



# Numerical Comparison of $Cu$ and $Al_2O_3$ Nanoparticles in an MHD Water-based Nanofluid

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## Authors' contributions

*This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.*

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## ABSTRACT

In this study, the impact of  $Cu$  and  $Al_2O_3$  nanoparticles in a water-based nanofluid are considered. The application of this can be found in biomedical sensors and drug delivery. Specifically, it investigates heat transfer in the MHD flow of two nanofluids ( $Cu$ -water and  $Al_2O_3$ -water) over an exponentially stretching surface. The study formulates a model and renders it dimensionless using Similarity Transformation. Numerical solutions are obtained using the MATLAB package bvp4c. The focus is on analysing the heat transfer rate variation with nanoparticle volume fraction. Results indicate that  $Cu$ -water nanofluid exhibits higher heat transfer rates and lower skin frictions compared to  $Al_2O_3$ -water nanofluid.

*Keywords: Heat transfer; MHD flow; hybrid nanofluid.*

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## NOMENCLATURE

Symbol	Meaning	Symbol	Meaning
$u, v, w$	Velocity in the $x, y, z$ -directions	$\mu_{nf}$	effective nanofluid viscosity
$T$	Dimensional fluid temperature	$\mu_{bf}$	base fluid viscosity
$T_\infty$	free stream temperature	$\mu_{np}$	nanoparticle viscosity
$T_w$	wall temperature	$\rho_{nf}$	effective nanofluid density
$g^*$	Acceleration due to gravity	$\rho_{bf}$	base fluid density
$k^*$	mean absorption coefficient	$\rho_{np}$	nanoparticle density
$B_0$	magnetic field strength	$\beta$	Coefficient of thermal expansion
$c_p$	Specific heat capacity	$\sigma_{nf}$	effective electrical conductivity
$\sigma^*$	Stefan–Boltzmann constant	$k_{nf}$	effective nanofluid thermal conductivity
$\phi$	nanoparticle volume fraction	$\alpha_{nf}$	effective nanofluid thermal diffusivity

## 1. INTRODUCTION

Maxwell [1] introduced the concept of enhancing thermal and electrical conductivity in fluids by incorporating solid particles. While initially successful, issues such as pipe clogging and erosion arose with the use of millimetre-sized particles. Choi and Eastman [2] proposed an alternative approach of dispersing nanoparticles in the fluid, termed nanofluids, which addressed these issues. Nanotechnology advancements have since facilitated significant progress in nanofluid preparation and application, particularly in engineering and industrial sectors. Nanofluids find utility in diverse areas, including oil well recovery [3], waste heat retrieval [4], and cooling systems for microchips and car radiators [5-7].

Additionally, when fluid flow occurs under magnetic influence, both thermal and electrical properties are further enhanced. The magnetohydrodynamic flow of nanofluids has garnered global attention, particularly for its applications in micro-technology advancement. Due to the complexity and resource-intensive nature of nanofluid preparation, theoretical and numerical studies are preferred over experimental approaches. Consequently, there is a greater emphasis on theoretical investigations of magnetohydrodynamic nanofluid flow compared to experimental research [8-10].

The passage of a fluid with electrical conductivity in a magnetic field induces an electric current, resulting in a loop of magnetic field generation. This phenomenon, known as magnetohydrodynamic (MHD) flow, is significant in various fields, such as biomedical applications, astrophysics, and liquid metal flows. Nayak et al. [11] investigated free convective MHD flow with varying magnetic field strength in three

nanofluids based on different base fluids, comparing heat transfer rates and skin friction. Elazem [12] numerically explored the effect of magnetic field strength on nanofluid flow when a plate is linearly stretched, focusing on heat and mass transfer rates. Irfan et al. [13] analysed the impact of time-dependent viscosity, thermal conductivity, surface stretching, and wall temperature on MHD nanofluid flow thermophysical properties. Noor et al. [14] studied the MHD flow of Jeffrey nanofluids, emphasising volume fraction effects rather than nanoparticle material. Results showed that increasing magnetic field intensity slowed down the flow, consistent with previous studies. Ahmed and Akbar [15] investigated the MHD flow of Williamson nanofluid over an ESS. Haroon et al. [16] examined the effects of uniform magnetism and heat radiation on nanofluid flow, noting that magnetism impedes fluid flow. Atif et al. [17] considered a micropolar-based nanofluid with thermal radiation and mixed convection, finding that magnetic field presence enhances flow temperature.

