British Journal of Mathematics & Computer Science
3(4): 750-771, 2013

SCIENCEDOMAIN *international*
www.sciencedomain.org

A Method for Two-Dimensional Cutting Stock Problem with Triangular Shape Items

W. N. P. Rodrigo[1*], W. B. Daundasekera[1] and A. A. I. Perera[1]

[1]*Department of Mathematics, Faculty of Science, University of Peradeniya, Sri Lanka.*

Research Article

Received: 06 June 2013
Accepted: 09 August 2013
Published: 21 August 2013

Abstract

Increasing cost of raw material and need to avoid industrial wastage, solving cutting stock problems became of great interest in the area of Operations Research. An optimum cutting stock problem can be defined as cutting a main sheet into smaller pieces while minimizing total wastage of the raw material or maximizing overall profit obtained by cutting smaller pieces from the main sheet. Objective of this study is to generate feasible cutting patterns for two-dimensional triangular shape cutting items. An algorithm is presented based on modified *Branch and Bound* Algorithm. A computer program is developed using Matlab software package to generate feasible cutting patterns. As a case study, four different sizes of triangular shape items with their demands are selected to cut from a main sheet with known dimensions. Applying proposed algorithm, demand is satisfied and total wastage is minimized. Proposed algorithm can be exploited to generate cutting patterns for rectangular and triangular cutting items at the same time and more suitable for medium size two dimensional cutting stock problem.

Keywords: Cutting stock problem, *Branch and Bound Algorithm*, pattern generation, Matlab software package.

1 Introduction

Increasing cost of raw material and need to avoid industrial wastage, solving cutting stock problems became of great interest in the area of Operations Research. Therefore, Operations Research plays a major role in minimizing wastage of raw material or to maximizing usage of the raw material. An optimum cutting stock problem can be defined as cutting a main sheet into smaller pieces while minimizing total wastage of the raw material or maximizing overall profit obtained by cutting smaller pieces from the main sheet. This often results in smaller pieces which cannot be used for the production and considered as waste. Therefore, cutting items should be planned and orient to minimize waste of raw materials. There are different arrangements to cut required items from the existing raw material to maximize the used area. Each arrangement is

**Corresponding author: nilukarodrigo@yahoo.co.uk;*

defined as a cutting pattern. These problems arise in industries such as garment, furniture, paper and sheet metal industries, glass etc.

Many researchers have worked on the cutting stock problem and used different approaches to solve the problem. Among them, Gilmore and Gomory [1] conducted some of the earliest research in this area and one-dimensional cutting stock problem is solved using *Linear Programming Technique*. In this study, unlimited numbers of raw materials with different lengths are assumed available in stock, and a mathematical model has been developed to minimize the total cutting cost of the stock length of the feasible cutting patterns and *Column Generation Algorithm* has been developed to generate feasible cutting patterns. Then, Gilmore has claimed that feasible cutting patterns are increased with the required cutting items. *Linear Programming Technique* is not applicable to solved mathematical model with too many variables. After two year Gilmore and Gomory [2] has made an approach for one-dimensional cutting stock problem as an extended of early study[1] and cutting stock problem has been described as a NP-hard problem. A new and rapid algorithm for the knapsack problem and changes in the mathematical formulation[1] has been evolved and Gilmore has explained the procedure of the *Knapsack Method* using a test problem. In addition Saad [3] has modified *Branch and Bound Algorithm* to find feasible cutting patterns for one-dimensional cutting stock problem and mathematical model has been developed to minimize the total cut loss. In the case study, Saad has selected four different types of steel coils to cut from the standard steel coil with the 130 cm length and width of the main coil and widths of the required coils are equal. *Branch and Bound Algorithm* has been explained using the example.

Also, Sirirat and Peerayuth [4] has proposed a mathematical model with column generation technique by a *Branch and Bound procedure* and *Heuristic based on the First Fit decreasing* method for one-dimensional cutting stock problem. Sirirat has made assumptions that different lengths of items to be cut from a stock, Each item has associated a certain length, and each cutting pattern for a stock are not limited in the number of knives, but the sum of the length of items do not exceed a length of stock.

Other than to one-dimensional cutting stock problem, most of the researches have conducted two dimensional cutting stock problem. Among them, Beasley [5] discussed the unconstrained two-dimensional cutting stock problem with guillotine cuts (a cut from one edge of the rectangle to the opposite edge which is parallel to the two remaining edges.) and staged cuts (the cuts at the first stage are restricted to be guillotine cuts parallel to one axis, then the cuts at the second stage are restricted to be guillotine cuts parallel to the other axis and the cuts at the third stage are restricted to be guillotine cuts parallel to the original axis etc.). Beasley has presented both *Heuristic* and *Optimal Based Dynamic Programming* for staged cutting and guillotine cuttings. Also, Coromoto et al. [6] has used a *Parallel Algorithm* and *Sequential Algorithm* to solve the mathematical model which maximizes the total profit incurred by cutting n number of rectangular pieces from a large rectangular main sheet. Coromoto has made an observation that all cutting patterns can be obtained by means of horizontal and vertical builds of meta-rectangles and used *Viswanathan and Bagchi Algorithm* to produce best horizontal and vertical builds.

Further Hassan et al. [7] has made an approach to cut regular and irregular shaped pieces from a rectangular main sheet of known dimensions and has transformed irregular shapes pieces into rectangles before the allocation is made. In the case study, Hassan considered male trouser with size 42 cm. Five types of pieces (two pieces of front side leg, two pieces of back side leg, belt, two pieces of side pockets and two pieces of back side pockets) need to be considered to tailor one trouser. For this task, imposed assumptions are the required pieces and main sheets should be in rectangular shape (if pieces are in irregular shapes it should be transformed to rectangular shape), the maximum length and width of each main sheet (fabric roll) are considered to be five meters and one meter respectively, all applied cuts are guillotine type and the cutting waste in each step cannot be more than the previous cutting step. Using *Simulated Annealing* process, single objective (minimizing the total cutting waste) two dimensional cutting stock problem has been solved.

