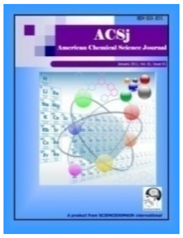

American Chemical Science Journal
4(3): 401-415, 2014

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Possessions of Chemical Reaction on MHD Heat and Mass Transfer Nanofluid Flow on a Continuously Moving Surface

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Authors' contributions

This work was carried out in collaboration among all three authors. Moreover the funding, computational suggestions, proof reading was also done by all three authors and approved the final manuscript.

Original Research Article

Received 19th July 2013
Accepted 10th January 2014
Published 3rd February 2014

ABSTRACT

A two-dimensional steady flow of an electrically conducting, viscous incompressible nanofluid past a continuously moving surface is considered in the presence of uniform transverse magnetic field with chemical reaction. A mathematical governing model has developed for the momentum, temperature and concentration boundary layer. Similarity transformations using to modify the boundary layer equations. Whereas this prominent transformations are used to transform the principal nonlinear boundary layer equations for momentum, thermal energy and concentration to a system of nonlinear ordinary coupled differential equations with fitting boundary conditions. The coupled differential equations are numerically simulated using the famous Nactsheim-Swigert shooting technique together with Runge-Kutta six order iteration schemes. Pertinent results with respect to embedded parameters are displayed graphically for the velocity, temperature and concentration profiles and were discussed quantitatively. Skin-friction, Heat transfer rate (Nusselt number) and mass transfer rate (Sherwood number) are illustrated for the various important parameters entering into the problem separately are discussed with the help of graphs. Finally for the accuracy of the present results a comparison with previously published research work are accomplished and proven an excellent agreement.

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Keywords: Nanofluid; magnetohydrodynamics (MHD); moving surface; chemical reaction.

1. INTRODUCTION

Magnetohydrodynamics (MHD) is concerning the mathematical and physical scaffold that introduces magnetic-dynamics in electrically conducting fluids (e.g. in plasmas and liquid metals). The applications of Magnetohydrodynamics (MHD) incompressible viscous flow in science and engineering involving heat and mass transfer under the influence of chemical reaction is of great importance to many areas of science and engineering. This frequently occurs in agriculture, engineering, plasma studies and petroleum industries.

The term nanofluid has been foremost introduced by Choi [1]. This novel fluid have been used potentially in numerous applications in heat and mass transfer, as well as microelectronics, fuel cells, pharmaceutical sections and hybrid-powered engines, engine cooling/vehicle thermal management etc.

In outlook of applications, Sakiadis [2,3] investigated the boundary-layer flow of a viscous fluid past a moving solid surface, while Tsou et al. [4] experimentally ascertained the results of Sakiadis by analyzing the effects of heat transfer on a continuously moving surface with constant velocity. Whereas, Soundalgekar and Murty [5] studied the heat transfer problem by assuming the plate due to variable temperature. Sakiadis work was again extended by Erickson et al. [6] to include suction or injection at the stretching sheet on a continuously moving surface with constant speed and investigated its effects on the heat and mass transfer in the boundary layer region. Wang [7] investigated the free convection flow on a vertical stretching surface also Gorla and Sidawi [8] also studied the problem of free convection on a vertical stretching surface with suction and blowing.

Pop et al. [9] obtained similarity solutions by considering viscosity as an inverse function of temperature and assuming constant velocity and temperature of the plate. Howell et al. [10] and Rao et al. [11] analyzed the momentum and heat transfer on a continuous moving surface in a power law fluid. Fang [12] studied similarity solutions of thermal boundary layer for a moving plate. Soundalgekar et al. [13] studied the flow of incompressible viscous fluid past a continuously moving semi-infinite plate by considering variable viscosity and variable temperature. Ibrahim et al. [14] studied the combined effect of wall suction and magnetic field on boundary layer flow with heat and mass transfer over an accelerating vertical plate. Makinde [15] studied the effect of temperature dependent viscosity on free convective flow past a vertical porous plate in the presence of a magnetic field, thermal radiation and a first order homogeneous chemical reaction. Recently Geetha and Moorthy [16] studied hydromagnetic flow and heat transfer on a continuously moving surface with Chemical Reaction.

Kang et al. [17] investigated the estimation of thermal conductivity of nanofluid using experimental effective Particle volume. The natural Convective Boundary layer flows of a nanofluid past a vertical plate have described by Kuznestov and Neild [18,19]. Bachok et al. [20] has showed the steady boundary layer flow of a nanofluid past a moving semi-infinite flat plate in a uniform free stream. Khan and Pop [21,22] formulated the problem of laminar boundary layer flow of a nanofluid past a stretching sheet. Also Khan et al. [23,24] investigated the effects of thermal radiation and magnetic field on boundary layer flow of a nanofluid past a stretching sheet using the model proposed by Nield and Kuznetsov [18], Kuznetsov and Nield [19] and Khan and Pop [21]. Recently there have been relatively few studies [25-32] that reports MHD boundary layer nanofluid flow as well.

