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# Infinite Linear Systems M/*G*/∞ and Multilinear Systems with M/*G*/*n*/0 Losses

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***Authors' contributions***

*This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.*

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## ABSTRACT

The method based on the description of the probabilities of states using a non-stationary Poisson flow allows using elementary reasoning to find not only a stationary, but also a non-stationary distribution of the number of requirements in the system.
To find a stationary distribution of the number of requirements in queuing systems (QS), the method of introducing additional variables leading to a piecewise linear Markov process is used.
The fact of invariance is shown: the stationary probabilities of pi states in queuing systems (QS) M/G/n/0 depend only on the average service time of the requirement and do not depend on the type of distribution G(x).

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## 1. INTRODUCTION

In an infinitely linear queuing system (QS) of the M/*G*/∞ type, every requirement that enters the system immediately begins to be serviced [1]. The intensity of the incoming flow is denoted by λ. Let the service time of each requirement be distributed according to an arbitrary law *G*(*x*).

There are various ways to study the M/*G*/∞ system. Let us consider a method based on the description of the probabilities of states using a non-stationary Poisson flow. The method allows using elementary reasoning to find not only stationary, but also non-stationary distribution of the number of requirements in the system.

For simplicity, we assume that at the initial moment 0 the system is free.

Consider on the interval (0, *t*) the flow of requirements that have not been serviced by the time *t*, the current time is denoted by *u*. This is a flow with no aftereffect, because due to the Poisson nature of the initial flow and the independence of the service times of the requirements from each other and the incoming flow, it follows that the numbers of unserved requirements received at the time t on disjoint segments of the time interval (0, *t*) are independent random variables.

Further, the flow is ordinary, since the probability of receipt at the interval Δ of more than one requirement that has not been serviced by the time *t* does not exceed the probability of receipt at this interval of more than one requirement and, therefore, has the order 0 (Δ).

However, the flow is not stationary, since the distribution of the number of requests received at the same length of time intervals, will depend on where this interval is located: the further the interval is from the moment *t*, the fewer requirements received on it will remain in the system by the moment *t*.

This means that the flow of unserved requirements by the time *t* will be non-stationary Poisson with time-dependent and intensity λ(*u*). The intensity λ(*u*) is easy to calculate: to do this, simply multiply the intensity of the initial flow λ by the probability 1 – *G*(*t* – *u*) that the requirement will not be serviced during *t* – *u*, i.e.

$$\lambda(u) = \lambda(1 - G(t - u)).$$

Finally, we obtain: the number of requirements in the system M/*G*/∞ at time t is the number of requirements received in the interval (0, *t*) of a non-stationary Poisson intensity flow

$$\lambda(u) = \lambda(1 - G(t - u)).$$

It follows from this that the number of requirements in the system at time t is distributed according to Poisson's law with the parameter

$$\Lambda(t) = \int_0^t \lambda\left(1 - G(t-u)\right)du = \lambda\int_0^t \left(1 - G(u)\right)du \quad , (1)$$

and the probabilities $p_i(t)$ that there are *i* requirements in the system at the moment *t* have the form

$$p_i(t) = \frac{\left(\Lambda(t)\right)^i}{i!} e^{-\Lambda(t)} \quad (2)$$

.

In order to obtain a stationary distribution of the number of requirements in the system M/*G*/∞, it is enough to aim *t* to infinity in formulas (1) and (2). If the average service time

$$M = \int_0^\infty x dG(x) = \int_0^\infty \left(1 - G(x)\right)dx$$

of course, that $\Lambda(t) \to \lambda M$ at $t \to \infty$ This means that the stationary distribution of the number of requirements in the system in this case will be Poisson with the parameter λ*M*.

If *M* = ∞, then over time the number of requirements in the system will tend to infinity on average.

A multilinear CFR with M/G/n/0 losses is sometimes called an Erlang system. It is assumed that the flow entering the system has intensity λ, and the service time of each requirement is distributed according to an arbitrary law *G*(*x*). The Erlang system also belongs to the number of non-Markov systems, its functioning cannot be described by a Markov process with continuous time and a discrete set of states.

To find a stationary distribution of the number of requirements in the system, we apply the method of introducing additional variables leading to a piecewise linear Markov process. Such variables, along with the number ν(*t*) of requirements in the system at time *t*, can serve, for example, the residual service times of the requirements that are on the devices at time *t*.

