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Stochastic Modelling of Life Insurance Reserving Process: Assessing Ruin Probability and Adjustment Factors

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Author's contribution

The sole author designed, analyzed, interpreted and prepared the manuscript.

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Abstract

The paper introduces a comprehensive stochastic model for the reserving process and the corresponding probability of ruin for a life insurance policy or, equivalently, for a portfolio of life policies. Within this framework, a discounted surplus process is established using a general probability space equipped with the natural filtration of past events and a suitable probability measure. Subsequently, it is demonstrated that the surplus process behaves as a submartingale and explores its impact on the probability of ruin, along with the inherent trade-off between the initial expense level and the adjustment factor applied to the net reserves of the life policy. Finally, a thorough numerical analysis is conducted focusing on a whole life insurance policy. In this specific case, a comprehensive range of values for the adjustment factor necessary to uphold the desired probability of ruin is ascertained, considering the corresponding values of the initial expense level.

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1 Introduction

The fundamental issue with a standard life insurance policy lies in the uneven distribution of expenses over time, refer to Segal [1]. Specifically, initial expenses are often excessive while renewal expenses are comparatively low. Consequently, companies tend to incur significant initial expenses, which diminish their initial capital, with the intention of gradually amortizing these expenses over time to restore their initial capital level and, consequently, achieve an appropriate level of financial stability corresponding to an appropriate low level of probability of ruin, as dictated by Solvency II framework.

Insurers typically address this issue by implementing a system of adjusted (lower) reserves for a brief period, usually a few years following the policy's inception, refer to Olivieri and Pitacco [2]. It is crucial that such adjustments adhere to and are approved within the relevant regulatory frameworks, such as Solvency II, refer to Burkhart et al. [3] and the new accounting standard IFRS17. Generally, the technical reserves governed by these frameworks are grounded in the concept of the best estimate plus the risk margin (or risk adjustment, as per IFRS17 terminology). The best estimate represents the "probability-weighted average of the future cash flows, considering the time value of money." This time value of money is reflected by the yield curve (required both in Solvency II and IFRS17 frameworks), which is derived from prevailing market conditions at each valuation time point. These reserves may be referred as net reserves. Under specific conditions, a modified yield curve, typically with higher rates, may be permitted, thereby allowing for the design of modified (adjusted) reserves. This adjusted curve may exclude certain elements that an insurance company typically experiences, particularly in the initial years of the policy.

Another method of modifying reserves entails the application of an adjustment factor, effectively decreasing the net reserves. This results in a series of adjusted reserves, which can then be used to derive the corresponding modified yield curve, and vice versa. Therefore, an adjustment factor or a sequence of adjustment factors is analogous to an adjusted yield curve, which represents a sequence of interest rates. Consequently, an adjustment factor that is accepted by the regulator leads to an accepted modified yield curve and vice versa. In our analysis, we adopt the approach of employing a single adjustment factor uniformly applied over time to all net reserve values. Throughout the remainder of the paper, we use the terms "modified reserve" and "adjusted reserve" interchangeably.

Expanding upon the context mentioned above and leveraging the foundational insights of Bühlmann [4] and Gerber [5], we intricately craft a probabilistic framework tailored specifically to address the nuances of an individual life insurance policy. Our methodology is deeply rooted in the principles espoused by Christiansen & Niemeyer [6], which emphasize the establishment of the Solvency Capital Requirement (SCR) and pertinent surplus valuation predicated on an acknowledged probability of ruin set at 0.5%. Throughout our analysis, we conscientiously adhere to the established framework governing surplus dynamics within the insurance landscape. Having meticulously delineated the intricacies of the surplus process, stakeholders within the insurance realm are empowered to conduct a comprehensive evaluation of the probability of ruin. Moreover, our approach facilitates the seamless integration of the concept of expected shortfall, a fundamental metric underscored by leading scholars such as those cited in Sandström [7], [8], and [9], and thoroughly examined through numerical illustrations presented in our study.

