frac{v}{c^2}\tilde{x}\,)\,;\\ &\quad (\text{w.r.t. } 1-\text{observer Peter}^{\,0} \text{ in } ct')\,;\\ \tilde{x}\,' &= \gamma\,(\tilde{x} - v\tilde{t}\,)\,;\ \tilde{y}\,' = \tilde{y} \text{ and } \tilde{z}\,' = \tilde{z}\,;\\ &\quad (\text{w.r.t. } 3-\text{observer Peter in } \Sigma') \end{aligned} \quad (31)$$

and

$$\begin{aligned} \tilde{t} &= \gamma\,(\tilde{t}\,' + \frac{v}{c^2}\tilde{x}\,')\,;\\ &\quad (\text{w.r.t. } 3-\text{observer Peter in } \Sigma')\,;\\ \tilde{x} &= \gamma\,(\tilde{x}\,' + v\tilde{t}\,')\,;\ \tilde{y} = \tilde{y}\,' \text{ and } \tilde{z} = \tilde{z}\,'\,;\\ &\quad (\text{w.r.t. } 1-\text{observer Peter}^{\,0} \text{ in } ct')\,. \end{aligned} \quad (32)$$

Systems (31) and (32) are the outward (or physical) manifestations on the flat four-dimensional proper metric spacetime (Σ', ct') in the context of SR, of systems (18) and (19) respectively, on the flat two-dimensional proper intrinsic metric spacetime $(\varnothing\rho', \varnothing c\varnothing t')$ in the context of ∅SR in our (or positive) universe.

System (26) or (27), or the explicit form (31) or (32), implies Lorentz invariance (LI) in SR in the positive universe namely,

$$c^2\tilde{t}^{\,2} - \tilde{x}^{\,2} - \tilde{y}^{\,2} - \tilde{z}^{\,2} = c^2\tilde{t}^{\,\prime 2} - \tilde{x}^{\,\prime 2} - \tilde{y}^{\,\prime 2} - \tilde{z}^{\,\prime 2}\,. \quad (33)$$

This is the outward manifestation on the flat four-dimensional spacetime of SR of the intrinsic Lorentz invariance (∅LI) (20) on page 74 on flat two-dimensional intrinsic spacetime of ∅SR.

Just as the ∅LT and its inverse of system (11) on page 71 and (14) on page 73, in the context of ∅SR, are made manifested in systems (26) and (27) respectively in SR in the positive universe, the ∅LT and its inverse of systems (21) and (22) in ∅SR, are made manifested in LT and its inverse in SR in the negative universe respectively as

$$\begin{aligned} -c\tilde{t}^{\,\prime *} &= -c\tilde{t}^{\,*}\sec\psi - (-\tilde{x}^{\,*})\tan\psi\,;\\ &\quad (\text{w.r.t. } 1-\text{observer Peter}^{\,0*} \text{ in } -ct'^{*})\,;\\ -\tilde{x}^{\,\prime *} &= -\tilde{x}^{\,*}\sec\psi - (-c\tilde{t}^{\,*})\tan\psi\,;\ -\tilde{y}^{\,\prime *} = -\tilde{y}^{*}\\ &\quad \text{and } -\tilde{z}^{\,\prime *} = -\tilde{z}^{*}\,;\\ &\quad (\text{w.r.t. } 3-\text{observer Peter}^{\,*} \text{ in } -\Sigma'^{*}) \end{aligned} \quad (34)$$

and

$$\begin{aligned} -c\tilde{t}^{\,*} &= -c\tilde{t}^{\,\prime *}\sec\psi + (-\tilde{x}^{\,\prime *})\tan\psi\,;\\ &\quad (\text{w.r.t. } 3-\text{observer Peter}^{\,*} \text{ in } -\Sigma'^{*})\,;\\ -\tilde{x}^{\,*} &= -\tilde{x}^{\,\prime *}\sec\psi + (-c\tilde{t}^{\,\prime *})\tan\psi\,; -\tilde{y}^{*} = -\tilde{y}^{\,\prime *}\\ &\quad \text{and } -\tilde{z}^{*} = -\tilde{z}^{\,\prime *}\,;\\ &\quad (\text{w.r.t. } 1-\text{observer Peter}^{\,0*} \text{ in } -ct'^{*})\,. \end{aligned} \quad (35)$$

