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Jeans Instability of Fine Dust Particle with Impact of Coriolis Force, Electrical Resistivity and Electron Plasma Frequency

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Authors' contributions

This work was carried out in collaboration among all authors. Author DLS designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Authors SS and PKP managed the analyses of the study. Author PKP managed the literature searches. All authors read and approved the final manuscript.

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ABSTRACT

The present paper deals with the theoretical investigation of the combined influence of Coriolis force, electrical resistivity, electron plasma frequency and fine dust particles on the linear Jeans instability of gaseous plasma by using the generalized magneto-hydrodynamic fluid model. The general dispersion relation is derived using normal mode analysis technique and reduced for both the longitudinal and transverse mode of propagation and further, it is discussed of the axis of rotation parallel and perpendicular to the magnetic field. We found that the stabilizing influence of

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Coriolis force and electron plasma frequency increases for a fine dust particle, but electrical resistivity has the destabilizing influence on them. The Jeans criterion of instability is modified by fine dust particle only transverse mode of propagation when rotation is perpendicular to the magnetic field. The Coriolis force and electron plasma frequency have stabilized the growth rate of the system but the electrical resistivity is destabilizing the system. These results are applicable to interstellar clouds and star formation region.

Keywords: Fine dust particle; Coriolis force; resistivity; electron plasma frequency; viscosity and magnetic field.

1. INTRODUCTION

The fine dust particle is an important component of an astrophysical system where self-gravitational instability plays a crucial role in the dynamics of the system, such as interstellar clouds. The self-gravitational instability of molecular clouds connected to the condensation of clouds, dark interiors molecular clouds, circumstellar shell, and giant star formation. James Jeans first discovered the gravitational instability of an infinite homogeneous gaseous plasma and pointed out that the self-gravitating fluid is unstable for all wave number which is less than critical Jean's wave number [1]. A detailed contribution of the self-gravitational instability with different assumptions on the rotating magnetized plasma has been examined by Chandrasekhar [2]. In cosmological situation, the fluid is not pure but may instead be permeated with fine dust particles. The influence of fine dust particle on the stability of superposed fluid might be of industrial and chemical engineering importance. The role of a fine dust particle in the laboratory, astrophysical plasma and in the interstellar clouds of the formation of stars as discussed by Alfven et al. [3]. Scanlon and Segal [4] have analyzed the problems of fine dust particles on the onset of B'enard convection. Sharma et al. [5] have carried out the investigation on the effect of a fine dust particle on the onset of B'enard problem (statically unstable configuration) in hydromagnetics. They concluded that the effect of the fine dust particle is destabilizing the layer and the magnetic field has a stabilizing effect. Sharma [6] has discussed the effect of a magnetic field on the gravitational instability of a medium in the presence of suspended particles. In this way, many researchers discussed the Jeans instability in the fine dust particles including various parameters such as [Kumar and Singh [7], Chhajlani and Sanghvi [8], Sharma and Sharma [9], Pensia et al. [10]. Recently Sutar and Pensia [11] have discussed the electron inertia effects on the gravitational instability under the influence of FLR corrections and suspended particles.

The magnetic field plays a crucial role in the formation of giant gas clouds and new stars in the intergalactic system. In interstellar medium a large amount of energy is injected by the stars, this phenomenon starts the formation of shock waves, but when these shock wave is weaken, they become large amplitude hydromagnetic Alfven waves. It is clear that magnetic fields can provide pressure support for the formation of interstellar clouds. It directly interacts only with the electrons, ions and charged grains in the gas medium.

On the other hand, it is well known that the Coriolis force plays a significant role in the formation of gas clouds and many astronomical objects. Acheson and Hide have [12] presented a comprehensive review of the investigations made on the rotating fluids. Gilman [13] is pointed out that sufficiently large rotation helps in the stabilizing of the magnetic fluid. Joshi and Pensia [14] analyzed the effect of rotation on Jeans instability of magnetized radiative quantum plasma. Prajapati et al. [15] have examined the self-gravitational instability of a rotating anisotropic heat-conducting plasma. Recently Bhakta et al. [16] have studied the small amplitude waves and linear firehose and mirror instabilities in rotating polytropic quantum plasma.

From the above discussion, we found that impact of fine dust particles on Jeans instability including the various parameters Coriolis force, electrical resistivity and electron plasma frequency of infinite homogeneous magnetized gaseous plasma. We hope our research help in understanding the various astronomical problems.

2. LINEARIZED PERTURBATION EQUATION

We consider an infinite homogeneous self-gravitating viscous rotating ionized plasma medium, including the electrical resistivity, fine dust particles (suspended particles) incorporating thermal conducting and electron plasma frequency in the presence of a magnetic field $\vec{H}(0,0,H)$.

2.1 Linearized Perturbation Equations of the Problem are,

The momentum transfer equation

$$\frac{\delta\vec{v}}{\delta t} = -\frac{\vec{\nabla}\delta P}{\rho} + \vec{\nabla}\delta\psi + \frac{K_s N}{\rho}(\vec{u}-\vec{v}) + \vartheta\left(\nabla^2 - \frac{1}{K_1}\right)\vec{v} + \frac{1}{4\pi\rho}(\vec{\nabla}\times\vec{h})\times\vec{H} + 2(\vec{v}\times\vec{\Omega}) \quad (1)$$

The equation of continuity

$$\frac{\partial\delta\rho}{\partial t} + \rho\vec{\nabla}.\vec{v} = 0 \quad (2)$$

Poisson's equation

$$\nabla^2\delta\psi + 4\pi G\delta\rho = 0 \quad (3)$$

$$\left(\tau\frac{\partial}{\partial t} + 1\right)\vec{u} = \vec{v} \quad (4)$$

The equation of thermal conductivity

$$\lambda\nabla^2\delta T = \rho C_p\frac{\partial\delta T}{\partial t} - \frac{\partial\delta P}{\partial t} \quad (5)$$

The gas equation

$$\frac{\delta P}{P} = \frac{\delta T}{T} + \frac{\delta\rho}{\rho} \quad (6)$$

$$\delta P = c^2\delta\rho \quad (7)$$

The idealized Ohm's law with the electron plasma frequency

$$\frac{\partial\vec{h}}{\partial t} = \vec{\nabla}\times(\vec{v}\times\vec{H}) + \eta\nabla^2\vec{h} + \frac{c'}{\omega_{pe}^2}\frac{\partial}{\partial t}\nabla^2\vec{h} \quad (8)$$

Where, $\vec{v}(v_x,v_y,v_z), \vec{u}(u_x,u_y,u_z), N, \rho, P, \psi, \vec{H}(0,0,H), \vec{\Omega}(\Omega_x,0,\Omega_z), T, G, c, \vartheta, C_p, \lambda, m, \rho_s, \omega_{pe}, K_1$ and η, denote respectively, the gas velocity, the particle velocity, the number density of the particle, density of the gas , pressure of the gas , Gravitational potential, magnetic field, rotation, temperature, Gravitational constant, velocity of sound in the medium, kinematic viscosity, specific heat at constant pressure, thermal conductivity, mass per unit volume of the particles and its density, plasma frequency of electron, permeability and electrical resistivity. The perturbations in fluid pressure, fluid density, gravitational potential, temperature and magnetic field are given as $\delta P, \delta\rho, \delta\psi, \delta T, \vec{h}(h_x,h_y,h_z)$, and c' is the speed of light. If we assume uniform particle-size, spherical shape and small relative velocities between the two phases, then the net effect of the particles on the gas is equivalent to an extra body force term per unit value written as $K_s N(\vec{u}-\vec{v}), K_s$ is the constant given by Stokes's drag formula $K_s(6\pi\rho\vartheta r)$ where r is the particle radius.