The paper is structured as follows: Section 2 delineates the fundamental assumptions and intricacies of the model, as well as formulating the surplus process. In Section 3, we establish the primary theoretical outcome for the surplus process as a submartingale and identify the relationship and boundaries for the probability of ruin when adapting the sequence of net or modified reserves. Section 4 entails a comprehensive numerical illustration, elucidating the trade-off between the initial expense level and the adjustment factor applied to the net reserves. Finally, Section 5 encapsulates the discussion and concludes the paper.

2 The model – Assumptions and General Framework

We consider a fixed probability space $(\Omega, \mathcal{F}, \mathbb{Q})$, (where Ω is the general sample space, $\mathcal{F}$ is the sigma algebra of the events and $\mathbb{Q}$ is the relevant probability measure) as the main framework to describe an individual life insurance policy. Additionally, we define the basic components as follows:

a) $(X_n)_{n=0}^{N}$, denotes the sequence of real valued random variables that corresponds to the sequence of net cash flows (benefits minus premiums) at each time point n = 0, 1, 2, …, N for the specific life policy. Actually, X_n represents the result, outflow (+) or inflow (-) as described below, at time n,

$$X_n = [benefits] - [premiums] \tag{1}$$

$$E\left[\sum_{j=0}^{N} X_j v^j\right] = 0$$

without losing the precision or generality of our approach, we may assume that in the premium (or benefit) part there is a fixed and flat small percentage covering the standard renewal (or claims handling) expenses occurred at each time point (at the date of the claim assessment)

b) $(Z_n)_{n=0}^{N^Z}$ denotes the sequence of real valued random variables that corresponds to the sequence of exceptional high initial expenses attributed at each time point n = 0, 1, 2, …, N^Z for the certain individual policy "adapted" to $(\mathcal{F}_t)_{t\geq 0}$ and such that.

$$E[\sum_{k=0}^{N^Z} Z_k \cdot v^k] = L_0 < 0$$

$$E[Z_k] < 0, \text{ k = 0, 1, 2, …,} N^Z \quad \mathbb{Q} \text{ - \textit{almost everywhere}.} \tag{2}$$

where L_0 is the initial cost incurred at the issue date of the policy.

N^Z, is the chosen period where the insurance company plans to amortize the initial cost L_0. Obviously, $N^Z < N$ (3)

c) $(\mathcal{F}_n)_{n=0}^{N}$, is a sequence of sub-sigma algebras of $\mathcal{F}$ and denotes all the available information up to and including time n with respect to the individual policy used by the company for valuation purposes at time n. We may assume that $\mathcal{F}_n$ is the σ-algebra generated by the information revealed from the previous years,

i.e. $\mathcal{F}_n = \sigma(X_1, X_2, \ldots, X_n, Z_1, Z_2, \ldots, Z_n)$ (4)

Hence, throughout this paper we additionally consider the following assumptions:

$$\mathcal{F}_n \subseteq \mathcal{F}_{n+m}, for\ all\ m, m \geq 0\ and\ \cup_{n=0}^{\infty} \mathcal{F}_n = \mathcal{F} \tag{5}$$

$$X_n\ and\ Z_n\ are\ \mathcal{F}_n - measurable\ for\ all\ n \geq 0 \tag{6}$$

d) $v^n, n = 0, 1, 2, \ldots$ denotes the discount factor where,

$$v^n = \frac{1}{(1+i)^n}, n = 0, 1, 2, \ldots \tag{7}$$

and i ($i \geq 0$) is the effective technical valuation rate of interest per unit time.

The curve (sequence) of $v^n, n = 0, 1, 2, \ldots$ reflects the time value of money as also required by the Solvency II legislation.

Under the framework of assumptions described above, we proceed with the definitions of net and modified reserves for this typical life policy.

