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Existence Solutions and Controllability of Nonlinear Fractional Integro-differential Systems with Nonlocal Conditions

Hamdy M. Ahmed[1*]

[1] *Higher Institute of Engineering, El-Shorouk Academy,*
P.O. 3 El-Shorouk City,Cairo,Egypt.

Research Article

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Abstract

We prove the existence of mild solutions of fractional integrodifferential equations with nonlocal conditions in Banach spaces. Sufficient conditions for controllability of fractional integrodifferential systems are established. The results are obtained by using resolvent operators and Schauder fixed point theorem. An example is provided to illustrate our results.

Keywords: Fractional calculus; nonlocal conditions; integrodifferential systems; mild solutions, resolvent operators; controllability; Schauder fixed point theorem

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1 Introduction

The nonlocal condition, which is a generalization of the classical condition, was motivated by physical problems. The pioneering work on nonlocal conditions is due to Byszewski [4]. In the last few years several papers have been devoted to the study of existence and uniqueness of solutions to nonlinear differential equations with nonlocal conditions. Among others, we refer to the papers of Balachandran and Chandrasekarn [6], Balachandran and Illamaran [5], Byszewski [3] and Ntouyas and Tsamatos [20]. Recently, there has been an increasing interest in studying the problem of controllability of integrodifferential systems (see [7-10]). On the other hand, there is also an increasing interest in the recent years related to dynamical fractional systems oriented towards the field of control theory concerning heat transfer, lossless transmission lines (see [24], [11], the use of discretizing devices supported by fractional calculus. Controllability results for linear fractional differential equations have been considered by a few authors (see [1], [2], [14], [17], [23], [25], [26], [27], [28],).
In this paper we study the existence of mild solution and controllability of the fractional integrodifferential equation with nonlocal condition in the following form

$$\frac{d^\alpha x(t)}{dt^\alpha} = A[x(t) + \int_0^t F(t-s)x(s)ds] + f(t, x(t)) + \int_0^t g(t, s, x(s), Q(s))ds, \quad t \in [0, T] = J, \tag{1.1}$$

*Corresponding author: *E-mail: Hamdy_17eg@yahoo.com*

$$x(0)+h(x(t_1),...,x(t_p)) = x_0 \tag{1.2}$$

where $0 < \alpha < 1$ and

$$Q(s) = \int_0^s k(s,\tau,x(\tau))d\tau.$$

Here A generates a strongly continuous semigroup in a Banach space X, $F(t)$ is a bounded operator for $t \in J$, and $f : J \times X \to X$, $k : \Delta \times X \to X$, $g : \Delta \times X \times X \to X$ and $h : X^p \to X$ are given functions. Here also $\Delta = \{(t,s) : 0 \leq s \leq t \leq T\}$, $F(t) \in B(X),\ t \in J$, $F(t) : Y \to Y$ and for $x(\cdot)$ continuous in Y, $AF(\cdot)x(\cdot) \in L^1(J,X)$. For $x \in X$, $F'(t)x$ is continuous in $t \in J$, where $B(X)$ is the space of all linear and bounded operators on X, and Y is the Banach space formed from $D(A)$, the domain of A, endowed with the graph norm .

This type of research has been considered in Balachandran and Park [10], when the equation (1.1)-(1.2) is given with conventional (classical) derivatives, also as several works; see for example [15, 16] and reference listed therein.

2 Preliminaries

In this section we give some basic definitions.

Definition 2.1 (see [21, 22]). The fractional integral of order $\alpha > 0$ with the lower limit zero for a function f can be defined as

$$I^\alpha f(t) = \frac{1}{\Gamma(\alpha)}\int_0^t \frac{f(s)}{(t-s)^{1-\alpha}}ds, \ \ t > 0$$

provided the right-hand side is pointwise defined on $[0,\infty)$, where Γ is the gamma function.
Definition 2.2 (see [21, 22]). The Caputo derivative of order α with the lower limit zero for a function f can be written as

$${}^cD^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)}\int_0^t \frac{f^{(n)}(s)}{(t-s)^{\alpha+1-n}}ds = I^{n-\alpha}f^{(n)}(t), \ \ t>0, \ 0 \leq n-1 < \alpha < n.$$