There are different arrangements to cut required pieces from the existing raw material to maximize the used area and each arrangement is defined as a cutting pattern. Rodrigo et al. [8,9] presented an algorithm based on *Branch and Bound Algorithm* to generate feasible cutting patterns. As a case study, manufacturing floor tiles with 3000 mm × 1400 mm dimensions was selected to cut four different sizes of rectangular shape tiles and twenty five different cutting patterns are generated and only two patterns are selected to cut the main sheet according to the requirements. As an extended of above paper, Rodrigo et al has modified earliest *Branch and Bound Algorithm* to represent Cartesian coordinate points of each item in each pattern within the main sheet.

In this paper, cutting items with triangular shapes are selected to cut from a rectangular shape main sheet. An algorithm based on *Branch and Bound Algorithm* and a computer program using Matlab software package to solve the algorithm are developed to generate feasible cutting patterns.

2 Material and Methods

Any firm's main objective is to maximize the annual contribution margin accruing from its production and sales. By reducing wastages and maximizing sales, productivity can be improved. Wastage can occur in many ways and at any step of production line cutting stock problem can be described under the raw material wastage.

According to the selection, a mathematical model to minimize the raw material wastage is formulated as follows:

Following notations are introduced to describe the model:

m = Number of items,
n = Number of patterns,
d_i = Demand for the i^{th} item.
p_{ij} = Number of occurrences of the i^{th} item in the j^{th} pattern,
c_j = Cutting loss for each j^{th} pattern,
x_j = Number of main sheets being cut according to the j^{th} pattern,

2.1 Mathematical Model (Gilmore and Gomory [1])

$$\text{Minimize} \quad z = \sum_{j=1}^{n} c_j x_j \quad (\text{Total Cutting Loss})$$

$$\text{Subject to} \quad \sum_{j=1}^{n} p_{ij} x_j \geq d_i \quad \text{for all } i = 1, 2, ..., m \quad (\text{Demand Constraints})$$

$$x_j,\ p_{ij} \geq 0 \text{ and integer for all } i,\ j,$$

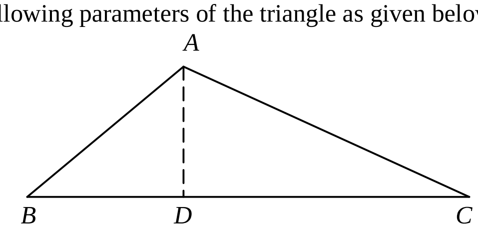
The number of occurrences of the i^{th} piece in the j^{th} pattern (p_{ij}) needs to be determined to find the optimum solution (minimum-waste arrangement) for the given mathematical model. Therefore, the develop algorithm is used to generate feasible cutting patterns.

Here, $\sum_{i=1}^{m} p_{ij} A_i \leq L \times W$ for all $j = 1, 2, ..., n$, where A_i is the area of the i^{th} item and L and W are length and width of the main sheet respectively.

In this study B&B algorithm is applied to generate cutting patterns (p_{ij}) and initial basic feasible solution can be found using Step 1 and Step 2. Then search tree method is applied. Here, lower bound of each item is zero and upper bound of each item is maximum number of pieces that can be cut in a main sheet for a particular item.

2.2 Modified *Branch and Bound Algorithm*

Here, we consider the following parameters of the triangle as given below:



$BC \geq AB$ and $BC \geq AC$,

$l_i = BC$ (length of the i^{th} item),

$w_i = AD$ (width of the i^{th} item),

$e_i = BD$

Step 1: Arrange required lengths, l_i, $i = 1, 2, \ldots, m$ in decreasing order, ie $l_1 > l_2 > \ldots > l_m$, where m = number of items.

Arrange required widths, w_i, $i = 1, 2, \ldots, m$ and lengths e_i, $i = 1, 2, \ldots, m$ according to the corresponding lengths l_i, $i = 1, 2, \ldots, m$.

Step 2: For $i = 1, 2, \ldots, m$ and $j = 1$ do Steps 3 to 5.

Step 3: set $a_{11} = \left\lfloor\!\left\lfloor L/l_1 \right\rfloor\!\right\rfloor$;

$$a_{ij} = \left\lfloor\!\left\lfloor \left(L - \sum_{z=1}^{i-1} a_{zj}\, l_z \right) \Big/ l_i \right\rfloor\!\right\rfloor \quad (1),$$

where L is the length of the main sheet.

Here, a_{ij} is the number of pieces of the i^{th} item in the j^{th} pattern along the length of the main sheet and $\lfloor\!\lfloor y \rfloor\!\rfloor$ is the greatest integer less than or equal to y.

Step 4:

If $a_{ij} > 0$, then set $b_{ij} = \left\lfloor\!\left\lfloor W/w_i \right\rfloor\!\right\rfloor$ (2)

else set $b_{ij} = 0$.

where W is the width of the main sheet.

Here, b_{ij} is the number of pieces of the i^{th} item in the j^{th} pattern along the width of the main sheet.

Step 5: Set $p_{ij} = (2a_{ij} - 1)b_{ij}$,

where p_{ij} is the number of pieces of the i^{th} item in the j^{th} pattern in the main sheet.