3. DISPERSION RELATION

We analyze these perturbations with normal mode analysis technique, we find the solution of equations (1-8). In a uniform system, we can get a plane-wave solution with all variables varying as,

$$exp\{i(k_x x + k_z z + \omega t)\} \quad (9)$$

Where k_x, k_z is the wave numbers of perturbation along the x-axis and z-axis respectively, such that $k_x^2 + k_z^2 = k^2$, σ is the growth rate of the perturbations, and ω is the frequency of harmonic disturbances, we substitute equations (2)-(9) in equation (1), we obtain the following algebraic equations for the components.

$$Q_1 v_x - 2\Omega_z v_y + \frac{ik_x}{k^2}\Omega_I^2 s = 0 \quad (10)$$

$$2\Omega_z v_x + Q_2 v_y - 2\Omega_z v_z = 0 \quad (11)$$

$$2\Omega_x v_y + M v_z + \frac{ik_z}{k^2}\Omega_I^2 s = 0 \quad (12)$$

Taking the divergence of equation (1) and using equations (2)-(9) we obtain

$$\frac{ik_x v^2 k^2}{a_1}v_x + 2iQ_4 v_y - Q_3 s = 0 \quad (13)$$

Where $s = \frac{\delta\rho}{\rho}$ is the condensation of the medium

$\gamma = \frac{C_p}{C_v} = \frac{C^2}{C'^2}$ ratio of the specific heat, $V = \frac{H}{\sqrt{4\pi\rho}}$ is the Alfven velocity,

$\Omega_s = \frac{K_s N}{\rho}$ has the dimensions of frequency, $\tau = \frac{m}{K_s}$ is the relaxation time,

$\beta = \tau\Omega_s = \frac{\rho_s}{\rho}$ is the mass concentration, $\sigma = i\omega$ is the growth rate of perturbation,

$\Omega_\vartheta = \vartheta\left(k^2 - \frac{1}{K_1}\right),\ a_1 = \sigma f + \Omega_m,\ f = \left(1 + \frac{C^2k^2}{\omega_{pe}^2}\right),$

$\theta = \frac{\lambda}{\rho C_p}$ is the thermometric conductivity,

$\Omega_m = \eta k^2$ is electrical resistivity, $C^2\left(\frac{\gamma p}{\rho}\right) and\ C'^2 = \left(\frac{p}{\rho}\right)$ where, $C\ and\ C'$ are the adiabatic and isothermal velocities of sound.

$M = \sigma + \Omega_\vartheta + \frac{\beta\sigma}{\sigma\tau+1},\ \Omega_j^2 = (C^2k^2 - 4\pi G\rho),\ \Omega_{j'}^2 = \left(C'^2k^2 - 4\pi G\rho\right),$

$Q_1 = \left(M + \frac{V^2K^2}{a_1}\right),\quad \Omega_I^2 = \frac{\sigma\Omega_j^2 + \theta_k\Omega_{j'}^2}{\sigma+\theta_k},\quad Q_2 = \left(M + \frac{V^2K_Z^2}{a_1}\right)$

$Q_4 = (k_x\Omega_z - \Omega_x k_z),\qquad Q_3 = \left(\sigma M + \Omega_I^2\right),\quad \theta_k = \gamma\theta k^2$

The solution of the determinant we obtained the following matrix relation, from equation (11)-(13) with $(v_x,\ v_y,\ v_z,\ s)$ having various coefficients,

$$X_{ij}Y_j = 0 \qquad ij = 1,2,3,4 \tag{14}$$

Where X_{ij} is the 4×4 matrix whose elements are

$X_{11} = \left(M + \frac{V^2K^2}{a_1}\right),\quad X_{12} = -2\Omega_z,\quad X_{13} = 0,\quad X_{14} = \frac{ik_x}{k^2}\Omega_I^2$

$X_{21} = 2\Omega_z,\quad X_{22} = \left(M + \frac{V^2K_Z^2}{a_1}\right),\quad X_{23} = -2\Omega_z,\quad X_{24} = 0$

$X_{31} = 0,\quad X_{32} = 2\Omega_x,\quad X_{33} = \left(\sigma + \Omega_\vartheta + \frac{\beta\sigma}{\sigma\tau+1}\right),\quad X_{34} = \frac{ik_z}{k^2}\Omega_I^2$

$X_{41} = \frac{ik_x v^2 k^2}{a_1},\quad X_{42} = 2i(k_x\Omega_z - \Omega_x k_z),\quad X_{43} = 0,\quad X_{44} = -\left(\sigma M + \Omega_I^2\right)$

Equation (14) has a non-trivial solution if the determinant of the matrix should vanish is to give the following dispersion relation.

$$M\left[\sigma M\{Q_1Q_2 + 4\Omega^2\} + \frac{4\Omega_x^2V^2k^2\sigma}{a_1} + \Omega_I^2\left\{Q_2^2 + 4\Omega^2 - \frac{4Q_4^2}{k^2}\right\}\right] = 0 \tag{15}$$

The dispersion relation (15) shows the combined influence of fine dust particles, electrical resistivity, thermal conductivity, electron plasma frequency, magnetic field, viscosity and Coriolis force on the self-gravitational instability of a homogeneous gaseous plasma. We found that the dispersion relation (15) is modified due to these parameters. This dispersion relation can predict the complete information about the acoustic wave, Alfven wave and Jeans gravitational instability of the gaseous plasmas considered. The above dispersion relation is very lengthy to analyze the effects of each parameter so we now reduced the dispersion relation (15) for two modes of propagation.

4. DISCUSSION

4.1 Longitudinal Propagation $(\boldsymbol{K} \parallel \boldsymbol{H})$

In this case, we have considered that all the perturbations are longitudinal to the direction of the magnetic field $(i.e. k_z = k,\ k_x = 0)$, and the dispersion relation (15) reduces to

$$\sigma M\left\{\left(M + \frac{k^2V^2}{a_1}\right)^2 + 4\Omega^2\right\} + \frac{4\sigma V^2k^2}{a_1}\Omega_x^2 + \Omega_I^2\left\{\left(M + \frac{k^2V^2}{a_1}\right)^2 + 4\Omega_z^2\right\} = 0 \tag{16}$$

This dispersion relation further reduced for simplicity for both parallel and perpendicular direction of the magnetic field on the rotational axis.