In the present analysis, it is proposed to investigate the heat and mass transfer flow for an electrically conducting incompressible nanofluid past a continuously moving plate with variable surface temperature in the presence of a uniform transverse magnetic field and chemical reaction. The governing equations are transformed into nonlinear ordinary differential equations and solved numerically using Nactsheim-Swigert [33] shooting technique. The velocity, temperature and concentration distributions are discussed and presented graphically, and also the skin-friction coefficient, the surface heat and mass transfer rate at the sheet are investigated.

2. MATHEMATICAL FORMULATION

A two-dimensional steady flow of an electrically conducting, viscous, incompressible nanofluid past a continuously moving surface with uniform velocity *U* in the presence of uniform transverse magnetic field of strength B_o is well thought-out. A uniform transverse magnetic field of strength B_o is applied parallel to the *y*-axis and chemical reaction is taking place in the flow as well. It is assumed that the induced magnetic field, the external electric field and the electric field are negligible due to the polarization of charges. Let the *x*-axis be taken along the surface and *y*-axis normal to it as shown in Fig. 1.

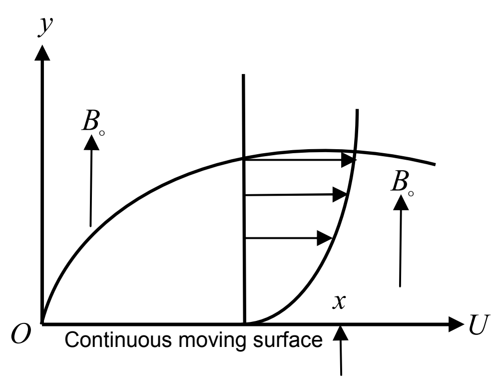

Fig. 1. Coordinate system for continuously moving surface

The properties of fluid considered to be isotropic and constant, except for the fluid viscosity which is assumed to be an inverse linear function [34] of temperature:

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}\left[1 + \gamma(T - T_\infty)\right] \Rightarrow \frac{1}{\mu} = a(T - T_\infty)$$

where $a = \frac{\gamma}{\mu_\infty}$ and $T_\gamma = T_\infty - \frac{1}{\gamma}$

Here μ be the coefficient of viscosity, μ_∞ is a reference viscosity, γ is a constant, T and T_∞ are the temperature of the fluid near and far away from the moving plate, a and T_γ are constants and their values depend on the reference state and the thermal property of the fluid. *i.e.,* in general $a > 0$ for liquids and $a < 0$ for gases. Under the above assumptions the governing equations are describing below:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\frac{\sigma_e B_0^2 u}{\rho}, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left(\alpha\frac{\partial T}{\partial y}\right)+\frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma_e B_0^2 u^2}{\rho C_p}+\tau\left\{D_B\left(\frac{\partial T}{\partial y}\cdot\frac{\partial C}{\partial y}\right)+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right\}, \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}+K_r\left(C-C_\infty\right). \tag{4}$$

And the boundary condition for the model is;

$$u=U=ax,\ v=0,\ T=T_w\left(x\right),\ C=C_w\left(x\right)\text{ at } y=0$$

$$u=0,\ T=T_\infty,\ C=C_\infty \text{ as } y\to\infty, \tag{5}$$

where, α is the thermal diffusivity, ρ is the density, σ_e is the electrical conductivity, k is the thermal conductivity, C_p is the specific heat at constant pressure, μ is the thermal viscosity, D_B is the brownian diffusion coefficient, D_T is the thermophoresis diffusion coefficient and K_r is the rate of chemical reaction.

The equations (2)-(4) can be transformed to the corresponding ordinary differential equations by introducing the following similarity transformations:

$$\begin{gathered}\eta=y\sqrt{\frac{a}{\upsilon_\infty}},\ \psi=x\sqrt{a\upsilon_\infty}\,f(\eta),\\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\varphi(\eta)=\frac{C-C_\infty}{C_w-C_\infty},\end{gathered} \tag{6}$$

where υ_∞ is a reference kinematic viscosity. It will be assumed that the temperature difference between the moving surface and the free stream varies as Ax^n.

$$\text{i.e. } T_w(x)-T_\infty=Ax^n. \tag{7}$$

Also the concentration difference between the moving surface and the free stream varies as Bx^n.

$$\text{i.e. } C_w(x)-C_\infty=Bx^n, \tag{8}$$

Where *A, B* is constants, *n* is exponent parameter and *x* is measured from the leading edge of the surface.

From the above transformations the non dimensional, nonlinear, coupled ordinary differential equations are obtained as:

$$f^{///} + ff^{//} - f^{/2} - \mathrm{Re}_m{}^2 f^{/} = 0, \tag{9}$$

$$\theta^{//} - nP_r f^{/}\theta + P_r f\theta^{/} + P_r E_c f^{//2} + \mathrm{Re}_m{}^2 P_r E_c f^{/2} + P_r N_b \theta^{/}\varphi^{/} + P_r N_t \theta^{/2} = 0, \tag{10}$$

$$\varphi^{//} + L_e f\varphi^{/} - nL_e f^{/}\varphi + \left(\frac{N_t}{N_b}\right)\theta^{//} - \gamma L_e \mathrm{Re}_x \varphi = 0, \tag{11}$$

Where the notation primes ($^{/}$) denote differentiation with respect to η and

$\mathrm{Re}_m = B_0\sqrt{\frac{\sigma_e}{\rho a}}$ (Magnetic parameter)