Step 6: Cutting Loss

(i) Cut loss along the length of the main sheet:

$$c_u = \left(L - \sum_{i=1}^{m} a_{ij}\, l_i \right) \times W$$

For $i = 1, 2, \ldots, m$

If $\left(L - \sum_{i=1}^{m} a_{ij}\, l_i \right) \geq w_i$ and $W \geq l_i$, then

(Considering 90^o rotation for the given cutting items.)

$$\text{set } A_{ij} = \left[\left[\left(L - \sum_{i=1}^{m} a_{ij}\, l_i \right) \Big/ w_i \right]\right];$$

$$B_{ij} = \begin{cases} \left[\left[W \big/ l_i \right]\right], & \text{if } A_{ij} > 0 \\ 0, & \text{otherwise.} \end{cases}$$

$$p_{ij} = p_{ij} + (2A_{ij} - 1)B_{ij}$$

else set $A_{ij} = 0;$
$B_{ij} = 0;$
$P_{ij} = P_{ij}.$

If $A_{ij} > 0$, then

$$\text{set } C_u = \left[\left(L - \sum_{i=1}^{m} a_{ij}\, l_i \right) - A_{ij}\, w_i \right] \times B_{ij}\, l_i ;$$

$$C_v = \left(L - \sum_{i=1}^{m} a_{ij}\, l_i \right) \times (W - B_{ij} l_i).$$

$$\text{else} \qquad C_u = \left(L - \sum_{i=1}^{m} a_{ij}\, l_i \right) \times W ,$$

where, A_{ij} and B_{ij} are the number of pieces of the i^{th} item in the j^{th} pattern along the length and width of the c_u rectangle respectively and C_u and C_v are the total cut loss area along the length and width of the c_u rectangle respectively.

(ii) Cut loss along the width of the main sheet:

$c_v = (a_{ij}\, l_i) \times k_{ij}$.
Here, $k_{ij} = W - (b_{ij}\, w_i);$
If $\left(b_{ij}\, w_i\right) = 0$, then
set $k_{ij} = 0$,

where k_{ij} is the remaining width of each item in each pattern

For $z \neq i$
If $(a_{ij}\, l_i) \geq l_z$ and $k_{ij} \geq w_z$, then

$$\text{set } A_{zj} = \left[\left[\left((a_{ij}\, l_i) \big/ l_z \right) \right]\right];$$

$$B_{zj} = \begin{cases} \left[\left[\left(k_{ij} / w_z \right)\right]\right], & \text{if } A_{zj} > 0 \\ 0, & \text{otherwise.} \end{cases}$$

$$p_{zj} = p_{zj} + (2A_{zj} - 1)B_{zj}$$

else set $A_{zj} = 0$;
$B_{zj} = 0$;
$p_{zj} = p_{zj}$.

If $A_{zj} > 0$, then
set $C_u = (a_{ij}\, l_i - A_{zj}\, l_z) \times B_{zj}\, w_z$;
$C_v = a_{ij}\, l_i \times (k_{ij} - B_{zj} l_z)$.

else $C_v = (a_{ij}\, l_i) \times k_{ij}$,

where, A_{zj} and B_{zj} are the number of pieces of the i^{th} item in the j^{th} pattern along the length and width of the c_v rectangle respectively and C_u and C_v are the total cut loss area along the length and width of the c_v rectangle respectively.

(iii) Cut loss within the triangular shape items in the main sheet:
If $a_{ij} \geq 0$, then
set $c_t = \frac{1}{2} e_i\, w_i + \frac{1}{2}(l_i - e_i)\, w_i$;
else set $c_t = 0$.
For $z \neq i$
If $e_i \geq l_z$ and $(e_i - (l_z - e_z))w_i / e_i \geq w_z$, then
set $E_{zj} = \left[\left[e_i / l_z \right]\right]$;

$$F_{zj} = \begin{cases} \left[\left[\left({w_i}/{w_z} \right)\right]\right], & \text{if } E_{zj} > 0 \\ 0, & \text{otherwise.} \end{cases}$$

$$p_{zj} = p_{zj} + (2E_{zj} - 1) F_{zj}\, b_{ij}.$$

else set $E_{zj} = 0;$
$F_{zj} = 0;$
$p_{zj} = p_{zj}.$

If $E_{zj} > 0,$ then

set $C_t = \left[\frac{1}{2} e_i\, w_i - \left(\frac{1}{2} E_{zj}\, l_z\, w_z\right)\right].$

else set $C_t = \frac{1}{2} e_i\, w_i\,.$

For $z \neq i$

If $(l_i - e_i) \geq l_z$ and $[(l_i - e_i) - e_z \;\;]w_i / (l_i - ei) \geq w_z$, then

set $E_{zj} = \left[\left[(l_i - e_i) / l_z \right]\right];$

$$F_{zj} = \begin{cases} \left[\left[\left({w_i}/{w_z} \right)\right]\right], & \text{if } E_{zj} > 0 \\ 0, & \text{otherwise.} \end{cases}$$

$$p_{zj} = p_{zj} + (2E_{zj} - 1) F_{zj}\, b_{ij}.$$

else set $E_{zj} = 0;$
$F_{zj} = 0;$
$p_{zj} = p_{zj}.$

If $E_{zj} > 0$, then

$$\text{set } C_t = \left[\frac{1}{2}(l_i - e_i)w_i - \left(\frac{1}{2}E_{zj}\, l_z\, w_z\right)\right].$$

$$\text{else set } C_t = \frac{1}{2}(l_i - e_i)w_i .$$

where, E_{zj} and F_{zj} are the number of pieces of the i^{th} item in the j^{th} pattern along the length and width of the c_t rectangle respectively and C_t is the total cut loss area of the triangular shapes .