4.2 Axis of Rotation Parallel to the Magnetic Field $(\Omega \parallel \boldsymbol{H})$

When the axis of rotation is along the magnetic field, i.e. $\Omega_x = 0$ and $\Omega_z = \Omega$. Then (16) reduces to

$$\left(\sigma M + \Omega_I^2\right)\left[\left(\sigma + \Omega_\vartheta + \frac{\beta\sigma}{\sigma\tau + 1} + \frac{k^2V^2}{a_1}\right)^2 + 4\Omega^2\right] = 0 \qquad (17)$$

Equation (17) shows the combined influence of electrical resistivity, fine dust particles, electron plasma frequency, viscosity, permeability, Coriolis force, magnetic field and thermal conductivity on the self-gravitational instability of the hydrodynamic fluid plasma. The dispersion relation (17) is the product of two independent factors. These factors show the mode of propagations incorporating different parameters as discussed below.

The first factor of (17) is identical to Chhajlani et al. [8]. It represents a stable damped mode modified due to the viscosity, permeability, fine dust particles, and thermal conductivity. The dynamical stability of the system examined by applying the Routh Hurwitz criterion in the first factor and get all the coefficient are positive of the dispersion relation (17). It can also see that, for the longitudinal mode propagation in the medium, Jean's condition of instability is unaffected by the fine dust particles and electron plasma frequency. The second factor of (17) gives, on substituting the value of M, a_1 the following Six-degree polynomial equation.

$$\begin{aligned}
\tau^2 f^2 \sigma^6 + 2\tau\sigma^5[f^2\{1 + \tau(\beta + \Omega_\vartheta)\} + \tau f \Omega_m] \\
&+ \sigma^4\left[f^2\{1 + \tau(\beta + \Omega_\vartheta)\}^2 + 2\tau f(\Omega_\vartheta f + \tau k^2V^2) + \tau^2 f^2 4\Omega^2\right. \\
&\left. + 4\tau f \Omega_m\left\{1 + \tau(\beta + \Omega_\vartheta) + \tau^2\Omega_m^2\right\}\right] \\
&+ \sigma^3\left[2\tau f k^2V^2 + \tau f^2 8\Omega^2 + 2f(f\Omega_\vartheta + \tau k^2V^2)\{1 + \tau(\beta + \Omega_\vartheta)\}\right. \\
&\left. + 2f\Omega_m\left\{(\tau\Omega_\vartheta + 1)^2 + \beta\tau(2 + \beta\tau + 2\tau\Omega_\vartheta) + \tau^2 4\Omega^2\right\}\right] \\
&+ \sigma^2\left[(f\Omega_\vartheta + \tau k^2V^2)^2 + 2fk^2V^2\{1 + \tau(\beta + \Omega_\vartheta)\} + f^2 4\Omega^2\right. \\
&+ 4f\Omega_m\Omega_\vartheta\{1 + \tau(\beta + \Omega_\vartheta)\} + \tau f\Omega_m 16\Omega^2 \\
&\left. + \Omega_m^2\left\{(\beta + \Omega_\vartheta)\left(2\tau^2k^2V^2 + \tau^2\Omega_\vartheta + \tau^2\beta + 2\tau + 1 + \tau^2 4\Omega^2 + 2\tau\Omega_\vartheta + 2k^2V^2\right)\right\}\right] \\
&+ \sigma\left[2k^2V^2(f\Omega_\vartheta + \tau k^2V^2) + 2f\Omega_m\left(\Omega_\vartheta^2 + 4\Omega^2\right) + 2\Omega_m^2\{1 + \tau(\beta + \Omega_\vartheta)\}(\Omega_\vartheta + k^2V^2)\right. \\
&\left. + 2\tau\Omega_m^2\left(\Omega_\vartheta k^2V^2 + \tau 4\Omega^2\right)\right] + \Omega_m^2\left(\Omega_\vartheta^2 + 2\Omega_\vartheta k^2V^2 + 4\Omega^2\right) + k^4V^4 \\
&= 0 \qquad (18)
\end{aligned}$$

The dispersion relation (18) is a non-gravitating Alfven mode modified by the presence of electrical-resistivity, fine dust particles, electron plasma frequency, viscosity, permeability, resistivity, and Coriolis force. We find that the condition of Jeans instability modified due to a magnetic field, permeability, viscosity, Coriolis force, and resistivity.

4.3 Axis of Rotation Perpendicular to the Magnetic Field$(\Omega \perp \boldsymbol{H})$

In this case, when a rotation is perpendicular to the magnetic field, we put $\Omega_x = \Omega \ and \ \Omega_z = 0$, this gives,

$$(Ma_1 + k^2V^2)\left[(Ma_1 + k^2V^2)\left(\sigma M + \Omega_I^2\right) + 4\sigma a_1 \Omega^2\right] = 0 \qquad (19)$$

The above dispersion relation is the product of two independent factors. These factors show the influence of different parameters as discussed below.

The first factor of (19) also represents a stable non-gravitating Alfven mode modified by fine dust particles, electron plasma frequency, and viscosity but this mode is not affected by Coriolis force.

Thus we see that Alfven mode is not affected by Coriolis force in the longitudinal mode of propagation when the axis of rotation is taken perpendicular to the direction of the magnetic field while this non-gravitating Alfven mode is affected by Coriolis force when the axis of rotation is chosen to parallel to the magnetic field. The second factor (19) gives on substituting the values of M, a_1 and Ω_I^2, the following seven-degree polynomial equation.