Definition 2.1. We define the net prospective reserve of the policy at time t as below,

$$V_t = E[X_{t+1}v + X_{t+2}v^2 + \ldots \mid \mathcal{F}_t], \ \ t \geq 0 \tag{8}$$

and additionally, the net discounted to zero (at inception date) reserve as below,

$$W_t = v^t V_t \tag{9}$$

Definition 2.2. We define the modified reserve of the policy at time t as below,

$$V_t^Z = E[(X_{t+1} + Z_{t+1})v + (X_{t+2} + Z_{t+2})v^2 + \ldots \mid \mathcal{F}_t], \quad t \geq 0 \tag{10}$$

and additionally, the modified discounted to zero (at inception date) reserve as below

$$W_t^Z = v^t V_t^Z \tag{11}$$

So, we proceed in the next section with the formal design and solution of the model.

3 The Model

3.1 Basic theoretical results

As a first step in our analysis, we derive recursion relationship between consecutive values of net and modified reserves while also confirming the relationship between the discounted net and discounted modified reserve using the following proposition.

Proposition 3.1. The following recursion relationship holds,

$$W_{t-1}^Z = E[(X_t + Z_t)v^t + W_t^Z | \mathcal{F}_{t-1}] \tag{12}$$

$$V_{t-1}^Z(1 + i) = E[(X_t + Z_t) + V_t | \mathcal{F}_{t-1}] \tag{13}$$

Proof: From definition (2.2) and relationships (10) and (11), we directly derive the relevant results above. □
Proposition 3.2. The sequence of expected values for discounted modified reserves is always dominated by the sequence of expected values for discounted net reserves.

$$EW_t^Z < EW_t, \quad t < N \quad \text{and} \quad EW_t^Z = EW_t, \quad t = N. \tag{14}$$

Proof: From definition (2.1) and (2.2) and subtracting relationships (8) and (10), we obtain

$$EW_t^Z - EW_t < E\sum_{j=1}^{N-t} Z_{t+j}v^{t+j} < 0 \ \Rightarrow \ EW_t^Z < EW_t \quad t < N$$

Obviously, $EW_t^Z - EW_t = 0 \Rightarrow EW_t^Z = EW_t$, t=N because of relationship (3). □

All relevant theoretical concepts used for our calculations may be found in any standard textbook for probability theory, see for example [10] or [11]. We proceed with the following two definitions of annual and accumulated loss processes both on a nominal and discounted basis [12].

Definition 3.1. – **Net annual & accumulated Loss** We define the discounted value of the marginal (annual) loss incurred at time t, on a net reserve basis as

$$L_t = Y_t + W_t - W_{t-1}, t \geq 1 \quad \& \quad L_0 = Y_0 + W_0, \ where \ Y_t = v^t X_t \tag{15}$$

and the discounted value of the total (accumulated) net loss incurred up and inclusive time t, on a net reserve basis as

$$M_t = \sum_{j=0}^{t} L_t \tag{16}$$

Definition 3.2. - Modified annual & accumulated Loss We define the discounted value of the marginal (annual) modified loss incurred at time t, on a modified basis, as

$$L_t^Z = Y_t + W_t^Z - W_{t-1}^Z, t \geq 1 \quad \& \quad L_0^Z = Y_0 + W_0^Z, \qquad where\ \ Y_t = v^t X_t \tag{17}$$

and the discounted value of the total (accumulated) modified loss incurred up and inclusive time t, on a modified basis, as

$$M_t^Z = \sum_{j=0}^{t} L_t^Z \tag{18}$$

Now, we proceed with the introduction of the following basic theorem.

Theorem 3.1. Given the N^Z duration for the amortization of the initial exceptional expenses of a life insurance policy under the typical framework of assumptions described above, it is proved that

a) $(M_t^Z)_{t\geq 0}$ is an $\mathcal{F}_t - submartingale$ (19)

b) $EM_t^Z < 0\ when\ \ t < N^Z$ and $EM_t^Z = 0\ when\ \ t = N^Z$ (20)

Proof

(a) To prove the $\mathcal{F}_t - submartingale$ property, it suffices to show that

$$E[M_t^Z|\mathcal{F}_{t-1}] \geq M_{t-1}^Z.$$

We start with the following equation.