Definition 2.3. A resolvent operator for problem (1.1)-(1.2) is a bounded operator valued function $R(t) \in B(X)$ for $t \in J$ having the following properties (see [13, 18]:
(a) $R(0) = I$(the identity operator on X),
(b) for all $x \in X,\ R(t)x$ is continuous for $t \in J$,
(c) $R(t) \in B(Y),\ t \in J$. For $y \in Y$, $R(t)y \in C([0,T],X) \cap C([0,T],Y)$ and

$$\begin{aligned}\frac{d^\alpha}{dt^\alpha}R(t)y &= A[R(t)y + \int_0^t F(t-s)R(s)yds] \\ &= R(t)Ay + \int_0^t R(t-s)AF(s)yds, \ t \in J.\end{aligned}$$

Definition 2.4. According to $([12, 29])$, a continuous solution $x(t)$ of the integral equation

$$x(t) = S_\alpha(t)[x_0 - h(x(t_1),....,x(t_p))] + \int_0^t (t-s)^{\alpha-1}T_\alpha(t-s)[f(s,x(s)) + \int_0^s g(s,\tau,x(\tau),Q(\tau))d\tau]ds \tag{2.1}$$

is called mild solution of the problem (1.1)-(1.2) where

$$S_\alpha(t)x = \int_0^\infty \xi_\alpha(\theta)R(t^\alpha\theta)xd\theta, \qquad T_\alpha(t)x = \alpha\int_0^\infty \theta\xi_\alpha(\theta)R(t^\alpha\theta)xd\theta$$

with ξ_α being a probability density function defined on $(0, \infty)$, that is
$\xi_\alpha(\theta) \geq 0, \theta \in (0, \infty)$ and $\int_0^\infty \xi_\alpha(\theta)d\theta = 1.$
Remark. $\int_0^\infty \theta\xi_\alpha(\theta)d\theta = \frac{1}{\Gamma(1+\alpha)}.$
Let $Y = C(J, X)$ and define the sets

$$X_r = \{x \in X : \|x\| \leq r\},\ Y_r = \{y \in Y : \|y\| = \sup_{t\in J} \|y(t)\| \leq r\}, \quad (2.2)$$

where r, positive constant, is defined by $r = M_1\|x_0\| + HM_1 + \frac{M_1T^\alpha}{\Gamma(\alpha+1)}(M_2 + M_3T).$
Further we assume the following hypotheses:
(i) The resolvent operator $R(t)$ is compact and there exists a constant $M_1 > 0$
such that $\|R(t)\| \leq M_1.$
(ii) The nonlinear operators $f : J \times X \to X, g : \Delta \times X \times X \to X,$ and $k : \Delta \times X \to X$ are continuous and there exist constants $M_2 > 0,\ M_3 > 0$ such that
$\|f(t, x(t))\| \leq M_2$ for $t \in J,\ x \in X_r,\ \|g(t, s, x(s), y(s))\| \leq M_3$ for $(t, s) \in \Delta,\ x,\ y \in X_r.$
(iii) The operator $h : X^p \to X$ is continuous and there exists a constant $H > 0$ such that
$\|h(x(t_1), ..., (x(t_p)))\| \leq H$ for $x \in Y_r,$
$h(\lambda x(t_1) + (1-\lambda)y(t_1), ..., \lambda x(t_p) + (1-\lambda)y(t_p)) = \lambda h(x(t_1), ..., x(t_p)) + (1-\lambda)h(y(t_1), ..., y(t_p))$ for $x,\ y \in Y_r.$
(iv) The set $\{y(0) : y \in Y_r,\ y(0) = x_0 - h(y(t_1), ..., y(t_p))\}$ is precompact in X.
Lemma 2.1. (see [29]). The operators $S_\alpha(t)$ and $T_\alpha(t)$ have the following properties:
(I) For any fixed $x \in X, \parallel S_\alpha(t)x \parallel \leq M_1 \parallel x \parallel, \parallel T_\alpha(t)x \parallel \leq \frac{\alpha M_1}{\Gamma(\alpha+1)} \parallel x \parallel;$
(II) $\{S_\alpha(t),\ t \geq 0\}$ and $\{T_\alpha(t),\ t \geq 0\}$ are strongly continuous;
(III) For every $t > 0,\ S_\alpha(t)$ and $T_\alpha(t)$ are also compact operators if $R(t),\ t > 0$ is compact.