$$
\begin{aligned}
&\tau^2\sigma^7 f+\sigma^6[\tau f\{2+\tau(2\Omega_\vartheta+2\beta+\theta_k)\}+\tau^2\Omega_m^2] \\
&+\sigma^5[f\{1+2\tau(2\Omega_\vartheta+\beta+\theta_k)\}+\tau^2\{f\Omega_j^2+k^2V^2+4f\Omega^2+f(\beta+\Omega_\vartheta)(\Omega_\vartheta+\beta+2\theta_k)\} \\
&+2\tau\Omega_m\{1+\tau(\beta+\Omega_\vartheta)\}+\tau^2\Omega_m\theta_k] \\
&+\sigma^4[f(2\Omega_\vartheta+\theta_k)+2\tau\{f\Omega_j^2+k^2V^2+f4\Omega^2+f(\beta+\Omega_\vartheta)(\Omega_\vartheta+\theta_k)+f\Omega_\vartheta\theta_k\} \\
&+\tau^2\{\theta_k(f\Omega_{j'}^2+k^2V^2+f4\Omega^2)+(\beta+\Omega_\vartheta)(f\Omega_j^2+k^2V^2+f\beta\theta_k+f\Omega_\vartheta\theta_k)\}+\Omega_m\{1+2\tau(2\Omega_\vartheta+\beta+\theta_k)\} \\
&+\tau^2\Omega_m\{\Omega_j^2+4\Omega^2+(\beta+\Omega_\vartheta)(\Omega_\vartheta+\beta+2\theta_k)\}] \\
&+\sigma^3[(f\Omega_j^2+k^2V^2+f4\Omega^2+f\Omega_\vartheta^2+2f\Omega_\vartheta\theta_k) \\
&+\tau\{(\beta+\Omega_\vartheta)(f\Omega_j^2+k^2V^2+2f\Omega_\vartheta\theta_k)+\Omega_\vartheta(f\Omega_j^2+k^2V^2)+2\theta_k(f\Omega_{j'}^2+k^2V^2+4f\Omega^2)\} \\
&+\tau^2\{k^2V^2\Omega_j^2+\theta_k(\beta+\Omega_\vartheta)(f\Omega_{j'}^2+k^2V^2)\}+\Omega_m(2\Omega_\vartheta+\theta_k) \\
&+2\tau\Omega_m\{\Omega_j^2+4\Omega^2+(\beta+\Omega_\vartheta)(\Omega_\vartheta+\theta_k)+\Omega_\vartheta\theta_k\}+\Omega_m\tau^2\{\theta_k(\Omega_{j'}^2+4\Omega^2)+(\beta+\Omega_\vartheta)(\Omega_j^2+\beta\theta_k+\Omega_\vartheta\theta_k)\}] \\
&+\sigma^2[\Omega_\vartheta(f\Omega_j^2+k^2V^2+f\Omega_\vartheta\theta_k)+\theta_k(f\Omega_{j'}^2+k^2V^2+f4\Omega^2)+\tau\{2k^2V^2\Omega_j^2+2\Omega_\vartheta\theta_k(f\Omega_{j'}^2+k^2V^2)\} \\
&+\tau^2k^2V^2\theta_k\Omega_{j'}^2+\tau\beta\theta_k(f\Omega_{j'}^2+k^2V^2)+\Omega_m(\Omega_j^2+4\Omega^2+\Omega_\vartheta^2+2\Omega_\vartheta\theta_k) \\
&+\Omega_m\tau\{(\beta+\Omega_\vartheta)(\Omega_j^2+2\Omega_\vartheta\theta_k)+\Omega_j^2\Omega_\vartheta+2\theta_k(\Omega_{j'}^2+4\Omega^2)+\tau^2\theta_k(\beta+\Omega_\vartheta)\Omega_j^2\}] \\
&+\sigma[k^2V^2\Omega_j^2+\Omega_\vartheta\theta_k(f\Omega_{j'}^2+k^2V^2)+2\tau k^2V^2\theta_k\Omega_{j'}^2 \\
&+\Omega_m\{\Omega_\vartheta(\Omega_j^2+\Omega_\vartheta\theta_k)+\theta_k(\Omega_{j'}^2+4\Omega^2)+\tau\theta_k\Omega_{j'}^2(\beta+2\Omega_\vartheta)\}]+\theta_k\Omega_{j'}^2(k^2V^2+\Omega_\vartheta\Omega_m)=0
\end{aligned}
\tag{20}
$$

This dispersion relation as represents the effect of the simultaneous inclusion of the fine dust particles, electron plasma frequency, electrical resistivity, thermal conductivity, viscosity, Coriolis force and magnetic field on the self-gravitational instability of the system for longitudinal propagation with the axis of rotation perpendicular to the magnetic field. The condition of instability and the expression of the critical Jeans wave number obtained from the constant term of (20), we found that the constant conditions are independent of the electron plasma frequency, but the growth rate of the system changed by them. We found that the condition of instability not affected by the viscosity and the presence of fine dust particles.

In order to study the effect of Coriolis force, fine dust particle and electron plasma frequency on the growth rate of the Jeans instability, we convert the dispersion relation (20) in dimensionless form dividing by $\sqrt{4\pi G\rho}$, thus equation (20) becomes

$$
\begin{aligned}
&\tau^{*2}\sigma^{*7}f^*+\sigma^{*6}[\tau^*f^*\{2+\tau^*(2\vartheta_k^*+2k_s^*+\lambda^*)\}+\tau^{*2}\eta^{*2}k^{*4}] \\
&+\sigma^{*5}[f^*\{1+2\tau^*(2\vartheta_k^*+k_s^*+\lambda^*)\} \\
&+\tau^{*2}\{f^*(k^{*2}-1)+k^{*2}V^{*2}+4f^*\Omega^{*2}+f(k_s^*+\vartheta_k^*)(\vartheta_k^*+k_s^*+2\lambda^*)\}+2\tau^*\{1+\tau^*(k_s^*+\vartheta_k^*)\} \\
&+\tau^{*2}\eta^*k^{*2}\lambda^*] \\
&+\sigma^{*4}[2f^*(\vartheta_k^*+\lambda^*)+2\tau^*\{f^*(k^{*2}-1)+k^{*2}V^{*2}+f^*4\Omega^{*2}+f^*(k_s^*+\vartheta_k^*)(\vartheta_k^*+\lambda^*)+f^*\vartheta_k^*\lambda^*\} \\
&+\tau^{*2}\{\lambda^*(f^*(k^{*2}-1)+k^{*2}V^{*2}+f^*4\Omega^{*2})+(k_s^*+\vartheta_k^*)(f^*(k^{*2}-1)+k^{*2}V^{*2}+f^*k_s^*\lambda^*+f^*\vartheta_k^*\lambda^*)\} \\
&+\eta^*k^{*2}\{1+2V^{*2}(2\vartheta_k^*+k_s^*+\lambda^*)\}+\tau^{*2}\eta^*k^{*2}\{(k^{*2}-1)+4\Omega^{*2}+(k_s^*+\vartheta_k^*)(\vartheta_k^*+k_s^*+2\lambda^*)\}] \\
&+\sigma^{*3}[(f^*(k^{*2}-1)+k^{*2}V^{*2}+f^*4\Omega^{*2}+f^*\vartheta_k^{*2}+2f^*\vartheta_k^*\lambda^*) \\
&+\tau^*\{(k_s^*+\vartheta_k^*)(f^*(k^{*2}-1)+k^{*2}V^{*2}+2f^*\vartheta_k^*\lambda^*)+\vartheta_k^*(f^*(k^{*2}-1)+k^{*2}V^{*2}) \\
&+2\lambda^*(f^*(k^{*2}-1)+k^{*2}V^{*2}+4f^*\Omega^{*2})\} \\
&+\tau^{*2}\{k^{*2}V^{*2}(k^{*2}-1)+\lambda^*(k_s^*+\vartheta_k^*)(f^*(k^{*2}-1)+k^{*2}V^{*2})\}+2\eta^*k^{*2}(\vartheta_k^*+\lambda^*) \\
&+2\tau^*\eta^*k^{*2}\{(k^{*2}-1)+4\Omega^{*2}+(k_s^*+\vartheta_k^*)(\vartheta_k^*+\lambda^*)+\vartheta_k^*\lambda^*\} \\
&+\eta^*k^{*2}\tau^{*2}\{\lambda^*((k^{*2}-1)+4\Omega^{*2})+(k_s^*+\vartheta_k^*)((k^{*2}-1)+k_s^*\lambda^*+\vartheta_k^*\lambda^*)\}] \\
&+\sigma^{*2}[\vartheta_k^*(f^*(k^{*2}-1)+k^{*2}V^{*2}+f^*\vartheta_k^*\lambda^*)+\lambda^*(f^*(k^{*2}-1)+k^{*2}V^{*2}+f^*4\Omega^{*2}) \\
&+\tau^*\{2k^{*2}V^{*2}(k^{*2}-1)+2\vartheta_k^*\lambda^*(f^*(k^{*2}-1)+k^{*2}V^{*2})\}+\tau^{*2}k^{*2}V^{*2}\lambda^*(k^{*2}-1) \\
&+\tau k_s^*\lambda^*(f^*(k^{*2}-1)+k^{*2}V^{*2})+\eta^*k^{*2}((k^{*2}-1)+4\Omega^{*2}+\vartheta_k^{*2}+2\vartheta_k^*\lambda^*) \\
&+\eta^*k^{*2}\tau^*\{(k_s^*+\vartheta_k^*)((k^{*2}-1)+2\vartheta_k^*\lambda^*)+(k^{*2}-1)\vartheta_k^*+2\lambda^*((k^{*2}-1)+4\Omega^{*2}) \\
&+\tau^{*2}\lambda^*(k_s^*+\vartheta_k^*)(k^{*2}-1)\}] \\
&+\sigma^*[k^{*2}V^{*2}(k^{*2}-1)+\vartheta_k^*\lambda^*(f^*(k^{*2}-1)+k^{*2}V^{*2})+2\tau^*k^{*2}V^{*2}\lambda^*(k^{*2}-1) \\
&+\eta^*k^{*2}\{\vartheta_k^*((k^{*2}-1)+\vartheta_k^*\lambda^*)+\lambda^*((k^{*2}-1)+4\Omega^{*2})+\tau^*\lambda^*(k^{*2}-1)(k_s^*+2\vartheta_k^*)\}] \\
&+\lambda^*(k^{*2}-1)(k^{*2}V^{*2}+\vartheta_k^*\eta^*k^{*2})\ =0
\end{aligned}
\tag{21}
$$