Using Eqs. (29) and (30) in systems (34) and (35) we obtain the LT and its inverse in the negative universe respectively as

$$
\begin{aligned}
-\tilde{t}'^{*} &= \gamma\,(-\tilde{t}^{*} - \frac{v}{c^2}(-\tilde{x}^{*}))\ ; \\
&\quad (\text{w.r.t. } 1-\text{observer Peter}^{0*} \text{ in } -ct'^{*})\ ; \\
-\tilde{x}'^{*} &= \gamma\,(-\tilde{x}^{*} - v(-\tilde{t}^{*}))\ ; -\tilde{y}'^{*} = -\tilde{y}^{*} \\
&\quad \text{and } -\tilde{z}'^{*} = -\tilde{z}^{*}\ ; \\
&\quad (\text{w.r.t. } 3-\text{observer Peter}^{*} \text{ in } -\Sigma'^{*})
\end{aligned} \tag{36}
$$

and

$$
\begin{aligned}
-\tilde{t}^{*} &= \gamma\,(-\tilde{t}'^{*} + \frac{v}{c^2}(-\tilde{x}'^{*}))\ ; \\
&\quad (\text{w.r.t. } 3-\text{observer Peter}^{*} \text{ in } -\Sigma'^{*})\ ; \\
-\tilde{x}^{*} &= \gamma\,(-\tilde{x}'^{*} + v(-\tilde{t}'^{*}))\ ;\ -\tilde{y}^{*} = -\tilde{y}'^{*} \\
&\quad \text{and } -\tilde{z}^{*} = -\tilde{z}'^{*}\ ; \\
&\quad (\text{w.r.t. } 1-\text{observer Peter}^{0*} \text{ in } -ct'^{*})\ .
\end{aligned} \tag{37}
$$

Systems (36) and (37) are the outward manifestations on the flat four-dimensional proper metric spacetime $(-\Sigma'^{*}, -ct'^{*})$ of SR, of systems (23) and (24) respectively, on the flat two-dimensional proper intrinsic metric spacetime $(-\varnothing\rho'^{*}, -\varnothing c\varnothing t'^{*})$ of $\varnothing$SR in the negative universe. Either the LT (34) or its inverse (35), or the explicit form (36) or (37), implies Lorentz invariance in SR in the negative universe namely,

$$(-c\tilde{t}^{*})^2 - (-\tilde{x}^{*})^2 - (-\tilde{y}^{*})^2 - (-\tilde{z}^{*})^2 = (-c\tilde{t}'^{*})^2 - (-\tilde{x}'^{*})^2 - (-\tilde{y}'^{*})^2 - (-\tilde{z}'^{*})^2\ . \tag{38}$$

This is the outward manifestation on the flat four-dimensional affine spacetime of SR of the intrinsic Lorentz invariance (25) on page 75 on flat two-dimensional intrinsic affine spacetime of $\varnothing$SR in the negative universe.

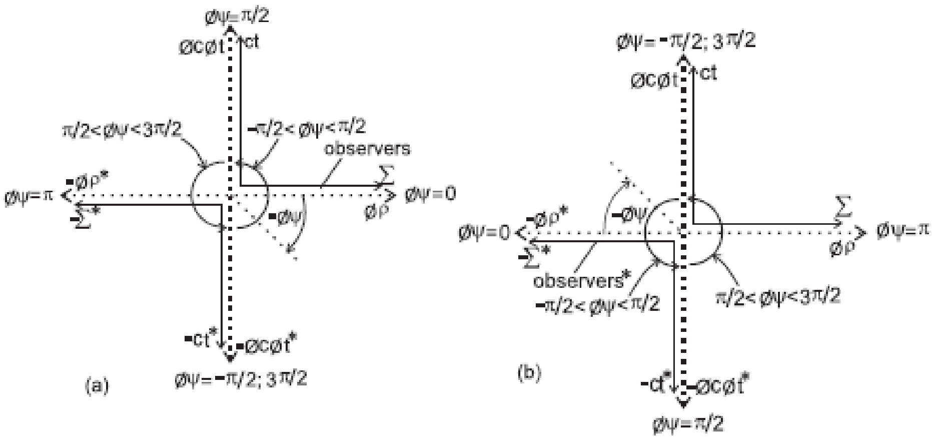

Fig. 10. The concurrent open intervals $(-\varnothing\pi/2, \varnothing\pi/2)$ and $(\varnothing\pi/2, 3\varnothing\pi/2)$ within which the intrinsic angle $\varnothing\psi$ can take on values: (a) with respect to 3-observers in 3-space in the positive universe and (b) with respect to 3-observers in 3-space in the negative universe

The restriction of the values of the intrinsic angle $\varnothing\psi$ to the spacetime half-hyperplane $(-\varnothing\pi/2 < \varnothing\psi < \varnothing\pi/2)$ with respect to observers in the positive universe in systems (11) and (14) and with respect to observers* in the negative universe in systems (21) and (22), is a temporary measure as indicated in those systems. The intrinsic angle $\varnothing\psi$ actually takes on values on the entire spacetime hyperplane $[-\varnothing\pi/2 \leq \varnothing\psi \leq 3\varnothing\pi/2]$ with respect to observers in the positive and negative universes, except that certain values of $\varnothing\psi$ namely, $-\varnothing\pi/2$, $\varnothing\pi/2$ and $3\varnothing\pi/2$, must be excluded, as shall be discussed more fully shortly. The values of $\varnothing\psi$ in the first cycle, as well as the negative senses of rotation (by negative intrinsic angle $-\varnothing\psi$), with respect to 3-observers in the 3-spaces in the positive and negative universes are illustrated in Figs. 10a and 10b respectively.