$E_c = \frac{U^2}{C_p\left(T_w - T_\infty\right)}$ (Eckert number)

$P_r = \frac{\upsilon_\infty}{\alpha}$ (Prandtl number)

$L_e = \frac{\upsilon_\infty}{D_B}$ (Lewis number)

$R_{e_x} = \frac{ax^2}{\nu_\infty}$ (Local Reynolds number)

$\gamma = \frac{\nu_\infty K_r}{U^2}$ (Chemical reaction parameter)

$N_b = \frac{(\rho c)_p D_B\left(\varphi_w - \varphi_\infty\right)}{\upsilon_\infty (\rho c)_f}$ (Brownian motion parameter)

$N_t = \frac{(\rho c)_p D_T\left(T_w - T_\infty\right)}{\upsilon_\infty T_\infty (\rho c)_f}$ (Thermophoresis parameter)

The corresponding boundary conditions are:

$$\left.\begin{array}{ll} f = 0, f^{/} = 1, \theta = 1, \varphi = 1, & at\ \eta = 0 \\ f^{/} = 0\ \theta = 0, \varphi = 0, & as\ \eta \to \infty \end{array}\right\}. \tag{12}$$

The physical quantities of the skin-friction coefficient, the reduced Nusselt number and reduced Sherwood number are calculated respectively by the following equations,

$$C_f\ \left(R_{e_x}\right)^{-\frac{1}{2}} = -f^{//}\left(0\right),$$
$$N_u\ \left(R_{e_x}\right)^{-\frac{1}{2}} = -\theta^{/}\left(0\right)\ \text{ and}$$
$$S_h\ \left(R_{e_x}\right)^{-\frac{1}{2}} = -\varphi^{/}\left(0\right). \tag{13}$$

Where $R_{e_x} = \dfrac{ax^2}{v_\infty}$ is the Local Reynolds number.

3. NUMERICAL“SHOOTING QUADRATURE” SOLUTIONS

The non-dimensional, nonlinear, coupled ordinary differential Eqs.(9) to (11) with boundary conditions (12) are solved numerically using the Nactsheim-Swigert shooting iteration technique [33]together with a sixth-order Runge-Kutta iteration scheme. In shooting methods, the missing (unspecified) initial condition at the initial point of the interval is assumed and the differential equation is integrated numerically asan initial value problem to the terminal point. The accuracy of the assumed missing initial condition is then verified via comparison with the computed value of the dependent variable at the terminal point with its given value there. If a difference exists, another value of the missing initial condition must be assumed and the process is repeated. This process is continued until the agreement between the calculated and the given condition at the terminal point is within the specified degree of accuracy.Extension of the iteration shell to considered system of differential eqns. is straightforward; there are three asymptotic boundary condition and hence three unknown surface conditions $f^{//}(0), \theta^{/}(0)$ and $\varphi^{/}(0)$.

4. NUMERICAL VALIDATIONS

The study investigates the problem of flow, heat and mass transfer for an electrically conducting incompressible nanofluid past a continuously moving plate with variable surface temperature in the presence of a uniform transverse magnetic field and the effects of chemical reaction. In order to investigate the physical representation of the problem, the numerical values of velocity $\left(f^{/}\right)$, temperature $\left(\theta\right)$ and concentration $\left(\varphi\right)$ against $\eta$with the boundary layer have been computed for different parameters the Magnetic parameter$\left(\mathrm{Re}_m\right)$, Prandtl number $\left(P_r\right)$, Eckert number $\left(E_c\right)$, Lewis number $\left(L_e\right)$, Chemical reaction parameter $\left(\gamma\right)$, Brownian motion parameter $\left(N_b\right)$, Thermophoresis parameter $\left(N_t\right)$,Local Reynolds number$\left(\mathrm{Re}_x\right)$ and exponent parameter $\left(n\right)$ respectively. Numerical results are reported in the Table 1 and Figs. 2-15.

For the accuracy of the present numerical results as the results for the reduced Nusselt number $-\theta^{/}\left(0\right)$ for different values of Prandtl number, the present results compared with Wang [7] and Gorla and Sidawi [8] by neglecting the Magnetic parameter, Eckert number , Chemical reaction parameter, Brownian motion parameter, Thermophoresis parameter, Local Reynolds number and exponent parameter respectively in Table1. andfrom thecomparison excellent agreement is observed.

Table 1. Comparison for the reduced Nusselt number $-\theta^{/}(0)$

P_r	**Wang [7]**	**Gorla and Sidawi [8]**	**Present Results** $Re_m = \gamma = n = 0$ $Re_x = N_t = N_b = 0$
0.07	0.0656	0.0656	0.0660
0.20	0.1691	0.1691	0.1693
0.70	0.4539	0.5349	0.4545
2.00	0.9114	0.9114	0.9117
7.00	1.8954	1.8904	1.8944
20.00	3.3539	3.3539	3.3542
70.00	6.4622	6.4622	6.4625

5. RESULTS AND DISCUSSIONS

In general, nanofluid-velocity is higher near the moving surface and decreases to zero far away from the plate surface satisfying the far field boundary conditions for all parameter values. Fig. 2 displays the dimensionless primary velocity distribution $f'(\eta)$ for different values of Re_m where $N_t = N_b = 0.5, P_r = 0.71, L_e = 5.0, E_c = 0.5, Re_x = 5.0, n = 1.0, \gamma = 0.5$ Then for above case It can be observed that velocity profiles are decreases as the Re_m increase. The magnetic field presents a damping effect on the velocity field by creating a drag force that opposes the fluid motion, causing the velocity to decrease.