Where the various nondimensional parameters defined as

$$\sigma^* = \frac{\sigma}{\sqrt{4\pi G\rho}},\ k_s^* = \frac{k_s N}{\rho\sqrt{4\pi G\rho}}, k^* = \frac{kC}{\sqrt{4\pi G\rho}}, \upsilon^* = \frac{\vartheta\sqrt{4\pi G\rho}}{C^2},\ K_1^* = \frac{k_1\sqrt{4\pi G\rho}}{C^2}\ , = \tau\sqrt{4\pi G\rho}\,,$$
$$V^* = \frac{V\sqrt{4\pi G\rho}}{C}, \Omega_j^{*2} = (k^{*2}-1),\ \Omega^* = \frac{\Omega}{\sqrt{4\pi G\rho}},$$
$$\lambda^* = \frac{\lambda}{\rho C_p\sqrt{4\pi G\rho}}\ ,\ \ = f\sqrt{4\pi G\rho},\ \ \eta^* = \frac{\eta\sqrt{4\pi G\rho}}{C^2},\qquad \vartheta_k^* = \upsilon^*\left(k^{*2} - \frac{1}{k_1^*}\right) \tag{22}$$

The variation of the growth rate σ^* with wave number k^* is shown in Figs.1-4.

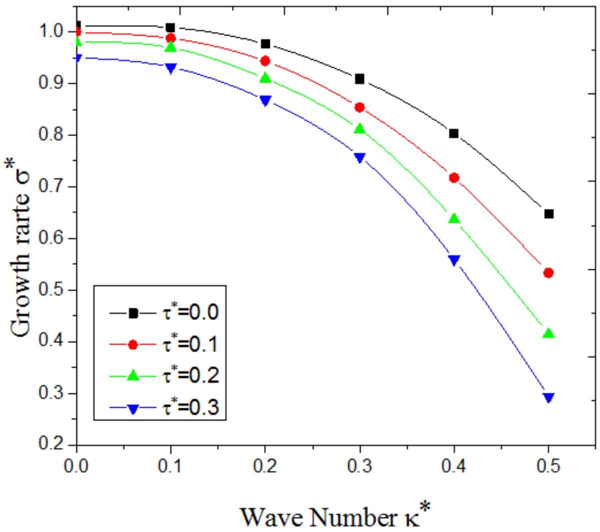

Fig. 1. The growth rate σ^*, in the longitudinal mode, is plotted against wave number k^* with variation in the fine dust particle τ^*= (0.0, 0.1, 0.2, 0.3), keeping the value of other parameters are one and $\Omega^* = 0.5$

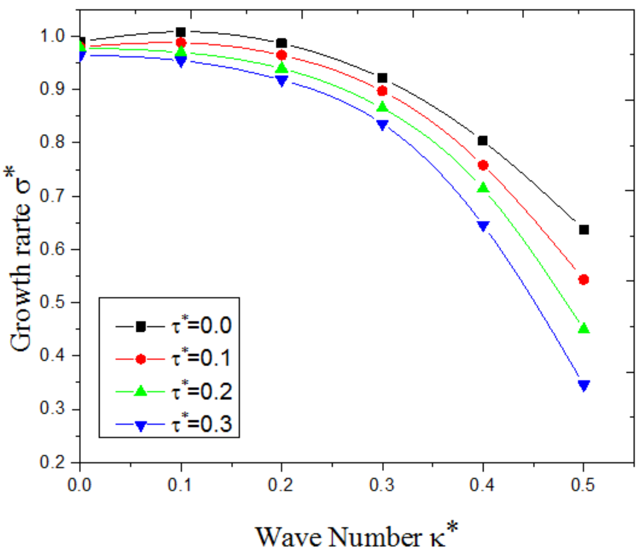

Fig. 2. The growth rate σ^*, in the longitudinal mode, is plotted against wave number k^* with variation in the fine dust particle τ^*= (0.0, 0.1, 0.2, 0.3), keeping the value of other parameters are one and $\Omega^* = 1.5$