It shall again be noted that, since the unprimed affine coordinates, $\tilde{x}$, $\tilde{y}$, $\tilde{z}$ and $c\tilde{t}$, of the particle's unprimed affine frame are embedded in the flat four-dimensional proper metric spacetime (Σ', ct') in the positive (or our) universe, and the unprimed affine coordinates, $-\tilde{x}^*, -\tilde{y}^*, -\tilde{z}^*$ and $-c\tilde{t}^*$, are embedded in the flat proper metric spacetime $(-\Sigma'^*, -ct'^*)$ in the negative universe in Figs. 8a and 8b and Figs. 9a and 9b, the unprimed affine coordinates, $\tilde{x}$, $\tilde{y}$, $\tilde{z}$, $c\tilde{t}$, $-\tilde{x}^*, -\tilde{y}^*$, $-\tilde{z}^*$ and $-c\tilde{t}^*$, correspond to the unprimed metric coordinates, x, y, z, ct, $-x^*, -y^*$, $-z^*$ and $-ct^*$ respectively, in which they are embedded, where $\tilde{x}$ and x are equal in length, $c\tilde{t}$ and ct are equal in length, etc. It must be noted that the unprimed metric coordinates, x, y, z and ct, are coordinates of the proper metric spacetime (Σ', ct') and the unprimed metric coordinates, $-x^*, -y^*, -z^*$ and $-ct^*$, are coordinates of the proper metric spacetime $(-\Sigma'^*, -ct'^*)$ in these notations.

The primed affine spacetime coordinates, $\tilde{x}', \tilde{y}'$, $\tilde{z}', c\tilde{t}'$, $-\tilde{x}'^*, -\tilde{y}'^*$, $-\tilde{z}'^*$ and $-c\tilde{t}'^*$, correspond to the primed metric coordinates, x', y', z', ct', $-x'^*, -y'^*$, $-z'^*$ and $-ct'^*$ respectively, in which they were embedded initially when the particle was at rest relative to the 3-observer in the 3-spaces, Σ' and $-\Sigma'^*$, where $\tilde{x}'$ and x' are equal in length, $c\tilde{t}'$ and ct' are equal in length, etc. and $-ct'^*$, are coordinates of the proper metric spacetime $(-\Sigma'^*, -ct'^*)$, in which they were embedded initially when the particle was at rest relative to the 3-observer* in $-\Sigma'^*$ in the negative universe.

The intrinsic special theory of relativity ($\varnothing$SR) involves the relative rotation of extended intrinsic affine spacetime coordinates, which leads to the transformations of the initial extended straight line proper (or primed) intrinsic affine coordinates into extended straight line relativistic (or unprimed) intrinsic affine coordinates in the context of $\varnothing$SR. They are then made manifested in the transformations of the initial extended straight line proper (or primed) affine spacetime coordinates into extended straight line relativistic (or unprimed) affine spacetime coordinates in the context of SR. However $\varnothing$SR does not change the initial flat proper intrinsic metric spacetime $(\varnothing\rho', \varnothing c\varnothing t')$ into a flat relativistic intrinsic metric spacetime $(\varnothing\rho, \varnothing c\varnothing t)$ and SR does not change the initial flat proper metric spacetime (Σ', ct') into a flat relativistic metric spacetime (Σ, ct). Consequently the proper (or primed) metric spacetime coordinates, x', y', z', ct', $-x'^*, -y'^*$, $-z'^*$ and $-ct'^*$, when the particle was initially at rest relative to the observer and the unprimed metric coordinates, x, y, z, ct, $-x^*, -y^*, -z^*$ and $-ct^*$, when the particle is in motion relative to the observer, are coordinates of the proper metric spacetimes, (Σ', ct') and $(-\Sigma'^*, -ct'^*)$, in the context of SR in our notations in this article, as mentioned above.

Although neither are the primed affine coordinates $\tilde{x}', c\tilde{t}', -\tilde{x}'^*$ and $-c\tilde{t}'^*$, rotated relative to the unprimed affine metric coordinates, $\tilde{x}, \tilde{t}, -\tilde{x}^*$ and $-c\tilde{t}^*$, nor are the primed metric coordinates, $x', ct', -x'^*$ and $-ct'^*$, rotated relative to unprimed metric coordinates, $x, ct, -x^*$ and $-ct^*$, in the context of SR in Figs. 8a and 8b and Figs. 9a and 9b, it follows from the two paragraphs leading to the preceding paragraph that the primed affine coordinates, $\tilde{x}', \tilde{y}'$, $\tilde{z}', c\tilde{t}'$, $-\tilde{x}'^*, -c\tilde{t}'^*$, $-\tilde{z}'^*$ and $-c\tilde{t}'^*$, in the derivations from (26) through (38) can be written in terms of the corresponding primed metric coordinates, x', y', z', ct', $-x'^*, -y'^*$, $-z'^*$ and $-ct'^*$, respectively . We must simply remove the the tilde label on the coordinates in those equations.