Let 's put

$$P_i\left(x_1,...,x_i;t\right)=P\left\{\nu\left(t\right)=i,\ \xi_1\left(t\right)<x_1,...,\xi_i\left(t\right)<x_i\right\}.$$

It can be shown that, given the finiteness of the average service time of the requirement $M=\int_0^\infty xdG\left(x\right)$ there is a stationary distribution

$$P_i\left(x_1,...,x_i\right)=\lim_{t\to\infty}P_i\left(x_1,...,x_i;\ \ t\right),$$

has a density

$$p_i\left(x_1,...,x_i\right)=\partial^i P_i\left(x_1,...,x_i\right)\Big/\left(\partial x_1\cdots\partial x_i\right).$$

For simplicity, assume that the distribution function *G*(x) of the service time also has a density $g\left(x\right)=G'\left(x\right)$.

For this case, consider all possible transitions from the state [1,2]

$$\left\{\nu\left(t\right)=i,\ \xi_1\left(t\right)=x_1,...,\xi_i\left(t\right)=x_i\right\}$$

for a "small" time Δ. During this time, the remaining service times of all requirements will decrease by Δ, with a probability of λΔ + o(Δ), a request may arrive and with a probability of

$$P\left(x_1,...,x_{k-1},\Delta,x_{k+1},..,x_i\right)=p\left(x_1,...,x_{k-1},0,x_{k+1},..,x_i\right)\Delta+o\left(\Delta\right)$$

the service of the *k* requirement, which had a residual service time of less may end.

In turn, the incoming request to the system has a service time enclosed in the interval (*y*, *y* + δ) with probability g(*y*)δ + 0(δ). Since the stationary mode is considered, the distributions of the piecewise linear process $\left(\nu\left(t\right),\xi_1\left(t\right),...,\xi_{\nu(t)}\left(t\right)\right)$ at moments *t* and *t* + Δ coincide and have a distribution density $p_i\left(x_1,...,x_i\right)$.

We believe that the received request can with equal probability occupy any free channel by number. Considering the above, we obtain for $p_i\left(x_1,...,x_i\right)$ when *i* > 0 and *i* < *n* the equation [3,4,5]

$$p_i(x_1,...,x_i) = p_i(x_1+\Delta,...,x_i+\Delta)(1-\lambda\Delta)+$$
$$+\lambda\Delta\frac{1}{i}\sum_{k=1}^{i} p_{i-1}(x_1,...,x_{k-1},x_{k+1},...,x_i)\,g(x_k)+$$
$$+\sum_{k=1}^{i+1} p_{i-1}(x_1,...,x_{k-1},0,x_k,...,x_i)\Delta+o(\Delta).$$

The equation for the stationary probability $p_0$ of the absence of requirements in the system has the form [6,7,8]:

$$p_0 = p_0(1-\lambda\Delta)+p_1(0)\Delta+o(\Delta).$$

Finally, if there are n requirements in the system, then the *n* requirements received in the system is lost. This remark allows us to write out the equation:

$$p_n(x_1,...,x_n) = p_n(x_1+\Delta,...,x_n+\Delta)+$$
$$+\frac{\lambda\Delta}{n}\sum_{k=1}^{n} p_{n-1}(x_1,...,x_{k-1},x_{k+1},...,x_n)\,g(x_k)+o(\Delta).$$

Transferring $p_o, p_i(x_1+\Delta,..,x_i+\Delta)$ and $p_n(x_1+\Delta,..,x_n+\Delta)$ to the left parts of the corresponding equations, dividing by Δ and aiming Δ to zero, we get [4,9,10]:

$$\lambda p_o = p_1(0),$$
$$-\left(\frac{\partial}{\partial x_1}+...+\frac{\partial}{\partial x_i}\right)p_i(x_1,...,x_i) = -\lambda p_i(x_1,...,x_i)+$$
$$+\sum_{k=1}^{i+1} p_{i+1}(x_1,...,x_{k-1},0,x_k,...,x_i)+$$
$$+\frac{\lambda}{n}\sum_{k=1}^{i} p_{i-1}(x_1,...,x_{k-1},x_{k+1},...,x_i)\,g(x_k) \qquad (0<i<n),$$
$$-\left(\frac{\partial}{\partial x_1}+...+\frac{\partial}{\partial x_n}\right)p_n(x_1,...,x_n) = \frac{\lambda}{n}\sum_{k=1}^{n} p_{n-1}(x_1,...,x_{k-1},x_{k+1},...,x_n)\,g(x_k). \qquad (3)$$

By direct verification, it is not difficult to verify that the functions

$$p_i(x_1,...,x_i) = \frac{\lambda^i}{i!}(1-G(x_1))\cdots(1-G(x_i))$$

are the solution of system (3), i.e. the densities of the stationary distribution of the process $(\nu(t), \xi_1(t),...,\xi_{\nu(t)}(t))$.