Step 7: Set $r = m - 1$.
While $r > 0$, do Step 8.
Step 8: While $a_{rj} > 0$
set $j = j + 1$ and do Step 9.
Step 9: If $a_{rj} \geq b_{rj}$, then generate a new pattern according to the following conditions:
For $z = 1, 2, \ldots, r - 1$
set $a_{zj} = a_{z\,j-1}$;
$b_{zj} = b_{z\,j-1}$.
For $z = r$
set $a_{z\,j} = a_{z\,j-1} - 1$;
if $a_{z\,j} > 0$, then set $b_{z\,j} = \left[\left[W/w_z\right]\right]$;

else set $b_{z\,j} = 0$.

For $z = r + 1, \ldots, m$
calculate $a_{z\,j}$ and $b_{z\,j}$ using Equations (1) and (2).

For $i = 1, \ldots, m$
set $p_{ij} = a_{ij} b_{ij}$.
Go to Step 5.
else generate a new pattern according to the following conditions:
For $z = 1, 2, \ldots, r - 1$
set $a_{zj} = a_{z\,j-1}$;
$b_{zj} = b_{z\,j-1}$.

For $z = r$
set $a_{zj} = a_{z\,j-1}$;
$b_{zj} = b_{z\,j-1} - 1$.

For $z = r + 1, \ldots, m$

calculate a_{zj} and b_{zj} using Equations (1) and (2).

For $i = 1, \ldots, m$

set $p_{ij} = a_{ij} b_{ij}$.

Go to Step 5.

Step 10: Set $r = r - 1$.

Step 11: STOP.

2.3 Illustrative Example

Following example will illustrate how to generate feasible cutting patterns with triangular cutting items by minimizing total cutting waste:

A floor tile manufacturing plant uses rectangular shaped marble sheets of length 50 cm and width 15 cm as raw material to cut tiles according to the given specifications. The company has received an order for floor tiles according to the dimensions given in Table 1:

Table 1. Required lengths of each item and demand

Item number	**1**	**2**	**3**	**4**
***BC* (cm^2)**	40	25	8	4
***AD* (cm^2)**	13	12	5	2
***BD* (cm^2)**	30	24	2	2
Demand (d_i)	6	30	125	500

Below illustrates the method described in the research paper to cut the main sheet according to the dimensions so that the total raw material wastage is minimized.

3 Results

Modified *Branch and Bound Algorithm* is applied to the above example to generate feasible cutting patterns as given below:

Step 1: For $i = 1, 2, 3, 4$, lengths $l_i = 40, 25, 8, 4$, $w_i = 13, 12, 5, 2$ and $e_i = 30, 24, 2, 2$.

Length (L) and width (W) of the raw material are 50 cm and 15 cm respectively.

Dimensions of each item:

Step 2: For $i = 1, 2, \ldots, 4$ and $j = 1$ do Steps 3 to 5.

Step 3: Set $a_{11} = \left[\left[L \middle/ l_1 \right]\right] = 1;$

$$a_{21} = \left[\left[\left[L - \left(l_1 a_{11}\right)\right] \middle/ l_2 \right]\right] = 0;$$

$$a_{31} = \left[\left[\left[L - (l_1 a_{11}) - (l_2 a_{21})\right] \Big/ l_3 \right]\right] = 1;$$

$$a_{41} = \left[\left[\left[L - (l_1 a_{11}) - (l_2 a_{21}) - (l_3 a_{31})\right] \Big/ l_4 \right]\right] = 0;$$

Step 4: $a_{11} > 0$, then set $b_{11} = \left[\left[W \big/ w_{11} \right]\right] = 1;$

$$a_{21} = 0, \text{ then } b_{21} = 0;$$

$$a_{31} = 1, \text{ then } b_{31} = 3;$$

$$a_{41} = 0, \text{ then } b_{41} = 0;$$

Step 5:

$$\text{Set Pattern 1} = \begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \end{bmatrix}.$$

Step 6:

(i) Cutting loss along the length of the main sheet:

$$c_u = \left[L - (l_1 a_{11}) - (l_2 a_{21}) - (l_3 a_{31}) - (l_4 a_{41})\right] \times W$$

$$c_u = 2 \times 15 = 30 \text{ cm}^2.$$

For $i = 1, 2, 3$

set $A_{ij} = 0;$

$B_{ij} = 0.$ (Conditions are not satisfied given in Step 6 part (i))

For $i = 4$, dimensions of Item 4 are of length (l_4) 4 cm and width (w_4) 2 cm and conditions are satisfied given in Step 6 part (i).

$$\text{set } A_{41} = \left[\left[2 \big/ w_4 \right]\right] = 1;$$

$$B_{41} = \left[\left[15 \big/ l_4 \right]\right] = 3.$$

$A_{41} > 0$, then

$$\text{set } C_u = \left[2 - (A_{41} w_4)\right] \times B_{41} l_4 = 0 \times 15 = 0 \text{ cm}^2;$$

$$C_v = 2 \times \left[15 - (B_{41} l_4)\right] = 2 \times 3 = 6 \text{ cm}^2;$$

$$C_t = 1\big/2\, l_4 w_4 = 4 \text{cm}^2.$$

$$\text{Pattern 1} = \text{Pattern 1} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 3 \\ 5 \end{bmatrix}.$$

(ii) Cutting loss along the width of the main sheet:

$$c_v = a_{11} l_1 \times [15 - (b_{11} w_1)] = 40 \times 2 = 80 \text{ cm}^2$$

For $i = 2, 3$

set $A_{ij} = 0$;

$B_{ij} = 0$. (Conditions are not satisfied given in Step 6 part (ii))