Interestingly Eqs. (26) – (38) when written in terms of the corresponding metric spacetime coordinates are the observable or experimentally verifiable equations of SR to 4-observers in the metric spacetimes, (Σ', ct') and $(-\Sigma'^{*}, -ct'^{*})$. They shall not be written here in order to conserve space. It is important to note however that when the Lorentz invariance (LI) of Eqs. (33) and (38) are written in terms of the corresponding metric coordinates, the resulting equations express LI on the flat proper metric spacetimes, (Σ', ct') and $(-\Sigma'^{*}, -ct'^{*})$, in the context of SR in our universe and the negative universe.

We have thus derived a new set of spacetime/intrinsic spacetime diagrams namely, Figs. 8a and 8b and their inverses Figs. 9a and 9b, in the context of scheme II in Table I, or in the two-world picture, for deriving intrinsic Lorentz transformation (∅LT) and its inverse, in terms of extended straight line intrinsic affine spacetime coordinates, $\varnothing\tilde{x}', \varnothing c\varnothing\tilde{t}'$ and $\varnothing\tilde{x}, \varnothing c\varnothing\tilde{t}$, on the flat two-dimensional proper intrinsic metric spacetime $(\varnothing\rho', \varnothing c\varnothing t')$ of the two-dimensional intrinsic special theory of relativity (∅SR) in both the positive and negative universes, and for deriving the Lorentz transformation (LT) and its inverse in terms of extended straight line affine spacetime coordinates, $\tilde{x}, \tilde{y}, \tilde{z}, c\tilde{t}$ and $\tilde{x}', \tilde{y}', \tilde{z}', c\tilde{t}'$, as outward (or physical) manifestations on the flat four-dimensional proper spacetime of SR of the intrinsic Lorentz transformation (∅LT) and its inverse of ∅SR, in both the positive and negative universes. Figures 8a and 8b and their inverses Figs. 9a and 9b in the two-world picture, must replace the existing Minkowski's diagrams of Figs. 3a and 3b in the context of scheme I in Table I, or in the one-world picture.

The skewness of the rotated spacetime coordinates in the Minkowski diagrams of Figs. 3a and 3b (and in the Loedel and Brehme diagrams of Figs. 4a and 4b), from which the LT and its inverse have sometimes been derived until now in the existing one-world picture, has been remarked to be undesirable earlier in this article, because the 'stationary' observer in the frame with rotated spacetime coordinates could detect the skewness of the coordinates of his frame as an effect of the uniform motion of his frame. Moreover the skewness of the rotated coordinates of the 'moving' frame vis-a-vis the non-skewed coordinates of the 'stationary' frame (in the Minkowski diagrams), gives apparent preference to one of two frames in uniform relative motion.

On the other hand, there is no skewness of the rotated primed intrinsic affine spacetime coordinates, $\varnothing\tilde{x}'$ and $\varnothing c\varnothing\tilde{t}'$ or of the unprimed intrinsic affine coordinates, $\varnothing\tilde{x}$ and $\varnothing c\varnothing\tilde{t}$, in Figs. 8a, 8b, 9a and 9b, as mentioned earlier. The diagrams of Figs. 8a, 8b, 9a and 9b in the two-world picture, do not give apparent preference to any one of the pair of intrinsic affine frames, $(\varnothing\tilde{x}', \varnothing c\varnothing\tilde{t}')$ and $(\varnothing\tilde{x}, \varnothing c\varnothing\tilde{t})$, in relative intrinsic motion, since both intrinsic frames have mutually pseudo-orthogonal intrinsic affine spacetime coordinates in each of those figures. Figures 8a and 8b and Figs. 9a and 9b do not contain rotated primed affine spacetime coordinates, $\tilde{x}', \tilde{y}', \tilde{z}', c\tilde{t}'$, relative to unprimed affine spacetime coordinates, $\tilde{x}$, $\tilde{y}$, $\tilde{z}$, $c\tilde{t}$. Hence they do not contain skewed primed or unprimed affine spacetime coordinates. The intrinsic coordinates of the flat proper intrinsic metric spacetimes, $(\varnothing\rho', \varnothing c\varnothing t')$ and $(-\varnothing\rho'^{*}, -\varnothing c\varnothing t'^{*})$, and the coordinates of the flat proper metric spacetimes, (Σ', ct') and $(-\Sigma'^{*}, -ct'^{*})$, are not rotated in Figs. 8a, 8b and Figs. 9a and 9b, much less the existence of the skewness of rotated intrinsic metric spacetime coordinates and the skewness of metric spacetime coordinates in those figures.

The ineradicable skewness of the coordinates of one or both of two frames in relative motion in the Minkowski diagrams of Figs. 3a and 3b, the Loedel diagramof Fig. 4a and the Brehme diagram of Fig. 4b, in the one-world picture (Scheme I in Table I), have been eliminated in Figs. 8a and 8b and Figs. 9a and 9b, in the two-world picture (Scheme II in Table I).