This study analyses the MHD flow of water-based nanofluids over an exponentially stretching surface in three dimensions, focusing on the effects of nanoparticle material and volume fraction. Specifically, the study compares the flow characteristics of two conducting nanofluids:  $Cu - H_2O$  and  $Al_2O_3 - H_2O$  nanofluids. By varying the volume fraction of nanoparticles, the project seeks to investigate the rate of heat transfer. The significance of using  $Cu$  and  $Al_2O_3$  The nanoparticle in the MHD flow of water-based nanofluid over an Exponentially stressing surface will be examined, shedding light on the impact of nanoparticle material and volume fraction on flow parameters.

## 2. METHODOLOGY

### 2.1 Governing Equations

The flow configuration depicted in Fig. 1 illustrates a steady, three-dimensional flow, with the magnetic field oriented along the z-axis. The nanofluid flows within the x-y plane, influenced by magnetisation perpendicular to the flow plane. In accordance with the steady nature of the flow, the continuity equation can be expressed as:

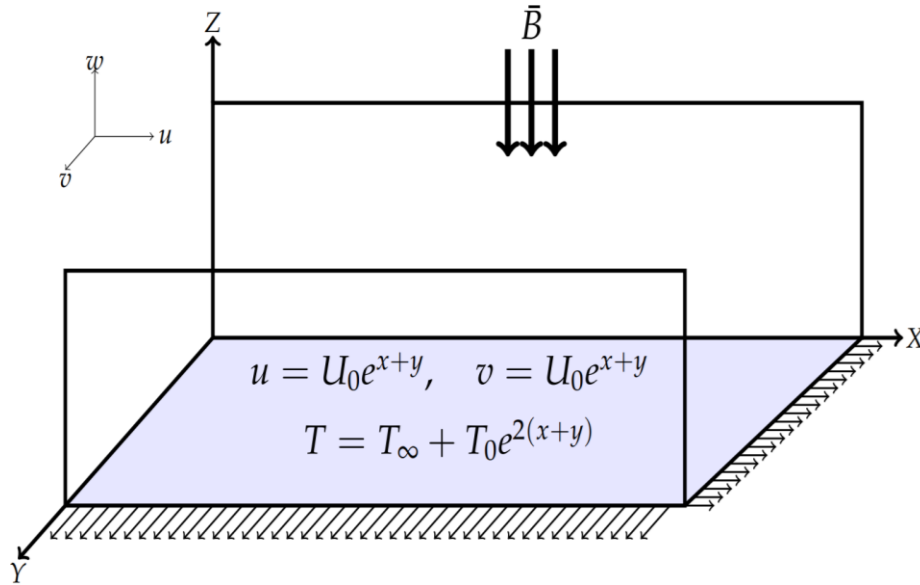
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0. \quad (1)$$

and the momentum and energy equations are

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 u}{\partial z^2} + g^* \beta (T - T_\infty) - \frac{\sigma_{nf} B_0^2 u}{\rho_{nf}} \quad (2)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 v}{\partial y^2} + g^* \beta (T - T_\infty) + \frac{\sigma_{nf} B_0^2 v}{\rho_{nf}} \quad (3)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \left( \alpha_{nf} + \frac{16\sigma^* T_\infty^3}{3k^* (\rho c_p)_{nf}} \right) \frac{\partial^2 T}{\partial z^2}, \quad (4)$$



**Fig. 1. Pattern of flow**

The effective dynamic viscosity, density, thermal diffusivity and heat capacity are defined in [18 - 21] as.

$$\mu_{nf} = (1 + 7.3\phi + 123\phi^2)\mu_{bf}, \quad \rho_{nf} = (1 - \phi)\rho_{bf} + \phi\rho_{np}, \quad (5)$$

$$\alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}}, \quad (\rho c_p)_{nf} = (\rho c_p)_{bf} \left( 1 - \phi + \phi \frac{(\rho c_p)_{np}}{(\rho c_p)_{bf}} \right) \quad (6)$$

$$k_{nf} = k_{bf}k_{np} \left[ 2 \frac{k_{bf}}{k_{np}} + \frac{\phi}{k_{np}} (k_{bf} - k_{np}) + k_{np} + 2k_{bf} - 2\phi(k_{bf} - k_{np}) \right]. \quad (7)$$