For $i = 4$, dimensions of Item 4 are of length (l_4) 4 cm and width (w_4) 2 cm and conditions are satisfied given in Step 6 part (ii).

$$\text{set } A_{41} = \left[\left[{}^{40}\!/_{l_4} \right]\right] = 10;$$

$$B_{41} = \left[\left[{}^{2}\!/_{w_4} \right]\right] = 1.$$

$A_{41} > 0$, then

$$\text{set } C_u = [40 - (A_{41} l_4)] \times B_{41} w_4 = 0 \times 2 = 0 \text{ cm}^2;$$

$$C_v = 40 \times [2 - (B_{41} w_4)] = 40 \times 0 = 0 \text{ cm}^2;$$

$$C_t = {}^{1}\!/_{2}\, l_4 w_4 = 4 \text{cm}^2.$$

$$\text{Pattern 1} = \text{Pattern 1} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 19 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 3 \\ 24 \end{bmatrix}.$$

(iii) Cut loss within the triangular shape items in the main sheet:

For $i = 1$, $a_{11} > 0$ and $z = 2, 3$ then

$$\text{set } e_1 = 30 > l_2 \text{ , and } {}^{(e_1 - (l_2 - e_2))w_1}\!/_{e_1} = 12.57 > w_2;$$

$$(l_1 - e_1) = 10 > l_3 \text{ , and } {}^{(l_1 - e_1 - e_3)w_1}\!/_{(l_1 - e_1)} = 8 > w_3.$$

i.e. For $z = 2, 3$ dimensions of Item 2 and Item 3 are of lengths (l_2, l_3) 25 cm, 8cm and widths (w_2, w_3) 12 cm, 5 cm respectively and conditions are satisfied given in Step 6 part (iii).

$$\text{set} \quad E_{21} = \left[\left[{e_1}\big/{l_2}\right]\right] = 1; \text{ and } E_{31} = \left[\left[{(l_1 - e_1)}\big/{l_3}\right]\right] = 1\,;$$

$$F_{21} = \left[\left[{w_1}\big/{w_2}\right]\right] = 1; \text{ and } F_{31} = \left[\left[{w_1}\big/{w_3}\right]\right] = 1.$$

$E_{21} > 0$, then

$$\text{set } C_t = \left[{1}\big/{2}\, e_1 w_1 - \left({1}\big/{2}\, E_{21} l_2 w_2\right)\right] = 45 \text{ cm}^2.$$

$$\text{Pattern 1} = \text{Pattern 1} + b_{11}\begin{bmatrix}0\\1\\0\\0\end{bmatrix} = \begin{bmatrix}1\\1\\3\\24\end{bmatrix}.$$

$E_{31} > 0$, then

$$\text{set } C_t = \left[{1}\big/{2}\,(l_1 - e_1) w_1 - \left({1}\big/{2}\, E_{31} l_3 w_3\right)\right] = 45 \text{ cm}^2.$$

$$\text{Pattern 1} = \text{Pattern 1} + \begin{bmatrix}0\\0\\1\\0\end{bmatrix} = \begin{bmatrix}1\\1\\4\\24\end{bmatrix}.$$

For $i = 1$, $a_{11} > 0$ and $z = 2, 4$ then

$$\text{set } e_1 = 30 > l_2 \text{ , and } {(e_1 - (l_2 - e_2))w_1}\big/{e_1} = 12.57 > w_2\,;$$

$$(l_1 - e_1) = 10 > l_4 \text{ , and } {(l_1 - e_1 - e_3)w_1}\big/{(l_1 - e_1)} = 8 > w_4\,.$$

i.e. For $z = 2, 4$ dimensions of Item 2 and Item 4 are of lengths (l_2, l_4) 25 cm, 4cm and widths (w_2, w_3) 12 cm, 2cm respectively and conditions are satisfied given in Step 6 part (iii).

$$\text{set} \quad E_{21} = \left[\left[{e_1}\big/{l_2}\right]\right] = 1; \text{ and } E_{31} = \left[\left[{(l_1 - e_1)}\big/{l_4}\right]\right] = 2\,;$$

$$F_{21} = \left[\left[{w_1}\big/{w_2}\right]\right] = 1; \text{ and } F_{31} = \left[\left[{w_1}\big/{w_4}\right]\right] = 1+ 4.$$

$E_{21} > 0$, then

$$\text{set } C_t = \left[{1}\big/{2}\, e_1 w_1 - \left({1}\big/{2}\, E_{21} l_2 w_2\right)\right] = 45 \text{ cm}^2.$$

$$\text{Pattern 1} = \text{Pattern 1} + b_{11}\begin{bmatrix}0\\1\\0\\0\end{bmatrix} = \begin{bmatrix}1\\1\\3\\24\end{bmatrix}.$$

$E_{41} > 0$, then

$$\text{set } C_t = \left[\tfrac{1}{2}(l_1 - e_1)w_1 - \left(\tfrac{1}{2} E_{41}\, l_4\, w_4\right)\right] = 45 \text{ cm}^2.$$

$$\text{Pattern 1} = \text{Pattern 1} + \begin{bmatrix}0\\0\\1\\0\end{bmatrix} = \begin{bmatrix}1\\1\\4\\24\end{bmatrix}.$$

For $i = 2, 4$ $a_{21}, a_{41} = 0$, then

set $C_t = 0 \text{ cm}^2$.