3 Existence of Mild Solutions

In this section, we can prove the existence of mild solution.

Theorem 3.1. Let hypotheses (i), (ii), (iii)and (iv) be satisfied. Then problem (1.1)-(1.2) has a mild solution on J.
Proof. We define the set Y_0 in Y by
$Y_0 = \{x \in Y : x(0) + h(x(t_1), ..., x(t_p)) = x_0,\ \|x(t)\| \leq r$ for $0 \leq t \leq T\}.$
Clearly, Y_0 is a bounded closed convex subset of Y. Define a mapping $\Psi : Y \to Y_0$ by

$$(\Psi x)(t) = S_\alpha(t)[x_0 - h(x(t_1), ..., x(t_p))] + \int_0^t (t-s)^{\alpha-1}T_\alpha(t-s)[f(s, x(s)) + \int_0^s g(s, \tau, x(\tau), Q(\tau))d\tau]ds.$$

Since

$$\begin{aligned}\|(\Psi x)(t)\| &\leq \|S_\alpha(t)x_0\| + \|S_\alpha(t)h(x(t_1), ..., x(t_p))\| \\ &+ \int_0^t (t-s)^{\alpha-1}\|T_\alpha(t-s)\|[\|f(s, x(s))\| + \int_0^s \|g(s, \tau, x(\tau), Q(\tau))\|d\tau]ds \\ &\leq M_1\|x_0\| + HM_1 + \frac{M_1T^\alpha}{\Gamma(\alpha+1)}(M_2 + M_3T) = r,\end{aligned}$$

then Ψ maps Y_0 into Y_0. Further, the continuity of Ψ from Y_0 into Y_0 follows from the fact that $f,\ g,\ ,k$ and h are continuous. Moreover Ψ maps Y_0 into a precompact subset of Y_0.
We prove that the set
$Y_0(t) = \{(\Psi x)(t) : x \in Y_0\}$ is precompact in X, for every fixed $t,\ 0 \leq t \leq T.$

For $t = 0$, the set $Y_0(0)$ is precompact in X.
Let $t > 0$ be fixed. Define, for $0 < \epsilon < t$,

$$(\Psi_\epsilon x)(t) = S_\alpha(t)[x_0 - h(x(t_1), ..., x(t_p))] + \int_0^{t-\epsilon} (t-s)^{\alpha-1} T_\alpha(t-s)[f(s, x(s)) + \int_0^s g(s, \tau, x(\tau), Q(\tau))d\tau]ds.$$

Since $R(t)$ is compact for every $t > 0$, the set $Y_\epsilon(t) = \{(\Psi_\epsilon x)(t) : x \in Y_0\}$ is precompact in X for every $\epsilon,\ 0 < \epsilon < t$.
Further, for $x \in Y_0$, we have

$$\begin{aligned}\|(\Psi x)(t) - (\Psi_\epsilon x)(t)\| &\le \| \int_{t-\epsilon}^t (t-s)^{\alpha-1} T_\alpha(t-s)[f(s, x(s)) + \int_0^s g(s, \tau, x(\tau), Q(\tau))d\tau]ds\| \\ &\le \frac{M_1 \epsilon^\alpha}{\Gamma(\alpha+1)}(M_2 + M_3 T),\end{aligned}$$

which implies that $Y_0(t)$ is totally bounded, that is $Y_0(t)$ is precompact in X.
We will show that $\Psi(Y_0) = S = \{(\Psi x) : x \in Y_0\}$ is an equicontinuous family of functions. For $0 \le t \le s$, we have