Fig. 3 depicts the dimensionless velocity distribution $f'(\eta)$ for different values of n where $N_t = N_b = 0.5, P_r = 0.71, L_e = 5.0, E_c = 0.5, Re_x = 5.0, Re_m = 0.3, \gamma = 0.5$ Then for above case It can be observed that velocity profiles are increases as the n increase.

Figs. 4 and 5displays the fluid temperatures attains its maximum value at the moving plate surface and decrease exponentially to the free stream zero value away from the plate and also it satisfying the boundary conditions. Fig. 4 shows the dimensionless temperature distribution $\theta(\eta)$ for different values of Re_m where $N_t = N_b = 0.5, P_r = 0.71, L_e = 5.0, E_c = 0.5, Re_x = 5.0, n = 1.0, \gamma = 0.5$ Then for above case It can be observed that temperature profiles are increases as the Re_m increase. As a results the thermal boundary layer thickness increases with an increase in the intensity of magnetic field.

Fig. 5 predicts the dimensionless temperature distribution $\theta(\eta)$ for different values of n where $N_t = N_b = 0.5, P_r = 0.71, L_e = 5.0, E_c = 0.5, Re_x = 5.0, Re_m = 0.3, \gamma = 0.5$ Then for above case It can be observed that temperature profiles are decreases as the n increase.