For $i = 3$, $a_{31} > 0$, and $z = 4$, then

set $e_3 = 2 < l_4$, then

$E_{41} = 0$;

$F_{41} = 0$.

$$\text{set } C_t = \left[\tfrac{1}{2} e_3\, w_3\right] b_{31} = 15 \text{ cm}^2, \text{ and}$$

Pattern 1 = Pattern 1.

$$\text{set } (l_3 - e_3) = 6 > l_4, \text{ and } \frac{(l_3 - e_3 - e_4)w_3}{(l_3 - e_3)} = 3.33 > w_4.$$

i.e. For $z = 4$ dimensions of Item 4 are of length (l_4) 4 cm and width (w_4) 2 cm respectively and conditions are satisfied given in Step 6 part (iii).

$$\text{set } E_{41} = \left[\left[\frac{(l_3 - d_3)}{l_4}\right]\right] = 1;$$

$$F_{41} = \left[\left[\frac{w_3}{w_4}\right]\right] = 1.$$

$E_{41} > 0$, then

$$\text{set } C_t = \left[\tfrac{1}{2}(l_3 - e_3)w_3 - \left(\tfrac{1}{2} E_{41}\, l_4\, w_4\right)\right] b_{31} = 33 \text{ cm}^2.$$

$$\text{Pattern 1} = \text{Pattern 1} + b_{31}\begin{bmatrix}0\\0\\0\\1\end{bmatrix} = \begin{bmatrix}1\\1\\4\\27\end{bmatrix}.$$

$$\text{Pattern 1} = \begin{bmatrix}1\\1\\4\\27\end{bmatrix}$$ and total cutting loss = 152 cm^2.

Step 7: Set $r = 4 - 1 = 3 > 0$

Step 8: $a_{31} = 1 > 0$, then

set $j = j + 1$ and go to Step 9.

Step 9:

$a_{31} < b_{31}$, then generate a new pattern j according to the following conditions :

$$\text{set } a_{12} = a_{11}\ ;\ b_{12} = b_{11}.$$
$$a_{22} = a_{21};\ b_{22} = b_{21}.$$
$$a_{32} = a_{31};\ b_{32} = b_{31} - 1.$$
$$a_{42} = \left[\left[\left[L - (a_{12}l_1) - (a_{22}l_2) - (a_{32}l_3)\right] \big/ l_4\right]\right] = 0.$$

Step 5:

$$\text{Set Pattern 2} = \begin{bmatrix}1\\0\\2\\0\end{bmatrix}.$$

Step 6:

(i) Cutting loss along the length of the main sheet:

$$c_u = \left[L - (l_1 a_{12}) - (l_2 a_{22}) - (l_3 a_{32}) - (l_4 a_{42})\right] \times W$$
$$c_u = 2 \times 15 = 30 \text{ cm}^2.$$

For i = 1, 2, 3

set $A_{ij} = 0$;

$B_{ij} = 0$. (Conditions are not satisfied given in Step 6 part (i))

For i = 4, dimensions of Item 4 are of length (l_4) 4 cm and width (w_4) 2 cm and conditions are satisfied given in Step 6 part (i).

$$\text{set } A_{42} = \left[\left[{}^{2}\!/_{w_4} \right]\right] = 1;$$

$$B_{42} = \left[\left[{}^{15}\!/_{l_4} \right]\right] = 3.$$

$A_{42} > 0$, then

$$\text{set } C_u = [2 - (A_{42} w_4)] \times B_{42}\, l_4 = 0 \times 15 = 0 \text{ cm}^2;$$

$$C_v = 2 \times [15 - (B_{42} l_4)] = 2 \times 3 = 6 \text{ cm}^2;$$

$$C_t = {}^{1}\!/_{2}\, l_4 w_4 = 4 \text{cm}^2.$$

$$\text{Pattern 2} = \text{Pattern 2} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 5 \end{bmatrix}.$$

(ii) Cutting loss along the width of the main sheet:

$$c_v = (a_{12} l_1 \times [15 - (b_{12} w_1)]) + (a_{32} l_3 \times [15 - (b_{32} w_3)])$$

$$= 40 \times 2 + 8 \times 5 = 120 \text{ cm}^2$$

Consider $a_{12} l_1 \times [15 - (b_{12} w_1)]$ rectangle:

For $i = 2, 3$

set $A_{ij} = 0$;

$B_{ij} = 0$. (Conditions are not satisfied given in Step 6 part (ii))

For $i = 4$, dimensions of Item 4 are of length (l_4) 4 cm and width (w_4) 2 cm and conditions are satisfied given in Step 6 part (ii).

$$\text{set } A_{42} = \left[\left[{}^{40}\!/_{l_4} \right]\right] = 10;$$

$$B_{42} = \left[\left[{}^{2}\!/_{w_4} \right]\right] = 1.$$

$A_{42} > 0$, then

$$\text{set } C_u = [40 - (A_{42} l_4)] \times B_{42}\, w_4 = 0 \times 2 = 0 \text{ cm}^2;$$

$$C_v = 40 \times [2 - (B_{42} w_4)] = 40 \times 0 = 0 \text{ cm}^2;$$

$$C_t = {}^{1}\!/_{2}\, l_4 w_4 = 4 \text{cm}^2.$$

$$\text{Pattern 2} = \text{Pattern 2} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 19 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 24 \end{bmatrix}.$$

Consider $a_{32}\, l_3 \times \left(15 - \left(b_{32}\, w_3\right)\right)$:

For $i = 1,\ 2$

set $A_{ij} = 0$;

$B_{ij} = 0$. (Conditions are not satisfied given in Step 6 part (ii))

For i = 4, dimensions of Item 4 are of length (l_4) 4 cm and width (w_4) 2 cm and conditions are satisfied given in Step 6 part (ii).

set $A_{42} = \left[\left[8/l_4 \right]\right] = 2$;

$B_{42} = \left[\left[5/w_4 \right]\right] = 2$.