$$\begin{aligned}\|(\Psi x)(t) - (\Psi x)(s)\| &\le \|(S_\alpha(t) - S_\alpha(s))x_0\| + \|(S_\alpha(t) - S_\alpha(s))h(x(t_1), ..., x(t_p))\| \\ &+ \| \int_0^t (t-\tau)^{\alpha-1} T_\alpha(t-\tau) - (s-\tau)^{\alpha-1} T_\alpha(s-\tau)[f(\tau, x(\tau)) + \int_0^\tau g(\tau, \nu, x(\nu), Q(\nu))d\nu]d\tau\| \\ &+ \| \int_t^s (s-\tau)^{\alpha-1} T_\alpha(s-\tau)[f(\tau, x(\tau)) + \int_0^\tau g(\tau, \nu, x(\nu), Q(\nu))d\nu]d\tau\| \\ &\le \|S_\alpha(t) - S_\alpha(s)\|(\|x_0\| + H) + (M_2 + TM_3) \int_0^t \|(t-\tau)^{\alpha-1} T_\alpha(t-\tau) - (s-\tau)^{\alpha-1} T_\alpha(s-\tau)\| d\tau \\ &+ \frac{M_1 (s-t)^\alpha}{\Gamma(\alpha+1)}(M_2 + TM_3).\end{aligned}$$

The right hand side of the above inequality is independent of $x \in Y_0$ and tends to zero as $s \to t$. It is also clear that S is bounded in Y. Thus by Arzela-Ascoli's theorem, S is precompact. Hence by the Schauder fixed point theorem, Ψ has a fixed point in Y_0 and any fixed point Ψ is a mild solution of the nonlocal Cauchy problem (1.1)-(1.2).

4 Controllability Results

In this section, we will establish a set of sufficient conditions for controllability of semilinear fractional integrodifferential system with nonlocal condition in the following form

$$\frac{d^\alpha x(t)}{dt^\alpha} = A[x(t) + \int_0^t F(t-s)x(s)ds] + (Bu)(t) + f(t, x(t)) + \int_0^t g(t, s, x(s), Q(s))ds,\ t \in [0, T] = J, \quad (4.1)$$

$$x(0) + h(x(t_1), ..., x(t_p)) = x_0, \quad (4.2)$$

where the state $x(\cdot)$ takes values in the Banach space X and the control function $u(\cdot)$ is given in $L^2(J, U)$, a Banach space of admissible control functions with U as a Banach space. Here B is a bounded linear operator from U into X. For system (4.1), there exists a mild solution of the following form

$$\begin{aligned}x(t) = S_\alpha(t)[x_0 - h(x(t_1), ..., x(t_p))] &+ \int_0^t (t-s)^{\alpha-1} T_\alpha(t-s)[(Bu)(s) \\ &+ f(s, x(s)) + \int_0^s g(s, \tau, x(\tau), Q(\tau))d\tau]ds. \quad (4.3)\end{aligned}$$

Definition 4.1. System (4.1) is said to be controllable with nonlocal condition (4.2) on the interval J if, for every $x_0,\ x_T \in X$, there exists a control function $u \in L^2(J,U)$ such that the mild solution $x(.)$ of (4.1) satisfies

$$x(0) + h(x(t_1), ..., x(t_p)) = x_0,\ x(T) = x_1.$$

To establish the result, we need the following additional hypothesis
(v) The linear operator W from $L^2(J,U)$ into X, defined by

$$Wu = \int_0^T (T-s)^{\alpha-1} T_\alpha(T-s)Bu(s)ds,\ \ u \in L^2(J,U) \qquad (4.4)$$

induces an inverse operator W^{-1} defined on $L^2(J,U)/kerW$, and there exists a constant $M_4 > 0$ such that $\|BW^{-1}\| \le M_4$.
Theorem 4.1. If the hypotheses (i)-(v) are satisfied, then the system (4.1)-(4.2) is controllable on J.
Proof. Using the hypothesis (v), for an arbitrary function $x(\cdot)$, define the control

$$u(t) = W^{-1}\{x_T - S_\alpha(T)[x_0 - h(x(t_1), ..., x(t_p))]$$
$$- \int_0^T (T-s)^{\alpha-1} T_\alpha(T-s)[f(s,x(s)) + \int_0^s g(s,\tau,x(\tau),Q(\tau))d\tau]ds\}(t). \qquad (4.5)$$