Although the negative universe is totally elusive to people in our (or positive) universe, just as our universe is totally elusive to people in the negative universe, from the point of view of direct experience, which is due to the existence of event horizons along the time dimension ct that shows up as a singularity for $\psi = \pi/2$ in the LT and its inverse of systems (26) and (27) in our universe, and along the time dimension $-ct^{*}$ that shows up as a singularity for $\psi = \pi/2$ in the LT and its inverse of systems (34) and (36) in the negative universe, we have now seen in the above that the intrinsic affine spacetime coordinates of the two universes unite in prescribing intrinsic Lorentz transformation and intrinsic Lorentz invariance on the flat two-dimensional intrinsic spacetime and, consequently, in prescribing Lorentz transformation and Lorentz invariance on flat four-dimensional spacetime in each of the two universes. It can thus be said that, there is intrinsic (that is, non-observable) interaction of the four-dimensional affine spacetime coordinates of the two universes in special relativity.

The singularities at $\varnothing\psi = \varnothing\pi/2$ and $\varnothing\psi = -\varnothing\pi/2$ or $\varnothing\psi = 3\varnothing\pi/2$ in systems (11) and (14) (of scheme II in Table I or in the two-world picture), correspond to the singularities at $\alpha = \infty$ and $\alpha = -\infty$ in the coordinate transformation of systems (5a) and (5b) in the Minkowski's one-world picture. Being smooth for all values of α, except for the extreme values, $\alpha = \infty$ and $\alpha = -\infty$, at its boundary, represented by the vertical line in Fig. 1a, which corresponds to a line along the $ct-$ and $-ct^*-$axes in Fig. 2a, the only (positive) Minkowski space including the time reversal dimension (to be denoted by $(\Sigma, ct; -ct^*)$), in Fig. 2a in the one-world picture, is usually considered to be sufficiently smooth.

Similarly being smooth for all values of the intrinsic angle $\varnothing\psi$ in the first cycle except, $\varnothing\psi = -\varnothing\pi/2, \varnothing\pi/2$ and $\varnothing\psi = 3\varnothing\pi/2$, along their interface in Fig. 2b, the positive Minkowski space including the time reversal dimension $(\Sigma, ct; -ct^*)$ and the negative Minkowski's space including time reversal dimension $(-\Sigma^*, -ct^*; ct)$, of the two-world picture in Fig.2b, must be considered to be sufficiently smooth individually.

An attempt to compose the positive Minkowski space including the time reversal dimension $(\Sigma, ct; -ct^*)$ and the negative Minkowski space including time reversal dimension $(-\Sigma^*, -ct^*; ct)$ into a singular space, over which $\varnothing\psi$ takes on values within the range $[-\varnothing\pi/2, 3\varnothing\pi/2]$ or $[0, 2\varnothing\pi]$, cannot work, because the resultant space possesses interior (and not boundary) discontinuities, at $\varnothing\psi = \varnothing\pi/2$ in the case of the range $[-\varnothing\pi/2, 3\varnothing\pi/2]$ and at $\varnothing\psi = \varnothing\pi/2$ and $\varnothing\psi = 3\varnothing\pi/2$ in the case of the range $[0, 2\varnothing\pi]$, thereby making the single space generated non-smooth. This implies that the larger spacetime of combined positive and and negative universes cannot be considered as a continuum of event domain, or as constituting a single world or universe.

The lines of singularity, $\varnothing\psi = \varnothing\pi/2$ and $\varnothing\psi = -\varnothing\pi/2$, along the vertical $ct-$ and $-ct^*-$ dimensions respectively, represent event horizons (the special-relativistic event horizons), to 3-observers in the 3-spaces, Σ and $-\Sigma^*$, in the positive and negative universes respectively. These event horizons at $\varnothing\psi = \varnothing\pi/2$ and $-\varnothing\pi/2$ show up as singularities in the intrinsic Lorentz transformation ($\varnothing$LT) and its inverse of systems (11) and (14) and, consequently, in the LT and its inverse of systems (26) and (27) in the positive universe. They show up as singularities in the $\varnothing$LT and its inverse of systems (21) and (22) and, consequently, in the LT and its inverse of systems (34) and (35) in the negative universe, as mentioned earlier.

The observers in 3-space on one side of the event horizons along the dimensions ct and $-ct^*$ in Fig. 5 or Fig. 7 cannot observe events taking place on the other side. This and the preceding two paragraphs makes a two-world interpretation of scheme II in Table I, with the larger spacetime/intrinsic spacetime diagram of Fig. 7 mandatory.