$A_{42} > 0$, then

set $C_u = \left[8 - \left(A_{42} l_4\right)\right] \times B_{42}\, w_4 = 0 \times 2 = 0 \text{ cm}^2$;

$C_v = 8 \times 5\left[2 - \left(B_{42} w_4\right)\right] = 8 \times 1 = 8 \text{ cm}^2$;

$$\text{Pattern 2} = \text{Pattern 2} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 8 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 32 \end{bmatrix}.$$

(iii) Cut loss within the triangular shape items in the main sheet:

For $i = 1$, $a_{11} > 0$ and $z = 2$, then

set $d_1 = 30 > l_2$, and $\left(d_1 - \left(l_2 - d_2\right)\right) w_1 / d_1 = 12.57 > w_2$;

$\left(l_1 - d_1\right) = 10 > l_3$, and $\left(l_1 - d_1 - d_3\right) w_1 / \left(l_1 - d_1\right) = 8 > w_3$.

i.e. For z = 2, 3 dimensions of Item 2 and Item 3 are of lengths (l_2, l_3) 25 cm, 8cm and widths (w_2, w_3) 12 cm, 5 cm respectively and conditions are satisfied given in Step 6 part (iii).

set $E_{22} = \left[\left[d_1/l_2 \right]\right] = 1$; and $E_{32} = \left[\left[\left(l_1 - d_1\right)/l_3 \right]\right] = 1$;

$F_{22} = \left[\left[w_1/w_2 \right]\right] = 1$; and $F_{32} = \left[\left[w_1/w_3 \right]\right] = 1$.

$E_{22} > 0$, then

$$\text{set } C_t = \left[\tfrac{1}{2} d_1 w_1 - \left(\tfrac{1}{2} E_{22} l_2 w_2\right)\right] = 45 \text{ cm}^2.$$

$$\text{Pattern 2} = \text{Pattern 2} + b_{12}\begin{bmatrix}0\\1\\0\\0\end{bmatrix} = \begin{bmatrix}1\\1\\2\\32\end{bmatrix}.$$

$E_{32} > 0$, then

$$\text{set } C_t = \left[\tfrac{1}{2}(l_1 - d_1) w_1 - \left(\tfrac{1}{2} E_{32} l_3 w_3\right)\right] = 45 \text{ cm}^2.$$

$$\text{Pattern 2} = \text{Pattern 2} + \begin{bmatrix}0\\0\\1\\0\end{bmatrix} = \begin{bmatrix}1\\1\\2\\32\end{bmatrix}.$$

For $i = 2, 4$ $a_{22}, a_{42} = 0$, then

set $C_t = 0$ cm^2.

For $i = 3$, $a_{32} > 0$, and $z = 4$, then

set $d_3 = 2 < l_4$, then

$E_{42} = 0$;

$F_{42} = 0$.

$$\text{set } C_t = \left[\tfrac{1}{2} d_3 w_3\right] b_{32} = 10 \text{ cm}^2, \text{ and}$$

$$\text{Pattern 2} = \text{Pattern 2}.$$

$$\text{set } (l_3 - d_3) = 6 > l_4, \text{ and } \frac{(l_3 - d_3 - d_4) w_3}{(l_3 - d_3)} = 3.33 > w_4.$$

i.e. For $z = 4$ dimensions of Item 4 are of length (l_4) 4 cm and width (w_4) 2 cm respectively and conditions are satisfied given in Step 6 part (iii).

$$\text{set } E_{42} = \left[\left[\frac{(l_3 - d_3)}{l_4}\right]\right] = 1;$$

$$F_{42} = \left[\left[\frac{w_3}{w_4}\right]\right] = 1.$$

$E_{42} > 0$, then

$$\text{set } C_t = \left[\tfrac{1}{2}(l_3 - d_3)w_3 - \left(\tfrac{1}{2}E_{42}\, l_4 w_4\right)\right] b_{32} = 22 \text{ cm}^2.$$

$$\text{Pattern 2} = \text{Pattern 2} + b_{32}\begin{bmatrix}0\\0\\0\\1\end{bmatrix} = \begin{bmatrix}1\\1\\3\\34\end{bmatrix}.$$

$$\text{Pattern 2} = \begin{bmatrix}1\\1\\3\\34\end{bmatrix} \text{ and total cutting loss} = 144 \text{ cm}^2.$$

The algorithm proceeds in the same manner to generate all the cutting patterns shown in Table 2 for the 50 cm × 15 cm standard dimension.

Below specified the search tree diagram that can be described the different patterns produced from the selected main sheet 50 cm×15 cm. The branches of level I show the multiples of the largest required length and corresponding width of the triangle (i.e. l_1 = 40 cm and w_1 = 13 cm) that can be produced from the main sheet, whereas the branches of level II show the multiples of the next largest required length and corresponding width of the triangle (i.e. l_1 = 25 cm and w_1 = 12cm), and so we branch along the tree as the required length decreases. Starting with the left branch of level I, 1 unit of item 1 is the maximum number of piece that can be produced from the main sheet. The two branches of level I show respectively, from left to right 1, and 0 pieces that can be cut. Nodes are representing cut loss at each level. Top to bottom of the tree gives one pattern and left to right designate all patterns. Here, last node stand for total cut loss.