Now we will show that, when using this control, the operator, defined by

$$(\Phi x)(t) = S_\alpha(t)[x_0 - h(x(t_1), ..., x(t_p))] + \int_0^t (t-s)^{\alpha-1} T_\alpha(t-s)[(Bu)(s)$$
$$+ f(s,x(s)) + \int_0^s g(s,\tau,x(\tau),Q(\tau))d\tau]ds, \qquad (4.6)$$

has a fixed point. This fixed point is then a solution of (4.1). Clearly $(\Phi x)(T) = x_T$, which means that the control u steers the semilinear fractional integrodifferential system from the initial state x_0 to final state x_T in time T provided we can obtain a fixed point of the nonlinear operator Φ.

Let $Y_0 = \{x \in Y : x(0) + h(x(t_1), ..., x(t_p)) = x_0,\ \|x(t)\| \le r'$, for $t \in J\}$ $\qquad (4.7)$
where r' is the positive constant. Then Y_0 is clearly a bounded, closed and convex subset of Y.
Define a mapping $\Phi : Y \to Y_0$ by

$$(\Phi x)(t) = S_\alpha(t)[x_0 - h(x(t_1), ..., x(t_p))] + \int_0^t (t-\eta)^{\alpha-1} T_\alpha(t-\eta)BW^{-1}\{x_T - S_\alpha(T)[x_0 - h(x(t_1), ..., x(t_p))]$$
$$- \int_0^T (T-s)^{\alpha-1} T_\alpha(T-s)[f(s,x(s)) + \int_0^s g(s,\tau,x(\tau),Q(\tau))d\tau]ds\}(\eta)d\eta$$
$$+ \int_0^t (t-s)^{\alpha-1} T_\alpha(t-s)[f(s,x(s)) + \int_0^s g(s,\tau,x(\tau),Q(\tau))d\tau]ds.$$

Consider

$$\|(\Phi x)(t)\| \le \|S_\alpha(t)\|[\|x_0\| + H] + \int_0^t (t-\eta)^{\alpha-1} \|T_\alpha(t-\eta)\| \|BW^{-1}\| \{\|x_T\| + \|S_\alpha(T)\|[\|x_0\| + H]$$

$$+\int_0^T (T-s)^{\alpha-1}\|T_\alpha(T-s)\|[\|f(s,x(s))\|+\|\int_0^s g(s,\tau,x(\tau),Q(\tau))d\tau\|]ds\}(\eta)d\eta$$
$$+\int_0^t (t-s)^{\alpha-1}\|T_\alpha(t-s)\|[\|f(s,x(s))\|+\|\int_0^s g(s,\tau,x(\tau),Q(\tau))d\tau\|]ds$$
$$\leq M_1(\|x_0\|+H)+\frac{M_1M_4T^\alpha}{\Gamma(\alpha+1)}[\|x_T\|+M_1(\|x_0\|+H)+\frac{M_1T^\alpha}{\Gamma(\alpha+1)}(M_2+M_3T)]+\frac{M_1T^\alpha}{\Gamma(\alpha+1)}(M_2+M_3T)=r'.$$

Since f and g are continuous and $\|(\Phi x)(t)\| \leq r'$, it follows that Φ is continuous and maps Y_0 into itself. Moreover, Φ maps Y_0 into a precompact subset of Y_0. To prove this, we first show that every fixed $t \in J$, the set $Y_0(t) = \{(\Phi x)(t) : x \in Y_0\}$ is precompact in X. This is clear for $t = 0$ since $Y_0(0)$ is precompact by assumption (iv).
Let $t > 0$ be fixed and for $0 < \epsilon < t$, define

$$(\Phi_\epsilon x)(t)=S_\alpha(t)[x_0-h(x(t_1),....,x(t_p))]+\int_0^{t-\epsilon}(t-\eta)^{\alpha-1}T_\alpha(t-\eta)BW^{-1}\{x_T-S_\alpha(T)[x_0-h(x(t_1),...,x(t_p))]$$
$$-\int_0^T (T-s)^{\alpha-1}T_\alpha(T-s)[f(s,x(s))+\int_0^s g(s,\tau,x(\tau),Q(\tau))d\tau]ds\}(\eta)d\eta$$
$$+\int_0^t (t-s)^{\alpha-1}T_\alpha(t-s)[f(s,x(s))+\int_0^s g(s,\tau,x(\tau),Q(\tau))d\tau]ds.$$