4.5 Reduction of LT and Its Inverse to Length Contraction and Time Dilation Formulae from the Point of View of what can be Measured with Laboratory Rod and Clock

Nature makes use of all the terms of the LT, system (26) or (31), and its inverse, system (27) or (32), to establish Lorentz invariance. However man cannot not detect all the terms of the LT and its inverse with his laboratory rod and clock. First of all, it is the last three equations of system (26) or (31) and the first equation of system (27) or (32), written by or with respect to the 'stationary' 3-observer (Peter) in the proper metric 3-space Σ' that are relevant for the measurements of distance in space with a rod in 3-space and of time duration by a clock kept in the metric 3-space Σ' respectively, of a special-relativistic event. Collecting those equations we have the following

$$\begin{aligned}\tilde{x}' &= \tilde{x}\sec\psi - c\tilde{t}\tan\psi\ ;\ \tilde{y}' = \tilde{y}\ ;\ \tilde{z}' = \tilde{z}\ \text{ and} \\ &\quad c\tilde{t} = c\tilde{t}'\sec\psi + \tilde{x}'\tan\psi\ ;\end{aligned} \tag{39}$$

(w.r.t. 3-observer Peter in Σ').

When the 3-observer Peter picks his laboratory rod to measure length, he will be unable to measure the term $-c\tilde{t}\tan\psi$ of the first equation of system (39) with his laboratory rod. Likewise when he picks his clock to measure time duration, he will be unable to measure the term $\tilde{x}'\tan\psi$ in the fourth equation of system (39) with his clock. Thus from the point of view of what can be measured with laboratory rod and clock by 3-observers in the metric 3-space Σ', system (39) reduces as follows

$$\tilde{x} = \tilde{x}'\cos\psi\ ;\ \tilde{y} = \tilde{y}'\ ;\ \tilde{z} = \tilde{z}'\ ;\ \text{and}\ \tilde{t} = \tilde{t}'\sec\psi\ . \tag{40}$$

System (40) becomes the following explicit form in terms of particle's speed relative to the observer by virtue of Eq. (30) on page 78,

$$\begin{aligned}\tilde{x} &= \tilde{x}'(1 - v^2/c^2)^{1/2}\ ;\ \tilde{y} = \tilde{y}'\ ;\ \tilde{z} = \tilde{z}'\ ;\ \text{and} \\ &\quad \tilde{t} = \tilde{t}'(1 - v^2/c^2)^{-1/2}\ .\end{aligned} \tag{41}$$

Systems (40) and (41) are length contraction and time dilation formulae, but written explicitly in terms of affine spacetime coordinates (for two frames in relative motion along their collinear $\tilde{x}-$ and $\tilde{x}'-$ axes) in SR. Showing that they pertain to the measurable sub-space of SR (while the LT and its inverse pertain to the larger or total space of SR), is the essential point being made here. It must be added that the affine spacetime coordinates in systems (40) and (41), being non-ponderable, are not measurable directly. It is the corresponding equations written in terms of the metric spacetime coordinates along which the affine coordinates lie (or are embedded) that can be measured.

4.6 The Generalized form of Intrinsic Lorentz Transformation in the Two-World Picture

Now let us rewrite the intrinsic Lorentz transformation (∅LT) and its inverse of system (11) on page 71 and system (14) on page 73 in the positive universe in the generalized forms in which they can be applied for all values of $\varnothing\psi$ in the concurrent open intervals $(-\varnothing\pi/2, \varnothing\pi/2)$ and $(\varnothing\pi/2, 3\varnothing\pi/2)$ in Fig. 10a, by factorizing out $\sec\varnothing\psi$ to have respectively as follows

$$\begin{aligned}\varnothing c\varnothing\tilde{t}' &= \sec\varnothing\psi(\varnothing c\varnothing\tilde{t} - \varnothing\tilde{x}\sin\varnothing\psi)\ ; \\ \varnothing\tilde{x}' &= \sec\varnothing\psi(\varnothing\tilde{x} - \varnothing c\varnothing\tilde{t}\sin\varnothing\psi)\end{aligned} \tag{42}$$

and

$$\begin{aligned}\varnothing c\varnothing\tilde{t} &= \sec\varnothing\psi(\varnothing c\varnothing\tilde{t}' + \varnothing\tilde{x}'\sin\varnothing\psi)\ ; \\ \varnothing\tilde{x} &= \sec\varnothing\psi(\varnothing\tilde{x}' + \varnothing c\varnothing\tilde{t}'\sin\varnothing\psi)\ ;\end{aligned} \tag{43}$$

for $\varnothing\psi \in (-\varnothing\pi/2, \varnothing\pi/2)$ and $(\varnothing\pi/2, 3\varnothing\pi/2)$.

The 3-observers in the proper Euclidean 3-space Σ' of the positive universe 'observe' intrinsic special relativity (∅SR) and, consequently, special relativity (SR), for intrinsic angles $\varnothing\psi$ in the range $(-\varnothing\pi/2, \varnothing\pi/2)$. However as Fig. 10a shows, 3-observers Σ' in the positive universe can construct ∅SR and, hence SR, relative to themselves for all intrinsic angles $\varnothing\psi$ in the concurrent open intervals $(-\varnothing\pi/2, \varnothing\pi/2)$ and $(\varnothing\pi/2, 3\varnothing\pi/2)$, by using the generalized ∅LT and its inverse of systems (42) and (43) and obtaining the LT and its inverse as the outward manifestations on flat four-dimensional spacetime, of the ∅LT and its inverse so derived, although they can observe special relativity for intrinsic angles $\varnothing\psi$ in $(-\varnothing\pi/2, \varnothing\pi/2)$ in Fig. 10a only.