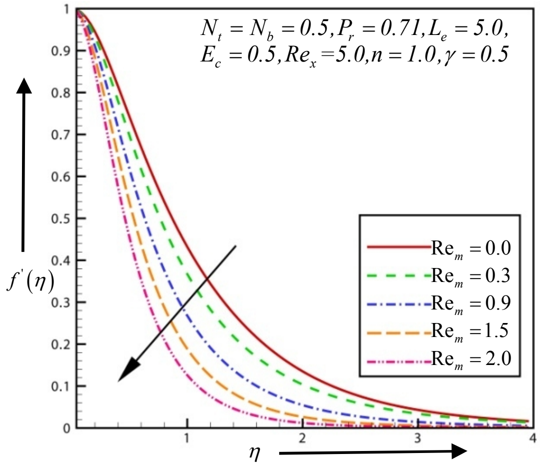

Fig. 2. Effects of magnetic parameter (Re_m) on velocity profiles

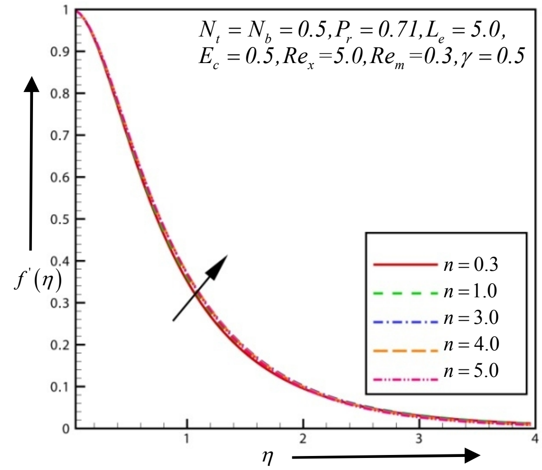

Fig. 3. Effects of exponent parameter (n) on velocity profiles

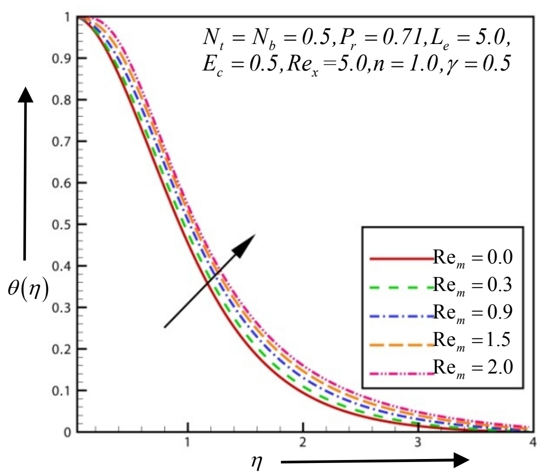

Fig. 4. Effects of magnetic parameter (Re_m) on temperature profiles

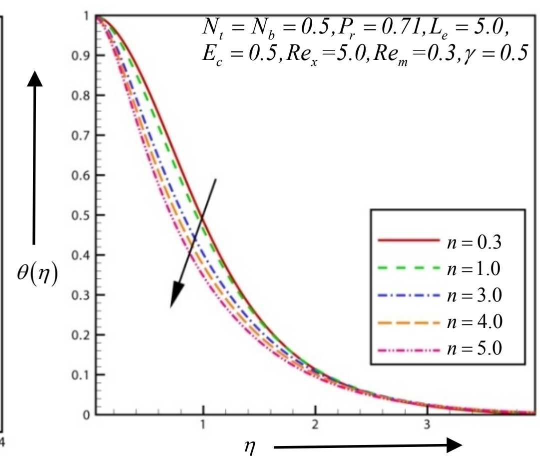

Fig. 5. Effects of exponent parameter (n) on temperature profiles

Figs. 6 to 8 portrays chemical species concentration profiles for varying values of physical parameters in the boundary layer. The species concentration is higher at the moving plate surface and decrease to zero far away from the plate also satisfying the boundary condition. Fig. 6 illustrates the dimensionless concentration distribution $\varphi(\eta)$ for different values of γ where $N_t = N_b = 0.5, P_r = 0.71, L_e = 5.0, E_c = 0.5, Re_x = 5.0, n = 1.0, Re_m = 0.3$ Then for above case It can be observed that concentration profiles are decreases as the γ increase. Fig. 7 presents the dimensionless concentration distribution $\varphi(\eta)$ for different values of Re_m where $N_t = N_b = 0.5, P_r = 0.71, L_e = 5.0, E_c = 0.5, Re_x = 5.0, n = 1.0, \gamma = 0.5$ Then for above case It can be observed that concentration profiles are decreases as the Re_m increase.

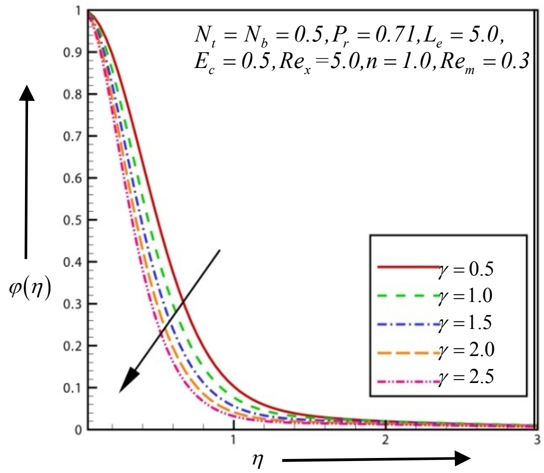

Fig. 6. Effects of chemical reaction parameter (γ) on concentration profiles

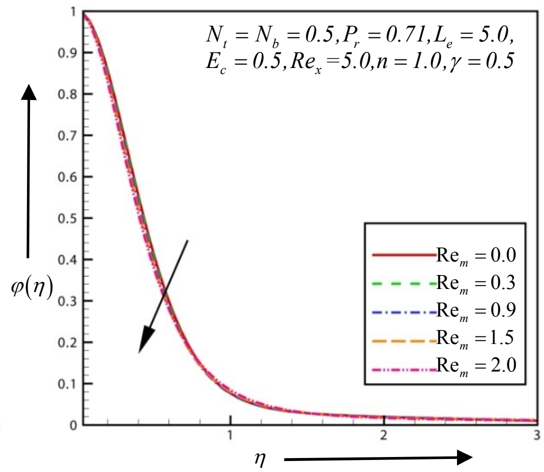

Fig. 7. Effects of magnetic parameter (Re_m) on concentration profiles

Fig. 8 displays the dimensionless concentration distribution $\varphi(\eta)$ for different values of n where $N_t = N_b = 0.5, P_r = 0.71, L_e = 5.0, E_c = 0.5, Re_x = 5.0, Re_m = 0.3, \gamma = 0.5$ Then for above case It can be observed that concentration profiles are decreases as the n increase.

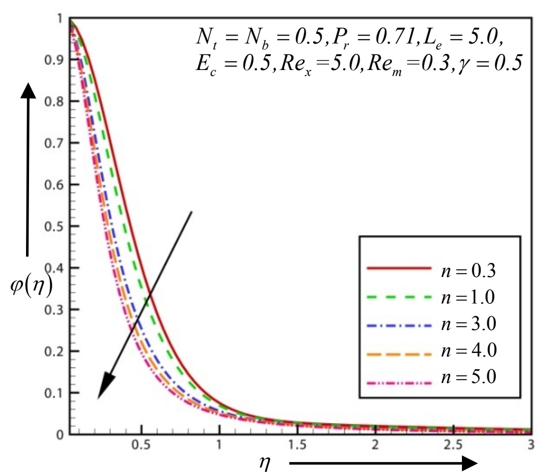

Fig. 8. Effects of exponent parameter (n) on concentration profiles

Since due to the physical interest of the problem, the skin-friction coefficient has been illustrates in Figs. 9 and 10. It depictsfor the different values of the magnetic field intensity and the power law index of the surface temperature and concentration variation.