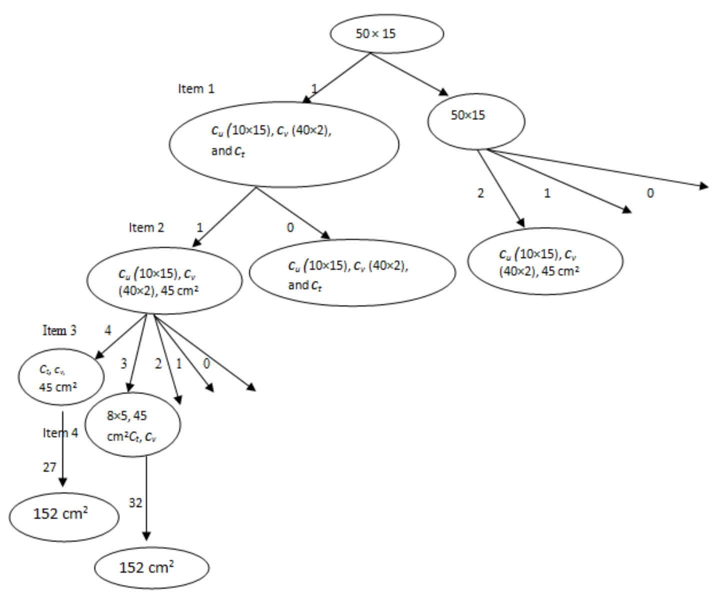


Table 2. Generated cutting patterns

Cutting	**Cutting patterns**													
Item	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	0	0	0	1	0	1	1	0	0	1	0	0	1
3	4	8	4	7	3	3	3	2	7	6	1	5	1	0
4	27	27	63	32	32	71	34	44	34	42	51	52	88	59
Cut loss	15	16	15	22	15	14	14	12	21	20	11	18	11	10
(cm^2)	2	8	8	2	2	6	4	4	4	2	6	2	8	4

Cutting	**Cutting patterns**													
Item	15	16	17	18	19	20	21	22	23	24	25	26	27	28
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	3	3	1	1	0	0	0	0	0	0	0	0	0
3	0	4	0	19	15	33	27	21	15	10	6	3	1	0
4	95	25	48	14	39	8	35	63	91	112	125	152	154	172
Cut loss (cm^2)	110	120	108	164	144	58	70	78	86	102	98	82	82	91

There are 28 feasible cutting patterns available to cut raw material with the dimensions 50 cm × 15 cm into required triangular shaped items. The mathematical model is developed to design generated cutting patterns so that waste (cut loss) will be minimized and the optimum solution to the model is given in Table 3:

Table 3. Optimum solution

Cutting	**Optimal Patterns**			**Demand**
Item	14	5	12	
1	1	0	0	6
2	1	3	0	30
3	0	0	33	125
4	59	48	8	500
Cut loss (cm^2)	104	108	58	
# of sheets from each pattern	6	8	4	

$$Z_{min} \text{ (Total cut loss)} = 1720 \text{ cm}^2.$$

4 Conclusion

In this study, a cutting stock problem is formulated as a mathematical model based on the concept of cutting patterns. As given in Table 2, twenty eight cutting patterns are generated and only three cutting patterns are selected as given in Table 3 to cut the main sheet according to the requirements. In this case study, the plant assumes that all the extra pieces from each item as wastage. Using the presented algorithm in this paper, both rectangular and triangular shape cutting items can be cut to minimize the cut loss. Also, total cut loss can be decreased if there are smaller cutting items with rectangular or triangular shaped.

Acknowledgment

My sincere thank goes out to my supervisors, Prof. W.B. Daundasekera and Prof. A. A. I. Perera, Department of Mathematics, Faculty of Science, University of Peradeniya, Sri Lanka for their guidance support and in valuable time given me to carry out this study a pleasurable success.

I may fail in my duty if I do not take this opportunity to express my deepest gratitude to my loving parents and husband for their encouragement and valuable support in this event come true. Finally, my thanks go any colleagues and others who helped me in many ways throughout this research.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] Gilmore PC, Gomory RE. A Linear Programming Approach to the Cutting Stock Problem. Operations Research. 1961;9(2):849-859.

[2] Gilmore PC, Gomory RE. A Linear Programming Approach to the Cutting Stock Problem – Part II. Operations Research. 1963;11(6):863–888.

[3] Saad MA. Suliman. Pattern generating procedure for the cutting stock problem", International Journal of Production Economics. 2001;74:293-301.

[4] Sirirat Wongprakornkul, Peerayuth Charnsethikul. Solving one-dimensional cutting stock problem with discrete demands and capacitated planning objective. Journal of Mathematics and Statistics. 2010;6(2):79-83.

[5] Beasley JE. Algorithms for Unconstrained Two-Dimensional Guillotine Cutting. Journal of Operations Research Science. 1985;36(4):297–306.

[6] Coromoto Leon, Gara Miranda, Casiano Rodriguez, Carlos Segura. 2D Cutting Stock Problem: *A* New Parallel Algorithm and Bounds. 25th November 2010.
Available: http://link.springer.com/chapter/10.1007/978-3-540-74466-5_85#.

[7] Hassan Javanshir, Shaghayegh Rezaei, Saeid Sheikhzadeh Najar, Gangi SS. Two Dimensional Cutting Stock Management in Fabric Industries and Optimizing the Large Object's Length. IJRRAS; August 2010.

[8] Rodrigo WNP, Daundasekera WB, Perera AAI. Pattern Generation for Two-Dimensional Cutting Stock Problem. International Journal of Mathematics Trends and Technology (IJMTT). 2012;3(2):54-62.

[9] Rodrigo WNP, Daundasekera WB, Perera AAI. Pattern Generation for Two-Dimensional Cutting Stock Problem with Location. Indian Journal of Computer Science and Engineering (IJCSE). 2012;3(2):354-368.

Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
www.sciencedomain.org/review-history.php?iid=240&id=6&aid=1894