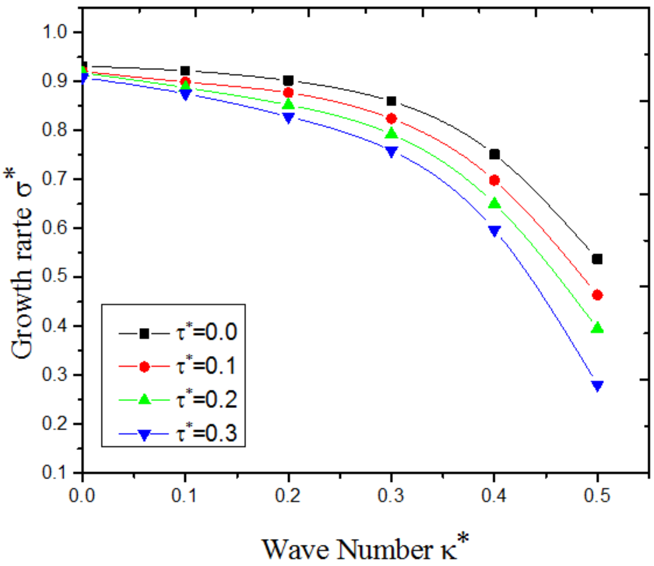

Fig. 3. The growth rate σ^*, in the longitudinal mode, is plotted against wave number k^* with variation in the fine dust particle τ^*= (0.0, 0.1, 0.2, 0.3), keeping the value of other parameters are one and $f^* = 0.5$

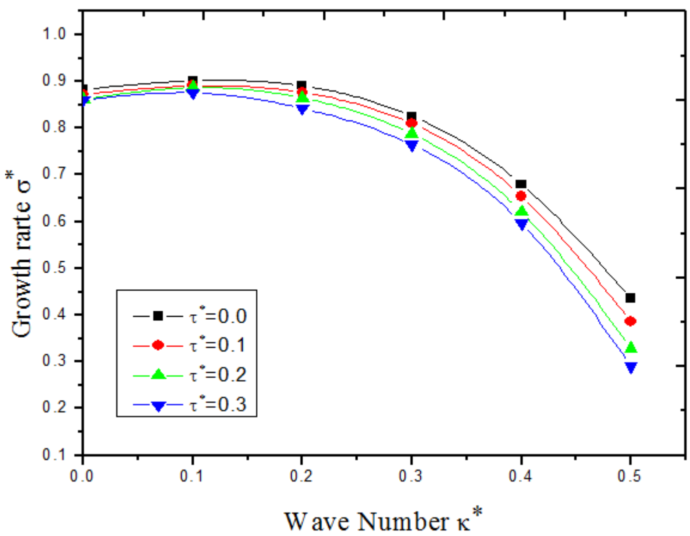

Fig. 4. The growth rate σ^*, in the longitudinal mode, is plotted against wave number k^* with variation in the fine dust particle τ^*= (0.0, 0.1, 0.2, 0.3), keeping the value of other parameters are one and $f^* = 1.5$

4.4 Transverse Propagation $(K \perp H)$

Similar to the longitudinal propagation, we assume that all the perturbations transverse to the direction of the magnetic field $(i.e. k_x = k,\quad kz=0$, Thus the dispersion relation (15) takes the form

$$\sigma\left[M\{Q_1M + 4\Omega^2\} + \frac{4\Omega_x^2 V^2 k^2}{a_1}\right] + \Omega_I^2(M^2 + 4\Omega_x^2) = 0 \quad (23)$$

The dispersion relation (23) shows the combined influence of electrical resistivity, electron plasma frequency, fine dust particles, Coriolis force, permeability, viscosity, magnetic field and thermal conductivity on the self-gravitational of infinite homogeneous gaseous plasma. If we ignore electron plasma frequency (23) the result is similar to given by Chhajlani and Sangvi [8]. Now we discuss this dispersion relation (23) in the case, when the axis of rotation parallel and perpendicular to the magnetic field.

4.5 Axis of Rotation Parallel to the Magnetic Field ($\boldsymbol{\Omega} \parallel \boldsymbol{H}$)

In the case of rotation is parallel to the magnetic field, here put $\Omega_x = 0 \; and \; \Omega_z = \Omega$ in the dispersion relation (23) and obtained

$$M\left[M\left\{\sigma Q_1 + \Omega_I^2\right\} + 4\sigma\Omega^2\right] = 0 \qquad (24)$$

This equation (24) shows the combined influence of electrical resistivity, electron plasma frequency, viscosity, Coriolis force, fine dust particles, and thermal conductivity on the self-gravitational instability of the hydro-magnetic fluid plasma. The dispersion relation (24) has two independent factors, each representing different modes of propagations. In the above equation, the first term of this dispersion relation represents the stability of the system is discussed in the previous case and the second factor of the dispersion relations (24) after simplification written as

$$\begin{aligned}
&\tau^2\sigma^7 f + \sigma^6\tau[2f\{1+\tau(\beta+\Omega_\vartheta)\} + \tau(f\theta_k + \Omega_m)] \\
&\quad + \sigma^5\left[\tau^2\left(f\Omega_j^2 + k^2V^2 + 4\Omega^2 f\right) + 2\tau\Omega_\vartheta f + f\{1+\tau(\beta+\Omega_\vartheta)\}\{1+\tau(\beta+\Omega_\vartheta+2\theta_k)\}\right. \\
&\quad \left. + \tau\Omega_m\{2 + 2\tau(\beta+\Omega_\vartheta) + \tau\theta_k\}\right] \\
&\quad + \sigma^4\left[\tau^2\theta_k\left(\Omega_{j'}^2 f + k^2V^2 + 4\Omega^2 f\right) + \tau\left(\Omega_j^2 f + k^2V^2 + 8\Omega^2 f + 2\Omega_\vartheta\theta_k f\right)\right. \\
&\quad + f\{1+\tau(\beta+\Omega_\vartheta)\}\left\{(2\Omega_\vartheta + \theta_k) + \tau\left(\Omega_j^2 f + k^2V^2 + \Omega_\vartheta\theta_k + \beta\theta_k\right)\right\} \\
&\quad \left. + \Omega_m\left\{\tau^2\left(\Omega_j^2 + 4\Omega^2\right) + 2\tau\Omega_\vartheta + \{1+\tau(\beta+\Omega_\vartheta)\}\{1+\tau(\beta+\Omega_\vartheta+2\theta_k)\}\right\}\right] \\
&\quad + \sigma^3\left[f\left(\Omega_\vartheta^2 + 4\Omega^2\right) + \{1+\tau(\beta+\Omega_\vartheta)\}\left\{\Omega_j^2 f + k^2V^2 + 2\Omega_\vartheta\theta_k f + \tau\theta_k\left(\Omega_{j'}^2 f + k^2V^2\right)\right\}\right. \\
&\quad + \tau\left\{\Omega_\vartheta\left(\Omega_j^2 f + k^2V^2\right) + \theta_k\left(\Omega_{j'}^2 f + k^2V^2 + 8\Omega^2 f\right)\right\} \\
&\quad + \Omega_m\left\{\tau^2\theta_k\left(\Omega_{j'}^2 + +4\Omega^2\right) + \tau\left(\Omega_j^2 + 8\Omega^2 + 4\Omega_\vartheta\theta_k\right)\right. \\
&\quad \left.\left. + \{1+\tau(\beta+\Omega_\vartheta)\}\left\{(2\Omega_\vartheta + \theta_k) + \tau\left(\Omega_j^2 + \Omega_\vartheta\theta_k + \beta\theta_k\right)\right\}\right\}\right] \\
&\quad + \sigma^2\left[\Omega_\vartheta\left(\Omega_j^2 f + k^2V^2 + \Omega_\vartheta\theta_k f\right) + \tau\theta_k\left(\Omega_{j'}^2 f + k^2V^2\right)\right. \\
&\quad + \theta_k\{1+\tau(\beta+\Omega_\vartheta)\}\left(\Omega_{j'}^2 f + k^2V^2\right) + 4\Omega^2 f\theta_k \\
&\quad + \Omega_m\left\{\left(\Omega_\vartheta^2 + 4\Omega^2\right) + \{1+\tau(\beta+\Omega_\vartheta)\}\left(\Omega_j^2 + \tau\theta_k\Omega_{j'}^2 + 2\Omega_\vartheta\theta_k\right) + \tau\Omega_\vartheta\Omega_j^2\right. \\
&\quad \left.\left. + \theta_k\left(\Omega_{j'}^2 + +8\Omega^2\right)\right\}\right] \\
&\quad + \sigma\left[\Omega_\vartheta\theta_k\left(\Omega_{j'}^2 f + k^2V^2\right) + \Omega_m\left\{\Omega_\vartheta\left(\Omega_j^2 + \Omega_\vartheta\theta_k\right) + \theta_k\Omega_{j'}^2\{1+\tau(\beta+\Omega_\vartheta)\}\right\}\right] \\
&\quad + \Omega_\vartheta\theta_k\Omega_{j'}^2\Omega_m = 0 \qquad (25)
\end{aligned}$$