$$E[M_t^Z|\mathcal{F}_{t-1}] = M_{t-1}^Z + E[L_t^Z|\mathcal{F}_{t-1}] \tag{21}$$

and using relationship (17)

$$E[L_t^Z|\mathcal{F}_{t-1}] = E[Y_t + W_t^Z - W_{t-1}^Z|\mathcal{F}_{t-1}] =$$

and the recursion relationship (12)

$$= E[Y_t + W_t^Z - (X_t + Z_t)v^t - W_t^Z|\mathcal{F}_{t-1}]$$

We derive that,

$$E[L_t^Z|\mathcal{F}_{t-1}] = -v^t E[Z_t|\mathcal{F}_{t-1}] \geq 0,$$

The last inequality holds because of the second item in relationships described in (2).

Hence, equation (21) becomes,

$$E[M_t^Z|\mathcal{F}_{t-1}] = M_{t-1}^Z + positive\ \ term$$

which results the initial requisite inequality,

$$E[M_t^Z|\mathcal{F}_{t-1}] \geq M_{t-1}^Z.$$

proving the $\mathcal{F}_t - submartingale$ property

(b) As regards the relationships described in (20), we write down the relationships (17) described in definition (3.2)

$$L_0^Z = W_0^Z + Y_0\ , L_1^Z = W_1^Z + Y_1 - W_0^Z,\ L_2^Z = W_2^Z + Y_2 - W_1^Z, \vdots\ L_t^Z = W_t^Z + Y_t - W_{t-1}^Z$$

Adding these equations, we obtain

$$M_t^Z = L_0^Z + L_1^Z + L_2^Z + \ldots + L_t^Z = (Y_0 + Y_1 + \ldots + Y_t) + W_t^Z.$$

and setting $\quad -W_t^* = Y_0 + Y_1 + \ldots + Y_t$
we get, $\quad M_t^Z = W_t^Z - W_t^*$

Taking expectation of both sides of the equation above, we obtain

$$EM_t^Z = EW_t^Z - EW_t^*$$

and since $EW_t^* = EW_t, t \geq 0$, we derive that

$$EM_t^Z = EW_t^Z - EW_t,$$

So, using proposition (3.2) we conclude that $EM_t^Z < 0, t < N\ and\ EM_t^Z = 0, t = N$. □

Before we proceed with the basic theorem for the probability of ruin, we present a short application of the discussion above, into a standard whole life insurance policy.

3.2 Application for a whole life insurance policy

We assume a whole life insurance policy with initial expenses are $L_0 < 0$.

So, $N = T_x$ (future lifetime of the policyholder aged x years old at the inception date).
Then,

$$X_t = \begin{cases} 1, & if\ S_{t-1} = 1 \wedge S_t = 0 \\ -P, & if\ \qquad S_t = 1 \end{cases} \quad \text{and} \quad Z_t = \begin{cases} 0, & if\ S_t = 0 \\ -z, & if\ S_t = 1 \end{cases}, \quad z > 0 \tag{22}$$

The net premium (P) is calculated from equation, $P = E\sum_{j=0}^{N} X_j v^j = 0$.

and additionally, as regards the expenses $\quad E\sum_{j=0}^{N} Z_j v^j = L_0$

or equivalently, assuming a flat expense loading z, i.e $z = Z_j,\ \ j = 0,1,2, \ldots$

$$-z \cdot \ddot{a}_x = L_0 \Rightarrow z = -\frac{L_0}{\ddot{a}_x} \tag{23}$$

where $\ddot{a}_x$ is the annuity value at the certain technical valuation rate i, for a life aged x. We then calculate (since, *it is easily verified that* $E[Z_{t+1}|S_t = 0] = 0$*)*,

$$E[Z_{t+1}|\mathcal{F}_t] = E[Z_{t+1}|S_t = 1] = 0 \cdot p[S_{t+1} = 0|S_t = 1] - z \cdot p[S_{t+1} = 1|S_t = 0] \Rightarrow$$
$$E[Z_{t+1}|\mathcal{F}_t] = -z \cdot p_{x+t} = \frac{L_0}{\ddot{a}_x} \cdot p_{x+t} \tag{24}$$