The BCs are

$$at \ z = 0; \quad u = v = U_0 \exp(x + y), \quad w = 0, \quad T = T_\infty + T_0 \exp(2x + 2y) \quad (8)$$

$$as \ z \rightarrow \infty; \quad u \rightarrow 0, \quad v \rightarrow 0, \quad T \rightarrow T_\infty \quad (9)$$

Factors of engineering applications are the skin friction and heat transfer rates;

$$Cf_x = \frac{\tau_x}{\rho_{nf}U_w^2}, \quad Cf_z = \frac{\tau_z}{\rho_{nf}U_w^2}, \quad \text{and} \quad Nu = \frac{zq_w}{\kappa_{nf}(T_w - T_\infty)}. \quad (10)$$

respectively, where the shear stress  $\tau$  is

$$\tau_x = \mu_{nf} \left. \frac{\partial u}{\partial z} \right|_{z=0} \quad \text{and} \quad \tau_z = \mu_{nf} \left. \frac{\partial v}{\partial z} \right|_{z=0}. \quad (11)$$

and the heat flux  $q_w$  is

$$q_w = -\kappa_{nf} \left. \frac{\partial T}{\partial y} \right|_{z=0}. \quad (12)$$

## 2.2 Nondimensionalisation

The similarity variables

$$\eta = \left( \frac{U_0}{2\nu_{bf}} \right)^{\frac{1}{2}} z \exp\left(\frac{x+y}{2}\right), \quad u = U_0 f' \exp(x+y), \quad v = U_0 g' \exp(x+y), \quad (13)$$

$$w = -\left( \frac{\nu_{bf} U_0}{2} \right)^{\frac{1}{2}} (f + \eta f' + g + \eta g') \exp\left(\frac{x+y}{2}\right), \quad T = T_\infty + \theta T_0 \exp(2x + 2y) \quad (14)$$

With these variables, the system reduces to

$$A_1 f'''' + 2Gr\theta - 2Mf' - 2f'(f' + g') + f''(f + g) = 0, \quad (15)$$

$$A_1 g'''' + 2Gr\theta + 2Mg' - 2g'(f' + g') + g''(f + g) = 0, \quad (16)$$

$$\left(1 + \frac{4}{3}R\right) A_2 \theta'' - 4Pr\theta(f' + g') + Pr(f + g)\theta' = 0. \quad (17)$$

with the initial and BCs

$$f' = 1; \quad g' = 1; \quad 0 = f + g; \quad \theta = 1; \quad at \ \eta = 0, \quad (18)$$

$$f' = 0; \quad g' = 0; \quad \theta = 0; \quad as \ \eta \rightarrow \infty. \quad (19)$$

where

$$A_1 = \frac{(1 + 7.3\phi + 123\phi^2)}{\left(1 - \phi + \phi \frac{\rho_{np}}{\rho_{bf}}\right)}, \quad Gr = \frac{g^* \beta T_0}{U_0^2}, \quad M = \frac{\sigma_{nf} B_0^2}{U_0 \rho_{nf} e^{x+y}}, \quad Pr = \frac{\nu_{bf}}{\alpha_{bf}}, \quad R = \frac{4\sigma^* T_\infty^3}{k^* (\rho c_p)_{nf} \alpha_{nf}}.$$

The quantities of engineering interests are;

$$Re^{-\frac{1}{2}}Cf_x = A_1f''(0), \quad Re^{-\frac{1}{2}}Cf_y = A_1g''(0), \quad Re^{\frac{1}{2}}Nu = \theta'(0) \quad (20)$$

### 2.3 Numerical Method

By rewriting

$$H_1 = f, H_2 = f', H_3 = f'',$$

$$H_4 = g, H_5 = g', H_6 = g'',$$

$$H_7 = \theta, H_8 = \theta',$$

equations (21) – (23) are put as a system of 1st-order ODEs

$$H'_1 = H_2, \quad H'_2 = H_3, \quad H'_3 = -\frac{1}{A_1}(2GrH_7 - 2MH_2 - 2H_2(H_2 + H_5) + H_3(H_1 + H_4)),$$

$$H'_4 = H_5, \quad H'_5 = H_6, \quad H'_6 = -\frac{1}{A_1}(2GrH_7 + 2MH_5 - 2H_5(H_2 + H_5) + H_6(H_1 + H_4)),$$

$$H'_7 = H_8, \quad H'_8 = -\frac{1}{\left(1 + \frac{4}{3}R\right)A_2}(-4PrH_4(H_2 + H_5) + Pr(H_2 + H_4)H_8).$$

with the conditions

$$H_2(0) = 1; \quad H_5(0) = 1; \quad H_1(0) + H_4(0) = 0; \quad H_7(0) = 1;$$

$$H_2(\infty) = 0; \quad H_5(\infty) = 0; \quad H_7(\infty) = 0.$$

The Shooting technique with the Runge-Kutta method is adopted in solving this system [22].