The equation (25) is seven-degree polynomial equations and shows the combined influence of fine dust particles, viscosity, Coriolis force, magnetic field, thermal conductivity, electron plasma frequency and electrical resistivity in the perpendicular direction when the axis of rotation is along to the magnetic field. The Jeans criterion of instability is determined by the constant term of equation (25) is modified by viscosity, thermal conductivity and electrical resistivity the critical Jeans wavenumber is given by

$$k < k_j = \left(\frac{4\pi G\rho}{C'^2 + V^2}\right)^{\frac{1}{2}} \qquad (26)$$

From (26) we note that the Jeans criteria of the instability are not affected by electron plasma frequency, Coriolis force, and suspended particles, but a magnetic field modifies Jean's condition.

4.6 Axis of Rotation Perpendicular to the Magnetic Field ($\boldsymbol{\Omega} \perp \boldsymbol{H}$)

In case when perturbation is perpendicular to the magnetic field, we put $\Omega_x = \Omega \; and \; \Omega_z = 0$ in equation (24) then we get

$$\left(M^2 + 4\Omega^2\right)\left\{\sigma Q_1 + \Omega_I^2\right\} = 0 \qquad (27)$$

The first factor of (27) gives as

$$\sigma^4\tau^2 + \sigma^3 2\tau\{1 + \tau(\beta + \Omega_\vartheta)\} + \sigma^2\left[\{1 + \tau(\beta + \Omega_\vartheta)\}^2 + 2\tau\Omega_\vartheta + \tau^2 4\Omega^2\right] + 2\sigma\left[\Omega_\vartheta\{1 + \tau(\beta + \Omega_\vartheta)\} + \tau 4\Omega^2\right] + \Omega_\vartheta^2 + 4\Omega^2 = 0 \quad (28)$$

The above equation is a four-degree polynomial equation and shows the combined influence of various parameters. The second factor of dispersion relation (27) simplification written as;

$$\begin{aligned}\sigma^5\tau f + \sigma^4[f\{1 + \tau(\beta + \Omega_\vartheta + \theta_k)\} + \tau\Omega_m] \\ + \sigma^3\left[f(\Omega_\vartheta + \theta_k) + \tau\{\Omega_j^2 f + k^2V^2 + \theta_k f(\beta + \Omega_\vartheta)\} + \Omega_m\{1 + \tau(\beta + \Omega_\vartheta + \theta_k)\}\right] \\ + \sigma^2\Big[\left(\Omega_j^2 f + k^2V^2 + \Omega_\vartheta\theta_k f\right) + \tau\theta_k\left(\Omega_{j'}^2 f + k^2V^2\right) \\ + \Omega_m\left\{\Omega_\vartheta + \theta_k + \tau\{\Omega_j^2 + \theta_k(\beta + \Omega_\vartheta)\}\right\}\Big] \\ + \sigma\left[\theta_k\left(\Omega_{j'}^2 f + k^2V^2\right) + \Omega_m\left\{\Omega_j^2 + \theta_k\left(\tau\Omega_{j'}^2 + \Omega_\vartheta\right)\right\}\right] = 0 \quad (29)\end{aligned}$$

Equation (29) is a five-degree polynomial equation and show the combined influence of various parameters, fine dust particles, viscosity, magnetic field, thermal conductivity, and electrical resistivity in the transverse mode of propagation when the axis of rotation perpendicular to the magnetic field. The Jeans instability of the system is modified by fine dust particle in a transverse mode of propagation. Now we can write (29) in non-dimensional form, for showing the influence of different parameters on the growth rate of instability, as:

$$\begin{aligned}\sigma^{*5}\tau^* f^* + \sigma^{*4}[f^*\{1 + \tau^*(k_s^* + \vartheta_k^* + \lambda^*)\} + \tau^*\eta^* k^{*2}] \\ + \sigma^{*3}[f^*(\vartheta_k^* + \lambda^*) + \tau\{(k^{*2} - 1) f^* + k^{*2}V^{*2} + \lambda^* f^*(k_s^* + \vartheta_k^*)\} \\ + \eta^* k^{*2}\{1 + \tau^*(k_s^* + \vartheta_k^* + \lambda^*)\}] \\ + \sigma^{*2}\Big[\left((k^{*2} - 1) f^* + k^{*2}V^{*2} + \vartheta_k^*\lambda^* f^*\right) + \tau^*\lambda^*\left((k^{*2} - 1) f^* + k^{*2}V^{*2}\right) \\ + \eta^* k^{*2}\{\vartheta_k^* + \lambda^* + \tau^*\{(k^{*2} - 1) + \lambda^*(k_s^* + \vartheta_k^*)\}\}\Big] \\ + \sigma^*\left[\lambda^*\left((k^{*2} - 1) f^* + k^{*2}V^{*2}\right) + \eta^* k^{*2}\{(k^{*2} - 1) + \lambda^*(\tau^*(k^{*2} - 1) + \vartheta_k^*)\}\right] \\ = 0 \quad (30)\end{aligned}$$

In Fig. 5 and 6, we have depicted the non-dimensional growth rate versus non-dimensional wave number of various value of fine dust particle.