where p_{x+t-1} , the standard survival probability from the relevant mortality table. i.e probability of survival for a life aged $x+t-1$ till age $x+t$. We proceed with the following calculations,

$$V_t^Z = E[(X_{t+1} + Z_{t+1})v + (X_{t+2} + Z_{t+2})v^2 + \ldots |S_t = 1] \Rightarrow$$
$$= V_t\ +\ E[Z_{t+1}v + Z_{t+2}v^2 + \ldots . |S_t = 1]$$

And using equation (24), we derive that,

$$V_t^Z = V_t + \frac{L_0}{\ddot{a}_x}(\ddot{a}_{x+t} - 1) = V_t + \frac{L_0}{\ddot{a}_x}\ddot{a}_{x+t} - \frac{L_0}{\ddot{a}_x} = V_t + \frac{L_0}{\ddot{a}_x}\ddot{a}_{x+t} - z \quad (25)$$

Now, provided that at the end of each year the reserves are calculated and kept aside only for alive persons who certainly will pay the relevant premium and consequently the portion of the premium (calculated as z) amortizing the relevant initial expenses, we add this to the last equation (24) resulting,

$$V_t^Z = V_t + L_0 \frac{\ddot{a}_{x+t}}{\ddot{a}_x} = V_t + L_0(1 - V_t) \quad (26)$$

The calculation above is based on a standard result from life insurance mathematics (e.g. refer to Neil(1977)), where, $V_t = 1 - \frac{\ddot{a}_{x+t}}{\ddot{a}_x}$

All our calculations above have been performed assuming the sum assured equal to one money unit. If we generally assume a sum assured equal to K then equation (26) is revised accordingly (see equation (27). So, in practice we normally set the modified reserve slightly lower than net reserve, using an adjustment factor (φ) which normally exceeds the ratio of initial expenses, $\frac{L_0}{K}$ as below

$$V_t^Z = V_t + \varphi \cdot (1 - V_t) \quad (27)$$

Typically, in a traditional whole life product the initial expenses are determined around 2.0% ($\frac{L_0}{K} \approx -2.0\%$) of the sum assured (thus, maximum might be -2.5% or 3.0%), while φ is normally greater.

3.3 Advanced theoretical results for the probability of ruin

In the remainder of this section, we examine the probability of ruin under a system of modified reserves, by proving the following strong result.

Theorem 3.2. Given the N^Z duration for the amortization of the initial exceptional expenses of a life insurance policy under the typical framework of assumptions described above, it is proved that:

$$M_t^Z \leq M_t \quad , \quad \mathbb{Q}\text{ - } almost\ everywhere \quad (\mathbb{Q} - a.e) \text{ , t = 0, 1, 2, …} \quad (28)$$

Note: This theorem directly implies part (b) of theorem (3.1)

Proof: We start with the calculation of annual modified loss

$$L_t^Z = Y_t + W_t^Z - W_{t-1}^Z \Rightarrow$$

$$L_t^Z = Y_t + W_t - W_{t-1} + v^t\{E[Z_{t+1}v + Z_{t+2}v^2 + \ldots |\mathcal{F}_t] - E[Z_t + Z_{t+1}v + Z_{t+2}v^2 + \cdots |\mathcal{F}_{t-1}]\} \Rightarrow$$

$$L_t^Z = L_t + v^t\{E[Z_{t+1}v + Z_{t+2}v^2 + \ldots |\mathcal{F}_t] - E[Z_t + Z_{t+1}v + Z_{t+2}v^2 + \cdots |\mathcal{F}_{t-1}]\}, t = 0,1,2, \ldots$$