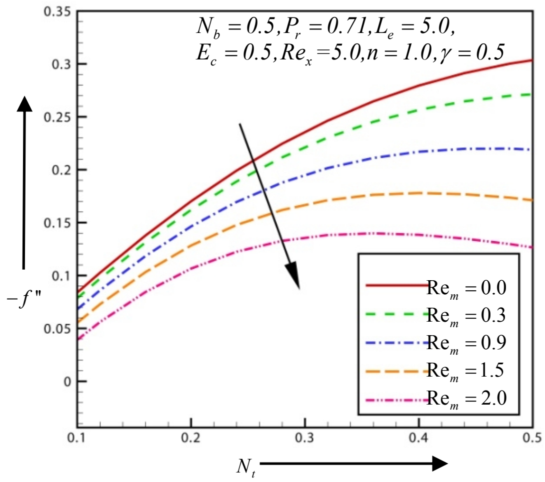

Fig. 9. Effects of Magnetic parameter (Re_m) on Skin-friction coefficient

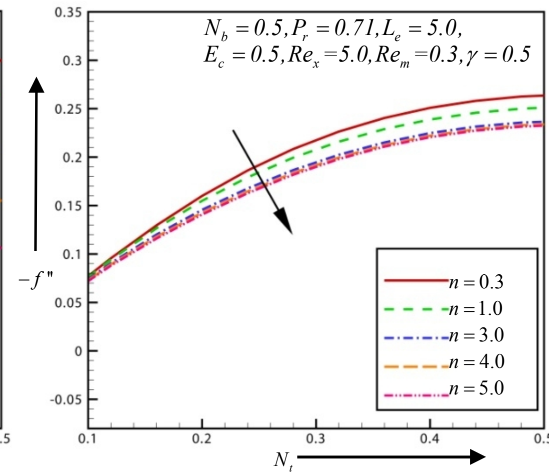

Fig. 10. Effects of Exponent parameter (n) on Skin-friction coefficient

Fig. 9 depicts the dimensionless Skin-friction coefficient for different values *of* Re_m *where* $N_b = 0.5, P_r = 0.71, L_e = 5.0, E_c = 0.5, Re_x = 5.0, n = 1.0, \gamma = 0.5$ *Then for above case It can be observed that Skin-friction coefficient are decreases as the* Re_m *increase.*

Fig.10 analysed the dimensionless Skin-friction coefficient for different values of n where $N_b = 0.5, P_r = 0.71, L_e = 5.0, E_c = 0.5, Re_x = 5.0, Re_m = 0.3, \gamma = 0.5$ Then for above case It can be observed that Skin-friction coefficient are decreases as the n increase.

The Surface heat transfer rate due to the physical interest of the problem illustrates Fig. 11 and 12 for the different values of the magnetic field intensity and the power law index of the surface temperature and concentration variation.

Fig. 11 shows the dimensionless heat transfer rate for different values of Re_m where $N_b = 0.5, P_r = 0.71, L_e = 5.0, E_c = 0.5, Re_x = 5.0, n = 1.0, \gamma = 0.5$ Then for above case It can be observed that heat transfer rate are increases as the Re_m increase.

Fig. 12 presents the dimensionless heat transfer rate for different values of n where $N_b = 0.5, P_r = 0.71, L_e = 5.0, E_c = 0.5, Re_x = 5.0, Re_m = 0.3, \gamma = 0.5$ Then for above case It can be observed that heat transfer rate are decreases as the n increase.

The rate of mass transfer due to physical interest of the problem depicts in Figs. 13-15 for the different values of the chemical reaction parameter, the power law index of the surface temperature and concentration variation and the magnetic field intensity.