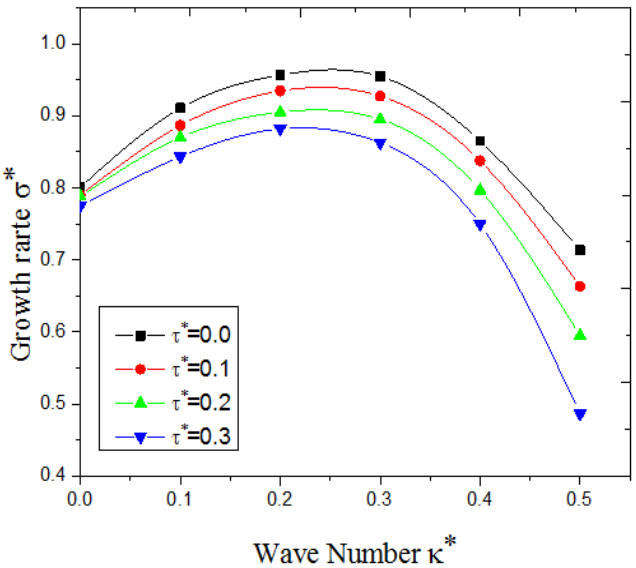

Fig. 5. The growth rate σ^*, in the transverse mode, is plotted against wave number k^* with variation in the fine dust particle τ^*= (0.0, 0.1, 0.2, 0.3), keeping the value of other parameters are one and $\eta^* = 0.5$

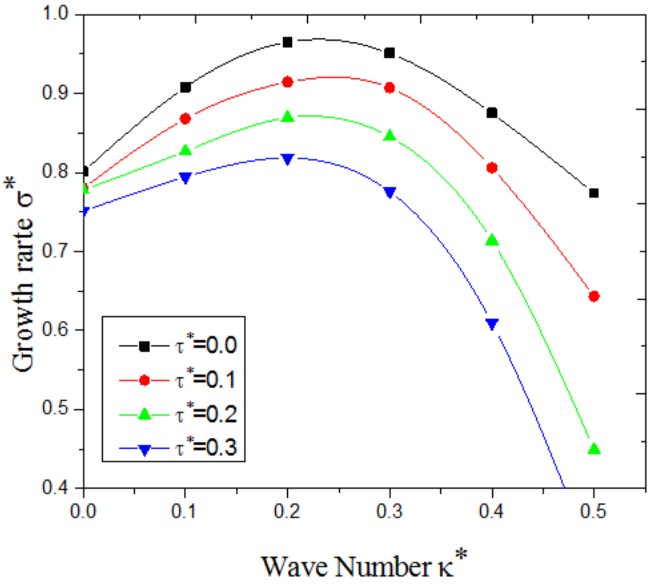

Fig. 6. The growth rate σ^*, in the transverse mode, is plotted against wave number k^* with variation in the fine dust particle τ^*= (0.0, 0.1, 0.2, 0.3), keeping the value of other parameters are one and $\eta^* = 1.5$

5. CONCLUSIONS

In the present paper, we have analyzed the problem of a self-gravitational instability of a viscous magnetized gaseous plasma, considering the effects of fine dust particles, including the parameters electrical resistivity, Coriolis force, and electron plasma frequency. The general dispersion relation is obtained, which modified due to the presence of these parameters. The general dispersion relation for a longitudinal and transverse mode of propagation examines the axis of rotation parallel and perpendicular to the direction of the magnetic field. We see that the Jeans criterion remains valid, but the expression of the Jeans wave number is modified.

In the longitudinal mode of propagation, it is found that the condition of Jeans instability is modified due to the presence of electrical resistivity and viscosity but it is not affected by electron plasma frequency and Coriolis force, but the growth rate of instability is affected by them. Form the graphical presentation, we conclude that fine dust particle has a stabilizing influence on the system and when we increase the value of Coriolis force the system is more stable as shown in Fig. 1-2, similarly, we increase the value of electron plasma frequency the system is more stable showing in Fig. 3-4.

In the transverse mode of propagation, we modified the condition of Jeans instability due to the presence of a magnetic field but it is not influenced by a fine dust particle. From Fig. 5-6 it is clear that the fine dust particle is stabilizing impact in the system but when we increase the value of electrical resistivity the system is unstable, so we conclude that the electrical resistivity has destabilized the system. The result of the present analysis may be useful in understanding the problem of wave propagation and Jeans instability in the gaseous plasma system. The results of the present study can be applied to understand the formation of molecular clouds, galaxies, giant stars, and many astrophysical objects.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

REFERENCES

1. Jeans JH. The stability of spherical nebula. Phil. Trans. Ray. Soc. London. 1902;1-53.
2. Chandrashekhar S. Hydrodynamic and Hydromagnetic Stability. Clarendon Press Oxford, Chapter. 1961;13.
3. Alfvén H, Carlqvist P. Interstellar clouds and the formation of stars. Astrophysics Space Science. 1978;487–509.
4. Scanlon JH, Segel LA. Some effects of suspended particles on the onset of

Bénard convection. Phys. Fluids. 1973; 1573.
5. Sharma RC, Prakash K, Dube SN. Effect of suspended particles on the onset of B'enard problem in hydromagnetics. Acta Physica Hungarica. 1976;3-10.
6. Sharma RC. Effect of magnetic field on the gravitational instability of self-gravitating homogeneous gas particle medium in the presence of suspended particles. Astrophysics and Space Science. 1977; 255-259.
7. Kumar P, Singh M. Hydrodynamic and Hydromagnetic stability of viscous-viscoelastic superposed fluids in presence of suspended particles. Physical Separation in Science and Engineering. 2007;1-6.
8. Chhajlani RK, Sanghavi RK. Suspended particles and the gravitational instability of rotating magnetized medium. Contrib. Plasma Physics. 1985;623-631.
9. Sharma RC, Sharma KC. Suspended particles and the gravitational instability of a rotating plasma. Astrophysics and Space Science. 1980;325-332.
10. Pensia R, Patidar AK, Shrivastava V, Kumar V, Prajapat V. The role of coriolis force and suspended particles in the fragmentation of matter in the central region of Galaxy. Journal of Bangladesh Academy of Sciences. 2012;1-12.
11. Sutar DL, Pensia RK. Electron inertia effects on the gravitational instability under the influence of FLR corrections and suspended particles. Physical Science International Journal. 2016;1-12.
12. Acheson DJ, Hide R. Hydromagnetics of rotating fluids. Rep. Prog. Physics. 1973;159.
13. Gilman PA. Instability of Magnetohydrostatic Stellar Interiors from Magnetic Buoyancy. I. Astrophysical Journal. 1970;1019.
14. Joshi H, Pensia RK. Effect of rotation on Jeans instability of magnetized radiative quantum plasma. Physics of Plasma. 2017;032113.
15. Prajapati RP, Parihar AK, Chhajlani RK. Self-gravitational instability of rotating anisotropic heat-conducting plasma. Physics of Plasmas. 2008;012107.
16. Bhakta S, Prajapati RP, Dolai B. Small amplitude waves and linear firehose and mirror instabilities in rotating polytropic quantum plasma. Physics of Plasmas. 2017;082113.

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