Thus, applying the equation above at each time point t and summing up, we obtain,

$$\begin{aligned} M_t^Z &= L_0^Z + L_1^Z + L_2^Z + \cdots + L_t^Z = L_0^Z + (L_1 + L_2 + \cdots + L_t) + \\ &+ v \cdot \{E[Z_2 v + Z_3 v^2 + \ldots |\mathcal{F}_1] - E[Z_1 + Z_2 v + Z_3 v^2 + \ldots |\mathcal{F}_0]\} + \\ &+ v^2 \cdot \{E[Z_3 v + Z_4 v^2 + \ldots |\mathcal{F}_2] - E[Z_2 + Z_3 v + Z_3 v^2 + \ldots |\mathcal{F}_1]\} + \\ &+ \ \ldots \ + \\ &+ v^t\{E[Z_{t+1}v + Z_{t+2}v^2 + \ldots |\mathcal{F}_t] - E[Z_t + Z_{t+1}v + Z_{t+2}v^2 + \ldots |\mathcal{F}_{t-1}]\} \end{aligned}$$

Consequently,

$$M_t^Z = M_t + L_0^Z + v^t E[Z_{t+1}v + Z_{t+2}v^2 + \ldots |\mathcal{F}_t] - E[Z_1 v + Z_2 v^2 + Z_3 v^3 + \ldots |\mathcal{F}_0]$$

Since, by definition

$$E[Z_1 v + Z_2 v^2 + Z_3 v^3 + \ldots |\mathcal{F}_0] = -L_0^Z \quad , \quad (L_0^Z = L_0 - Z_0).$$

we obtain,

$$M_t^Z = M_t + v^t \cdot E[Z_{t+1}v + Z_{t+2}v^2 + \ldots |\mathcal{F}_t] \text{ , t = 0, 1, 2, …} \quad (29)$$

But

$$E[Z_{t+1}v + Z_{t+2}v^2 + \ldots |\mathcal{F}_t] = E\{E[Z_{t+1}v|\mathcal{F}_t] + E[Z_{t+2}v^2|\mathcal{F}_{t+1}] + E[Z_{t+3}v^3|\mathcal{F}_{t+2}] + \ldots |\mathcal{F}_t]$$

and since by definition $E\left[Z_{t+j}\middle|\mathcal{F}_{t+(j-1)}\right] < 0,\ j = 1, 2, \ldots \mathbb{Q} - a.e$, it follows that
$E[Z_{t+1}v + Z_{t+2}v^2 + \ldots |\mathcal{F}_t] < 0, t = 0, 1, 2, \ldots \mathbb{Q} - a.e$ and finally proved that
$M_t^Z \leq M_t,\ t = 0, 1, 2, \ldots \mathbb{Q} - a.e$ □

Now, we can proceed with the basic theorem dealing with the probability of ruin.

Theorem 3.3. We assume an insurer covering a typical life insurance policy exhibiting exceptionally high initial expenses with a specific duration N. The insurer adopts a system of modified reserves as described above. We consider the two surplus processes $(S_t)_{t\geq 0}$ and $(S_t^Z)_{t\geq 0}$ potentially adopted by the insurer and the respective times T, T^*(less than N) and probabilities of ruin Ψ_N , Ψ_N^*, typically defined as follows,

$$S_t = u + M_t \quad and \quad S_t^Z = u^Z + M_t^Z,$$
$$T = min\{t: S_t \leq 0\} \quad \text{and} \quad T^* = min\{t: S_t^Z \leq 0\}$$

$$\Psi_N = \mathbb{Q}\{\omega: T \leq N\}\ and\ \Psi_N^* = \mathbb{Q}\{\omega: T^* \leq N\}$$

it is proved that given that $u^Z = u$

(a) $S_t^Z \leq S_t$, $\mathbb{Q}$ - *almost everywhere* $(\mathbb{Q} - a.e)$, t = 0, 1, 2, … (28)

(b) $T < T^*\ \ and\ \ \Psi_N < \Psi_N^*$ (29)

Proof: Obviously derived from last theorem (3.2)

4 Numerical Application – Whole Life Annuity

In this section, we provide a full numerical example that elucidates the theoretical results derived before and supports further our investigation by finding actual figures for the probability of ruin under different scenarios for initial expenses and the adjustment factor.