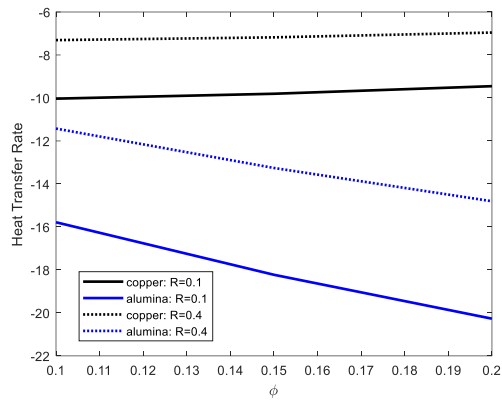
### 3. RESULTS AND DISCUSSION

The investigation of heat transfer rates and skin friction for copper-water and alumina-water nanofluids is depicted graphically in Figs. 2 and 3. In both figures, the black lines represent copper-water nanofluid, while the blue lines represent alumina-water nanofluid. Each nanofluid is examined under two conditions: low thermal radiation and high thermal radiation. Fig. 2 illustrates that heat transfer rates increase with thermal radiation, with copper-water nanofluid exhibiting higher heat transfer rates compared to alumina-water nanofluid. This difference can be attributed to the higher specific heat of alumina nanoparticles compared to copper nanoparticles.

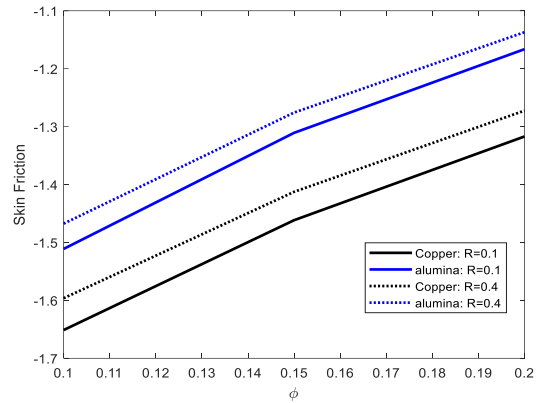
Regarding skin friction, it is low at low thermal radiation for both nanofluids but increases at high thermal radiation (see Fig. 3). Further analysis of Fig. 3 reveals that skin friction is higher during alumina-water nanofluid flow compared to copper-water flow. Consequently, while the heat transfer rate is higher in Cu- $H_2O$  nanofluid

compared to  $Al_2O_3-H_2O$  nanofluid, skin friction is higher in  $Al_2O_3-H_2O$  than in Cu- $H_2O$ .

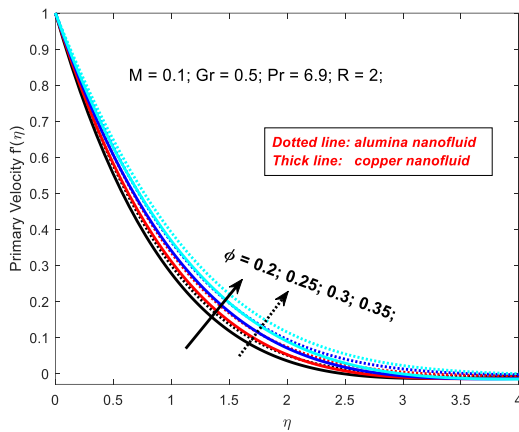
Volume fraction refers to the ratio of the volume of copper or alumina nanoparticles suspended in a specific volume of water. Figs. 4 and 5 analyse the flow response by varying the volume fraction ( $\phi$ ). It is important to note that suspending a higher volume of nanoparticles in the fluid can lead to aggregation, which may cause wear on pipe linings. Due to the lower density of alumina nanoparticles compared to copper nanoparticles, the flow velocity is higher in the  $Al_2O_3 - H_2O$  nanofluid than in the copper-water nanofluid, as depicted in Fig. 4. Additionally, upon closer examination of Fig. 4, it can be generally inferred that flow velocity increases as the volume fraction increases. However, flow temperature decreases as more volume fraction of nanoparticles are suspended in the base fluid, as shown in Fig. 5.