The stationary probability $p_i$ of the presence of exactly $i$ requirements in the system is given by the formula [11,12,13]

$$p_i = P_i(\infty,...,\infty) = \int_0^\infty ... \int_0^\infty p_i(x_1,...,x_i)\,dx_1 \cdots dx_i = \frac{(\lambda M)^i}{i!} p_o$$

where $p_o$ is determined from the normalization condition $p_o + ... + p_n = 1$, i.e.

$$p_o = \left(1 + \frac{\lambda M}{1!} + \frac{(\lambda M)^2}{2!} + ... + \frac{(\lambda M)^n}{n!}\right)^{-1}.$$

A particularly important role in practical calculations is played by the stationary probability of $P$, which coincides with the stationary probability $p_n$ that all channels are occupied.

It is easy to see that the stationary probabilities $p_i$ for the M/*G*/*n*/0 system are determined by the same expressions as the stationary probabilities $p_i$ for the M/M/*n*/0 system with the exponential distribution parameter of the service time $\mu = 1/M$.

Thus, it can be argued about the fact of invariance: the stationary probabilities of the states of $p_i$ in the CFR M/*G*/*n*/0 depend only on the average service time of the requirement and do not depend on the type of distribution *G*(*x*) [14,15,16,17,18].

## 2. CONCLUSION

1. A particularly important role in practical calculations is played by the stationary probability of $P$, which coincides with the stationary probability $p_n$ that all channels are occupied.

2. Stationary probabilities $p_i$ for the M/*G*/*n*/0 system are determined by the same expressions as the stationary probabilities $p_i$ for the M/M/*n*/0 system with the exponential distribution parameter of the service time $\mu = 1/M$.

3. It can be argued about the fact of invariance: the stationary probabilities of the states of $p_i$ in the CFR M/*G*/*n*/0 depend only on the average service time of the requirement and do not depend on the type of distribution *G*(*x*).

## COMPETING INTERESTS

Authors have declared that no competing interests exist.

## REFERENCES


1. Gnedenko BV, Kovalenko IN. Vvedenie v teoriyu queuing / M.: Nauka; 1966.
2. L Mills. Statistical methods/ Trans. see – M.: Gosstatisdat; 1965.
3. Wentzel ES. Introduction to operations research: Textbook, Moscow: Sovetskoe Radio Publ. 1964:390.
4. Ventzel ES, Ovcharov LA. Applied problems of probability theory. 1983;416.
5. I Likesha, Lyaga Y. Basic tables of mathematical statistics/ Trans. from Czech- M.: Finance and Statistics; 1985.
6. Popov AM, Sotnikov VN. Die ekonomiko-mathematischen methoden und die modelle. Die monografie. Monograph / Germany; 2015.
7. Kokren GU. Metody selektivnogo issledovaniya [Methods of selective research]; 1990.
8. G Kramer. Mathematical methods of statistics/ Trans. see – M.: Mir; 1975.
9. Raifa G, Shleifer R. Prikladnaya teoriya statisticheskikh resheniy [Applied theory of statistical solutions]; 1992.
10. G. Cochran.U. Methods of selective research / Trans. see – M.: Statistics; 1976.
11. Wentzel ES, Ovcharov LA. Probability theory and its engineering applications. 2nd ed. – Moscow: Higher School. 2000;480.
12. Wentzel ES. Operations research: tasks, principles, methodology. 2nd ed. – Moscow: Nauka, 1988;208.

13. Gini K. Average values/ Trans. from Italian. – M.: Statistics; 1970.
14. Popov AM, Sotnikov VN, Valiev RM. Ekonomiko-matematicheskie metody i modeli v mashinostroenii [Economic and mathematical methods and models in mechanical engineering]. Monograph. / M.: OOO "Nauka-Inform; 2017.
15. Popov AM, Sotnikov VN. Probability theory and mathematical statistics. Textbook and workshop. 2nd ed., ispr. and add. / Moscow: Yurayt; 2017.
16. Draper N, Smith G. Applied regression analysis/ Translated from English – M.: Finance and Statistics; 1987.
17. Finni D. Vvedenie v teoriyu planirovaniya eksperimentov [Introduction to the theory of experiment planning]; 1992.
18. Dubrovsky SA. Applied multidimensional statistical analysis.– M.: Finance and Statistics; 1982.



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