Since $R(t)$ is compact for every $t > 0$, the set $Y_\epsilon(t) = \{(\Phi_\epsilon x)(t) : x \in Y_0\}$ is precompact in X for every ϵ, $0 < \epsilon < t$. Furthermore, for $x \in Y_0$, we have

$$\|(\Phi x)(t)-(\Phi_\epsilon x)(t)\| \leq \int_{t-\epsilon}^t (t-\eta)^{\alpha-1}\|T_\alpha(t-\eta)\|\|BW^{-1}\|\{\|x_T\|+\|S_\alpha(T)\|[\|x_0\|+H]$$
$$+\int_0^T (T-s)^{\alpha-1}\|T_\alpha(T-s)\|[\|f(s,x(s))\|+\|\int_0^s g(s,\tau,x(\tau),Q(\tau))d\tau\|]ds\}(\eta)d\eta$$
$$+\int_{t-\epsilon}^t (t-s)^{1-\alpha}\|T_\alpha(t-s)\|[\|f(s,x(s))\|+\|\int_0^s g(s,\tau,x(\tau),Q(\tau))d\tau\|]ds$$
$$\leq \frac{M_1M_4\epsilon^\alpha}{\Gamma(\alpha+1)}[\|x_T\|+M_1(\|x_0\|+H)+\frac{M_1T^\alpha}{\Gamma(\alpha+1)}(M_2+M_3T)]+\frac{M_1\epsilon^\alpha}{\Gamma(\alpha+1)}(M_2+M_3T)$$

which implies that $Y_0(t)$ is totally bounded, that is, precompact in X.
We want to show that $\Phi(Y_0) = \{\Phi x : x \in Y_0\}$ is an equicontinuous family of functions. For that, let $t_2 > t_1 > 0$.

$$\|(\Phi x)(t_1)-(\Phi x)(t_2)\| \leq \|S_\alpha(t_1)-S_\alpha(t_2)\|[\|x_0\|+H]+\|\int_0^{t_1}(T_\alpha(t_1-\eta)(t_1-\eta)^{\alpha-1}-T_\alpha(t_2-\eta)(t_2-\eta)^{\alpha-1})BW^{-1}$$
$$\times\{x_T-S_\alpha(T)[x_0-h(x(t_1),...,x(t_p))]$$
$$-\int_0^T T_\alpha(T-s)(T-s)^{\alpha-1}[f(s,x(s))+\int_0^s g(s,\tau,x(\tau),Q(\tau))d\tau]ds\}(\eta)d\eta$$
$$-\int_{t_1}^{t_2} T_\alpha(t_2-\eta)(t_2-\eta)^{\alpha-1}BW^{-1}\{x_T-S_\alpha(T)[x_0-h(x(t_1),...,x(t_p))]$$

$$- \int_0^T T_\alpha(T-s)(T-s)^{\alpha-1}[f(s,x(s)) + \int_0^s g(s,\tau,x(\tau),Q(\tau))d\tau]ds\}(\eta)d\eta\|$$

$$+\| \int_0^{t_1} (T_\alpha(t_1-s)(t_1-s)^{\alpha-1} - T_\alpha(t_2-s)(t_2-s)^{\alpha-1})[f(s,x(s)) + \int_0^s g(s,\tau,x(\tau),Q(\tau))d\tau]ds$$

$$- \int_{t_1}^{t_2} T_\alpha(t_2-s)(t_2-s)^{\alpha-1}[f(s,x(s)) + \int_0^s g(s,\tau,x(\tau),Q(\tau))d\tau]ds\|$$

$$\leq \|S_\alpha(t_1) - S_\alpha(t_2)\|(\|x_0\| + H) + \int_0^{t_1} \|T_\alpha(t_1-\eta)(t_1-\eta)^{\alpha-1} - T_\alpha(t_2-\eta)(t_2-\eta)^{\alpha-1}\|$$