Likewise the $\varnothing$LT and its inverse in the negative universe of system (23) on page 75 and system (24), shall be written in the generalized forms in which they can be applied for all intrinsic angles $\varnothing\psi$ in the concurrent open intervals $(-\varnothing\pi/2, \varnothing\pi/2)$ and $(\varnothing\pi/2, 3\varnothing\pi/2)$ in Fig. 10b respectively as

$$\begin{aligned} -\varnothing c\varnothing\tilde{t}\,'^{*} &= \sec\varnothing\psi(-\varnothing c\varnothing\tilde{t}^{\,*} - (-\varnothing\tilde{x}^{\,*})\sin\varnothing\psi)\;; \\ -\varnothing\tilde{x}\,'^{*} &= \sec\varnothing\psi(-\varnothing\tilde{x}^{\,*} - (-\varnothing c\varnothing\tilde{t}^{\,*})\sin\varnothing\psi)\;. \end{aligned} \tag{44}$$

and

$$\begin{aligned} -\varnothing c\varnothing\tilde{t}^{\,*} &= \sec\varnothing\psi(-\varnothing c\varnothing\tilde{t}\,'^{*} + (-\varnothing\tilde{x}\,'^{*})\sin\varnothing\psi)\;; \\ -\varnothing\tilde{x}^{\,*} &= \sec\varnothing\psi(-\varnothing\tilde{x}\,'^{*} + (-\varnothing c\varnothing\tilde{t}\,'^{*})\sin\varnothing\psi)\;. \end{aligned} \tag{45}$$

for $\varnothing\psi \in (-\varnothing\pi/2, \varnothing\pi/2)$ and $(\varnothing\pi/2, 3\varnothing\pi/2)$.

The 3-observers* in the proper Euclidean 3-space $-\Sigma'^{*}$ of the negative universe 'observe' intrinsic special relativity ($\varnothing$SR) and, hence special relativity (SR), for intrinsic angles $\varnothing\psi$ in the open interval $(-\varnothing\pi/2, \varnothing\pi/2)$ in Fig. 10b. Again as Fig. 10b shows, 3-observers* in $-\Sigma'^{*}$ in the negative universe can construct $\varnothing$SR and, hence SR, relative to themselves for all intrinsic angles $\varnothing\psi$ in the concurrent open intervals $(-\varnothing\pi/2, \varnothing\pi/2)$ and $(\varnothing\pi/2, 3\varnothing\pi/2)$, by using the generalized $\varnothing$LT and its inverse of $\varnothing$SR of systems (44) and (45) and obtaining the LT and its inverse of SR as the outward manifestations on flat the four-dimensional spacetime of $\varnothing$LT and its inverse so constructed, although they can observe special relativity for intrinsic angles $\varnothing\psi$ in $(-\varnothing\pi/2, \varnothing\pi/2)$ in Fig. 10b only.

The fact that the intrinsic Lorentz transformation ($\varnothing$LT) and its inverse represent continuous rotation of intrinsic affine spacetime coordinates, $\varnothing\tilde{x}\,'$ and $\varnothing c\varnothing\tilde{t}\,'$, of the the particle's proper (or primed) intrinsic affine frame relative to the intrinsic affine spacetime coordinates, $\varnothing\tilde{x}$ and $\varnothing c\varnothing\tilde{t}$, respectively, of the particle's relativistic (or unprimed) intrinsic affine frame, through all intrinsic angles $\varnothing\psi$ in the closed range $[0, 2\varnothing\pi]$, while avoiding rotation by $\varnothing\psi = \varnothing\pi/2$ and $\varnothing\psi = 3\varnothing\pi/2$, is clear from the concurrent open intervals $(-\varnothing\pi/2, \varnothing\pi/2)$ and $(\varnothing\pi/2, 3\varnothing\pi/2)$ of the intrinsic angle $\varnothing\psi$ in Figs. 10a and 10b, over which the generalized $\varnothing$LT and its inverse of systems (42) and (43) in the positive universe and systems (44) and (45) in the negative universe can be applied.

We shall not be concerned with the explanation of how the intrinsic affine coordinates, $\varnothing\tilde{x}\,'$ and $\varnothing c\varnothing\tilde{t}\,'$, of the particle's primed intrinsic affine frame can be rotated continuously relative to the intrinsic affine coordinates, $\varnothing\tilde{x}$ and $\varnothing c\varnothing\tilde{t}$, of the particle's unprimed intrinsic affine frame through intrinsic angles $\varnothing\psi$ in the range $[0, 2\varnothing\pi]$, while avoiding $\varnothing\psi = \varnothing\pi/2$ and $\varnothing\psi = 3\varnothing\pi/2$ in this paper.