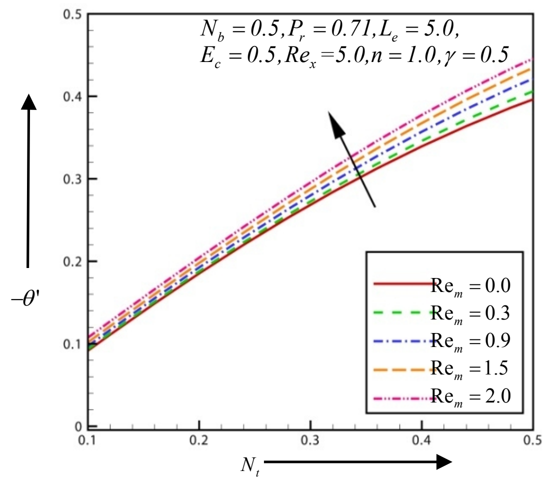

Fig. 11. Effects of Magnetic parameter (Re_m) on heat transfer rate

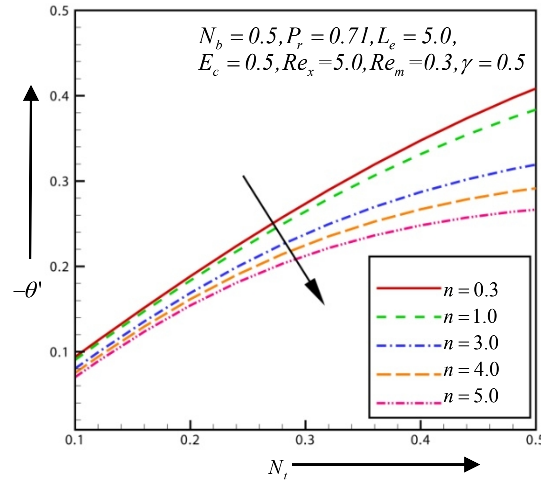

Fig. 12. Effects of Exponent parameter (n) on heat transfer rate

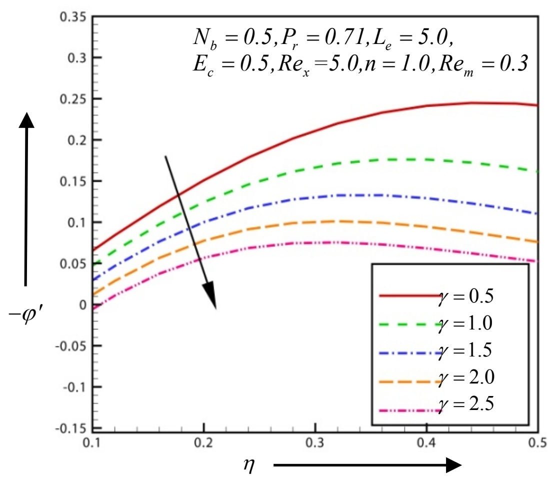

Fig. 13. Effects of chemical reaction parameter (γ) on mass transfer rate

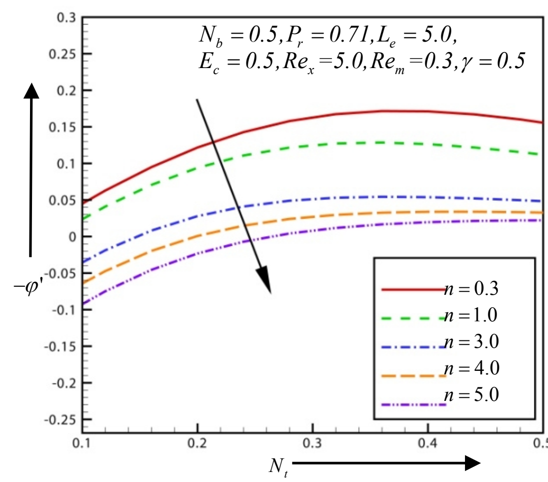

Fig. 14. Effects of exponent parameter (n) on mass transfer rate

Fig. 13 depicts the dimensionless mass transfer rate for different values of γ where $N_b = 0.5, P_r = 0.71, L_e = 5.0, E_c = 0.5, Re_x = 5.0, n = 1.0, Re_m = 0.3$ Then for above case It can be observed that mass transfer rate are decreases as the γ increase.

Fig. 14 presents the dimensionless mass transfer rate for different values of n where $N_b = 0.5, P_r = 0.71, L_e = 5.0, E_c = 0.5, Re_x = 5.0, Re_m = 0.3, \gamma = 0.5$ Then for above case It can be observed that mass transfer rate are decreases as the n increase.

Fig. 15 displays the dimensionless mass transfer rate for different values of Re_m where $N_b = 0.5, P_r = 0.71, L_e = 5.0, E_c = 0.5, Re_x = 5.0, n = 1.0, \gamma = 0.5$ Then for above case It can be observed that mass transfer rate are decreases as the Re_m increase.

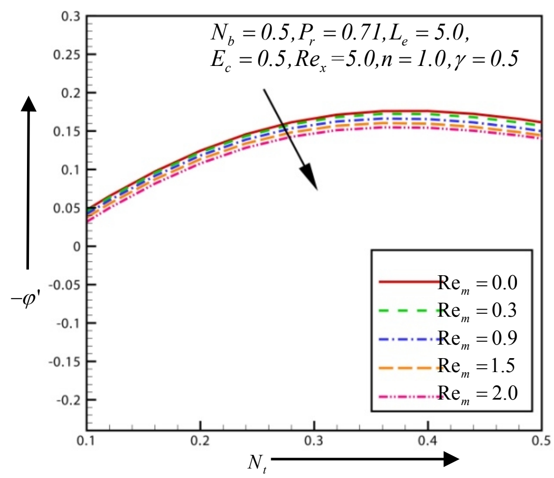

Fig. 15. Effects of magnetic parameter (Re_m) on mass transfer rate

6. CONCLUSION

MHD boundary layer flow flow of a nanofluid on a continuously moving surface with chemical reaction has been studied numerically. The comparison part has provides the accuracy of the present study. The results revealed that velocity and concentration decreases, whereas the temperature increase with increase in the magnetic field intensity parameter. Also, an increase in the power law indexes of the surface temperature and concentration variation the velocity increase, whereas the temperature and concentration decrease gradually. The concentration also decreases as increase chemical reaction. Furthermore an increase in the power law index cause decrease of skin-friction coefficient and the heat and mass transfer rate at the moving plate surface.

7. NOMENCLATURE

a	Non-dimensional constant
A, B	Constants
B_0	Magnetic field induction
C	concentration
C_f	Skin-friction coefficient
c_p	Specific heat at constant pressure
D_B	Brownian diffusion coefficient
D_T	Thermophoresis diffusion coefficient
K_r	Rate of chemical reaction
L_e	Lewis number
n	Exponent parameter
N_u	Nusselt number
N_b	Brownian motion parameter

N_t	Thermophoresis parameter
P	Fluid pressure
P_r	Prandtl number
Re_m	Magnetic parameter
R_{e_x}	Local Reynolds number
S_h	Sherwood number
T	Fluid temperature
u, v	Velocity components in the *x* and *y* directions
U	Uniform velocity
x, y	Cartesian coordinates

Greek symbols

μ , ν_∞	Dynamic and kinematic viscosities
μ_∞	Reference viscosity
$(\rho c)_p$	Effective heat capacity of the nanofluid
$(\rho c)_f$	Heat capacity of the fluid
α	Thermal diffusivity
β	Co-efficient of thermal expansion
γ	Chemical reaction parameter
η	Similarity variable
ψ	Stream function
$f'(\eta)$	Dimensionless velocity
$\theta(\eta)$	Dimensionless temperature
$\varphi(\eta)$	Dimensionless concentration

Superscript

/	Differentiation with respect to η

Subscripts

w	Surface condition
∞	Condition far away the surface

ACKNOWLEDGEMENTS

The authors are very thankful to the editor and reviewers for their constructive comments and valuable suggestions to improve this research paper.