We assume a whole life individual policy with the following details:

Description		Value
Technical interest rate	:	1.0%
Mortality Table (unisex table approved by Bank of Greece)	:	EAE2012P
Age of the insured life at the inception date	:	50 years old
Sum assured (constant for the (whole period)	:	10,000€
Net Annual Premium (paid in advance for all the period)	:	*238.56* €
Initial expenses (% of sum assured)	:	2.0%
Gross Annual Premium (paid in advance for all the period)	:	243.62 €
Initial Reserve (ξ% of the sum assured)	:	10.0%
Adjustment Factor for modified reserves (φ%)	:	3.0%
We use a simulation framework of 10.000 trials and obtain the following results.		

Table 1. List of cases

1st case: No initial expenses – No adjustment for reserves		
Probability of ruin	:	34.4%
Expected Shortfall, (% sum assured)	:	2,757 € (27.6%)
2nd case: With initial expenses – No adjustment for reserves		
Probability of ruin	:	37.6%
Expected Shortfall, (% sum assured)	:	3,013 € (30.1%)
3rd case: With initial expenses – With adjustment for reserves		
Probability of ruin	:	34.2%
Expected Shortfall, (% sum assured)	:	2,624 € (26.2%)

Additionally, we performed different sensitivity scenarios for the initial expense factor (ξ%) and the adjustment factor (φ %), calculating the relevant probability of ruin. These results are summarized in the following table.

Table 2. Summarization of reselt

ξ% \\ φ%	0%	1%	2%	3%	4%	5%	6%
0%	34.4%	-	-	-	-	-	-
1%	36.1%	35.0%	34.1%	33.8%	-	-	-
2%	37.6%	-	35.1%	34.2%	33.9%	-	-
3%	38.8%		-	-	35.7%	34.3%	32.9%

5 Conclusions

The current paper presents a model featuring a system of modified reserves tailored for a life insurance policy, which commonly experiences an uneven distribution of expenses throughout the policy duration. This model delineates the surplus process and can effectively serve as an internal model for a life insurance company, as mandated by the standard legislative framework of Solvency II.

After establishing the foundational framework of assumptions and details, we derive several key theoretical results. These results shed light on the relationship between the surplus process assuming net reserves versus modified reserves, and the corresponding probabilities of ruin. Additionally, they elucidate the trade-off between the level of initial expenses and the magnitude of adjustment (lowering) factor for the technical reserves when calculating the probability of ruin.

The primary findings are as follows:

1. The probability of ruin of the surplus process under the assumption of a typical life insurance policy is exacerbated due to the disparity in expense distribution, particularly between initial expenses and renewal expenses.
2. The probability of ruin of the surplus process under the assumption of a typical life insurance policy may be improved if the decision maker of the insurer adopts a system of modified (lower) reserves.
3. The extent to which reserves are reduced to counterbalance the deterioration in ruin is directly correlated with the magnitude of initial expenses. Therefore, the decision-maker must either decrease reserves or augment the initial capital if reducing reserves is not permissible within the regulatory framework. Of course, a combined action (lowering reserves and increasing initial capital) may also be acceptable by the legislative basis.

In our numerical example, we have developed two insightful tables. That first table presents the pertinent probabilities of ruin and the expected shortfall across three potential scenarios, considering the level of initial expenses as a percentage (ξ%) of the relevant sum assured and the adjustment (lowering) factor (φ%) of the net reserves. The second table presents different sensitivity scenarios for ξ and φ factors. In order to preserve the initial level of probability of ruin, a certain approximate relation holds, as described below,

$$\varphi\% \approx 150\% \times \xi\%$$

Therefore, life insurance decision-makers can utilize the empirical rule above or similar simulation results to strategize their approach concerning reserve adjustment levels aimed at preserving the initial probability of ruin.

Competing Interests

Author has declared that no competing interests exist.

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