$$\times M_4[\|x_T\| + M_1(\|x_0\| + H) + \frac{M_1 T^\alpha}{\Gamma(\alpha+1)}(M_2 + M_3 T)]d\eta$$

$$+ \frac{M_1(t_2-t_1)^\alpha}{\Gamma(\alpha+1)}\|M_4[\|x_T\| + M_1(\|x_0\| + H) + \frac{M_1 T^\alpha}{\Gamma(\alpha+1)}(M_2 + M_3 T)]$$

$$+ \int_0^{t_1} \|T_\alpha(t_1-s)(t_1-s)^{\alpha-1} - T_\alpha(t_2-s)(t_2-s)^{\alpha-1}\|(M_2 + M_3 T)ds + \frac{M_1(t_2-t_1)^\alpha}{\Gamma(\alpha+1)}(M_2 + M_3 T). \quad (4.8)$$

The compactness of $R(t),\ t > 0$, implies that $R(T)$ is continuous in the uniform operator topology for $t > 0$. Thus, the right hand side of (4.8), which is independent of $x \in Y_0$, tends to zero as $t_2 \to t_1$. So $\Phi(Y_0)$ is equicontinuous family of functions. Also $\Phi(Y_0)$ is bounded in Y, and so by Arzela- Ascoli theorem, $\Phi(Y_0)$ is precompact. Hence, from the Schauder fixed-point theorem, Φ has a fixed point in Y_0. Any fixed point of Φ is a mild solution of (4.1) on J satisfying $(\Phi x)(t) = x(t) \in X$. Thus, system (4.1) is controllable on J.

5 Example

Consider a control system governed by the following fractional partial differential equation with nonlocal condition

$$\partial_t^\alpha z(t,x) = \partial_x^2[z(t,x) + \int_0^t b(t-s)z(s,x)ds] + Bu(t) + P(t,z(t,x)) + \int_0^t q(t,s,z(s,x), \int_0^s e(s,\tau,z(\tau,x))d\tau)ds,$$

$$z(0,t) = z(1,t) = 0,\ x \in I = (0,1),\ t \in J \quad (5.1)$$

$$z(x,0) + \sum_{i=0}^{p} c_i z(x,t_i) = z_0(x),\ x \in I$$

where ∂_t^α is the Caputo fractional partial derivative of order $0 < \alpha < 1$, $c_i,\ (i = 1,...,p)$ is given positive constant, b is continuous and bounded.
Here $B : U \to X$ is a linear operator such that there exists an inverse operator W^{-1} on $L^2(J;U)/kerW$ is defined by

$$Wu = \alpha \int_0^T \int_0^\infty \theta(T-s)^{\alpha-1}\xi_\alpha(\theta)R((T-s)^\alpha\theta)Bu(s)d\theta ds. \quad (5.2)$$

The resolvent operator $R(t)$ is compact (see [19]) and $P : J \times X \to X$, $e : \Delta \times X \to X$ and $q : \Delta \times X \times X \to X$ are all continuous and uniformly bounded.

To write the above system into the abstract form of (4.1), let $X = U = L^2(J, R)$ and $Aw = w_{xx}$ with domain $D(A) = \{w \in X : w_{xx} \in X, w(0) = w(1) = 0\}$.
Let $f(t, w)(x) = P(t, w(x)), \ \ (t, w) \in J \times X$,
$k(t, s, w)(x) = e(t, s, w(x))$,
$g(t, s, w, \sigma)(x) = q(t, s, w(x), \sigma(x)), \ x \in I$.
Therefore, with the above choices, the system (5.1) can be written to the abstract form $(4.1) - (4.2)$ and all conditions of theorem 4.1 are satisfied. Thus by theorem 4.1, fractional control system is controllable on J.

6 Conclusion

In this paper, we have presented, by using resolvent operators and Schauder fixed point theorem, the existence of mild solutions of fractional integrodifferential equations with nonlocal conditions in Banach spaces. Sufficient conditions for controllability of fractional integrodifferential systems are established. we provided example to illustrate our results.

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Competing Interests

The author declares that no competing interests exist.

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