4.7 Non-existence of Light Cones in the Two-World Picture

The concept of light cone does not exist in the two-world picture. This follows from the derived relation, $\sin\varnothing\psi = \varnothing v/\varnothing c$ (Eq. (16) on page 74), which makes the intrinsic speed $\varnothing v$ of intrinsic motion of the primed and unprimed intrinsic particle's affine frames relative to the observer, for every pair of particle and observer, lower than the intrinsic light speed $\varnothing c$ ($\varnothing v < \varnothing c$), for all values of $\varnothing\psi$ in the concurrent open intervals $(-\varnothing\pi/2, \varnothing\pi/2)$ and $(\varnothing\pi/2, 3\varnothing\pi/2)$ in our universe in Fig. 10a, in the context of $\varnothing$SR and, consequently speed v of relative motion of every pair of particle and observer lower than the speed of light c ($v < c$), for all intrinsic angles $\varnothing\psi$ in the concurrent open intervals $(-\varnothing\pi/2, \varnothing\pi/2)$ and $(\varnothing\pi/2, 3\varnothing\pi/2)$ in Fig. 10a. The intrinsic angle, $\varnothing\psi = \varnothing\pi/2$, corresponds to intrinsic speed, $\varnothing v = \varnothing c$ and $\varnothing\psi = -\varnothing\pi/2$ or $\varnothing\psi = 3\varnothing\pi/2$ corresponds to $\varnothing v = -\varnothing c$, which are excluded from ∅SR. They correspond to speed, $v = c$ and $v = -c$ respectively, which are excluded from SR. The speed, $v \leq c$, is naturally negative in the fourth quadrant by virtue of the natural time reversal without parity inversion in that quadrant.

We therefore have a situation where all intrinsic angles $\varnothing\psi$ in the closed range $[0, 2\varnothing\pi]$, except $\varnothing\psi = \varnothing\pi/2$ and $\varnothing\psi = 3\varnothing\pi/2$ (in Fig. 10a), are accessible to intrinsic special relativity (∅SR) with intrinsic timelike geodesics and, consequently, to SR with timelike geodesics, with respect to observers in the positive universe. All intrinsic angles $\varnothing\psi$ in the closed interval $[0, 2\varnothing\pi]$, except $\varnothing\psi = \varnothing\pi/2$ and $\varnothing\psi = 3\varnothing\pi/2$ (in Fig. 10b), are likewise accessible to ∅SR with intrinsic timelike geodesics and, hence, to SR with timelike geodesics, with respect to observers* in the negative universe.

Intrinsic spacelike geodesics for which $\varnothing v > \varnothing c$ and spacelike geodesics for which $v > c$ do not exist for any value of the intrinsic angle $\varnothing\psi$ in the four quadrants, that is, for $\varnothing\psi$ in the closed range $[0, 2\varnothing\pi]$, on the hyperplane and intrinsic hyperplane formed by the larger spacetime and larger intrinsic spacetime of combined positive and negative universes in Fig. 7. Since the existence of light cones requires regions of spacelike geodesics outside the cones on the hyperplane, the concept of light cones does not exist in the two-world picture.

An additional argument against the existence of light cones is that, the temporary systems (4a) and (4b), usually derived directly from the Minkowski's diagrams of Figs. 3a and 3b, being invalid as inverse LT and LT respectively, thereby necessitating their replacements by systems (5a) and (5b), are not to be considered of relevance in SR anymore. The relation (4d) derived from system (4a) and the corresponding relation, $\tan\phi = -v/c$, from system (4b), which, for $v = c$, are lines on the surfaces of the future light cone and the past light cone, $\phi = \pi/4$ and $\phi = -\pi/4$ respectively, must also be considered of no relevance in SR. Moreover the final systems (5a) and (5b) adopted in the Minkowski's one-world picture do not imply light cones.

5 CONCLUSIONS

Although SR has been re-formulated on a two-world background in this paper, the theory remains unchanged in each of the two universes. However there is now a broader view and improvement of SR. The addition of the fourth quadrant of the spacetime hyperplane with the time reversal dimension to the spacetime of our universe, is prospective of important impacts on both theoretical and experimental particle physics. The replacement of the "hyperbolic projections" of coordinates on spacetime hyperplane in the Minkowski geometry by their trigonometric counterparts and expressing length contraction and time dilation as coordinate projections, $l = l'\cos\psi$ and $t = t'\sec\psi$; $\cos\psi = (1 - v^2/c^2)^{1/2}$, should also impact experimental particle physics and practices in astronomy. There is also the non-observable parallel intrinsic special relativity (∅SR) on the flat two-dimensional intrinsic spacetime embedded in spacetime, which helps to determine SR in spacetime, as part of the broader view and improvement of SR brought about by this paper.

Although the possibility of the existence of a two-world picture (or symmetry) in nature has been exposed in this paper, there is the need for further theoretical justification than contained in this initial paper and possible experimental confirmation ultimately, in order to conclude the definite existence of the two-world picture. The next natural step is to investigate the signs of mass and other physical parameters, as well as the possibility of invariance of the natural laws in the negative universe.

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COMPETING INTERESTS

No competing interests are involved in this work.

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