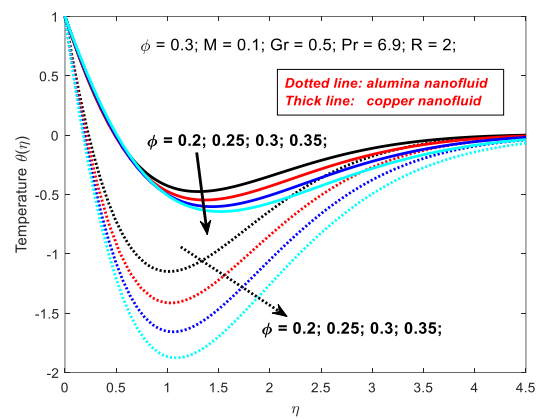
**Fig. 2.  $Nu$  against  $\phi$  at low and high thermal radiation**



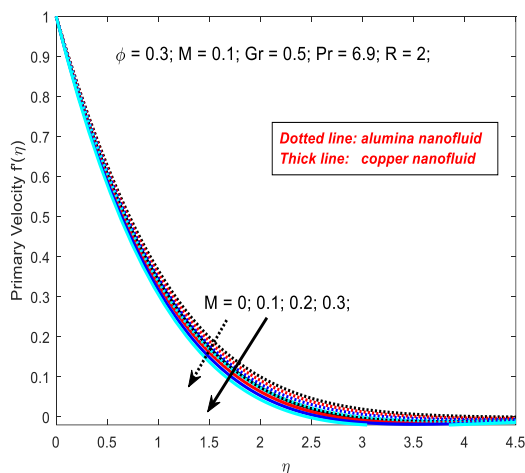
**Fig. 3.  $C_f$  against  $\phi$  at low and high thermal radiation**



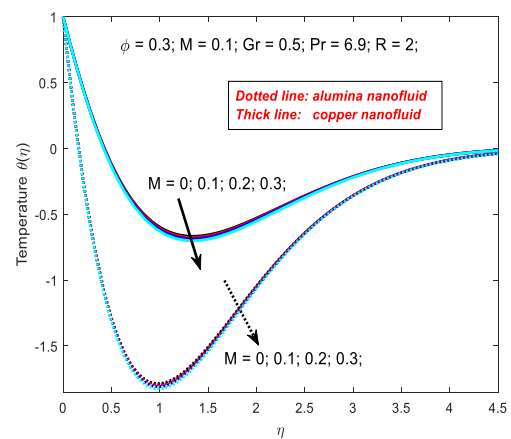
**Fig. 4. Primary velocity against  $\phi$**



**Fig. 5. Temperature against  $\phi$**



**Fig. 6. Primary velocity with  $M$**



**Fig. 7. Temperature with  $M$**

The effects of magnetic field strength on the flow of copper-water and alumina-water nanofluids are analysed, with results depicted in Figs. 6 and 7. It's essential to note that Lorentz force is generated when flow occurs in a magnetic field,

acting against the flow and thereby reducing velocity as the magnitude of the magnetic field strength increases. As expected, the velocity profile decreases with increasing magnetic field strength, as illustrated in Fig. 6. Additionally, from Fig. 6, it's observed that the alumina-water

nanofluid flows at a higher velocity than the Cu- $H_2O$  nanofluid. Moreover, the  $Al_2O_3-H_2O$  nanofluid exhibits a higher flow temperature compared to the Cu- $H_2O$  nanofluid, as depicted in Fig 7.

#### 4. CONCLUSION

The project analyses the heat transmission during the flow of copper-water and alumina-water nanofluids in a magnetic field. The surface undergoes horizontal exponential stretching while thermal radiation is applied perpendicular to the flow. Mathematical equations are reformulated dimensionally and solved using the RK4-Sh method. The outcomes of the study reveal the following:

1. Heat transfer rates increase with thermal radiation, with copper-water nanofluid exhibiting higher rates compared to alumina-water nanofluid.
2. Skin friction increases with thermal radiation, with alumina-water nanofluid demonstrating higher friction compared to copper-water nanofluid.
3. Flow velocity is enhanced as the volume fraction increases, with alumina-water nanofluid exhibiting higher velocity.
4. Flow temperature decreases with increasing volume fraction of nanoparticles in the base fluid.

#### COMPETING INTERESTS

Authors have declared that no competing interests exist.

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