COMPETING INTERESTS

The authors declare that they have no competing interests.

REFERENCES

1. Choi SUS. Enhancing thermal conductivity of fluids with nanoparticles, development and applications of non- Newtonian flows. D.A. Siginer and H. P. Wang, eds., ASME MD- vol. 231 and FED-vol. 66, USDOE, Washington, DC (United States). 1995;99-105.
2. Sakiadis BC. Boundary layer behavior on continuous solid surfaces: I. boundary layer equations for two dimensional and axisymmetric flows. AIChE J. 1961;7:26–28.
3. Sakiadis BC. Boundary layer behavior on continuous solid surfaces: II. Boundary layer on a continuous flat surface. AIChE J. 1961;7:221–225.
4. Tsou FK, Sparrow EM, Goldstein RJ. Flow and heat transfer in the boundary layer on a continuous moving surface.Int J Heat and Mass Transfer. 1967;10:219-235.
5. Soundalgekar VM, MurtyTV. Heat transfer in flow past a continuous moving plate with variable temperature. Warme-und stoffubertrag-ung. 1980;14:91-93.
6. Erickson LE, Fan LT, Fox VG. Heat and Mass transfer on moving continuous flat plate with suction or injection. Ind. Engg. Chem. Fundam. 1966;5:19-25.
7. Wang CY. Free convection on a vertical stretching surface. Journal of Applied mathematics and Mechanics (ZAMM). 1989;69:418-420.
8. Gorla RSR, Sidawi I. Free convection on a vertical stretching surface with suction and blowing. Applied Scientific Research. 1994;52:247-252.
9. Pop I, Gorla RSR, Rashidi M. The effect of variable viscosity on flow and heat transfer to a continuously moving flat plate. Int J Eng Sci. 1992;30:1-6.
10. Howell TG, Jeng DR, De Witt KJ. Momentum and heat transfer on continuous moving surface in a power law fluid. Int. J. Heat and Mass Transfer. 1997;40:1853-1861.
11. Rao JH,Jeng DRDR, De Witt KJ. Momentum and heat transfer in a power- law fluid with arbitrary injection / suction at a moving wall. Int J Heat Mass Transf. 1999;42:2837-2847.
12. Fang T. Similarity solutions for a moving flat plate thermal boundary layer. Actaech. 2003;163:161-172.
13. Soundalgekar VM, Takhar HS, DasUN, Deka RK,Sarmah A. Effect of variable viscosity on boundary layer flow along a continuously moving plate with variable surface temperature. Heat and Mass Transfer. 2004;40:421-424.
14. Ibrahim SY, Makinde OD. Chemically reacting MHD boundary layer flow of heat and mass transfer past a moving vertical plate with suction. Scientific Research and Essays. 2010;5:2875-2882.
15. Makinde OD, OguluA. The effect of thermal radiation on the heat and mass transfer flow of a variable viscosity fluid past a vertical porous plate permeated by a transverse magnetic field. Chemical Engineering Communications. 2008;195:1575-1584.
16. Geetha P, Moorthy MBK. MHD Boundary Layer Flow of Heat and Mass Transfer on a Continuously Moving Surface with Chemical Reaction. European Journal of Scientific Research. 2011;56:354-363.
17. Kang HU, Kim SH, Oh JM. Estimation of Thermal Conductivity of Nanofluid Using Experimental Effective Particle Volume. Exp. Heat Transfer. 2006;19:181-191.
18. Kuznetsov AV, Nield DA. The Cheng–Minkowycz Problem for Natural Convective Boundary-layer Flow in a Porous Medium Saturated by a Nanofluid. Int J of Heat and Mass Transfer. 2009;52:5792-5795.
19. Kuznetsov AV, Nield DA. Natural Convective Boundary- layer Flow of a Nanofluid Past a Vertical Plate. Int J of Thermal Sci. 2010;49:243-247.
20. Bachok N, Ishak A, Pop I. Boundary-layer Flow of Nanofluids over a Moving Surface in a Flowing Fluid. Int J of Thermal Sci. 2010;49:1663-1668.

21. Khan WA, Pop I. Boundary-Layer Flow of a Nanofluid Past a Stretching Sheet. Int J of Heat and Mass Trans. 2010;53:2477-2483.
22. Khan WA, Pop I. Free Convection Boundary Layer Flow Past a Horizontal Flat Plate Embedded in a Porous Medium Filled with a Nanofluid. J Heat Trans. 2011;133.
23. Khan MS, Alam MM, Ferdows M. Finite Difference Solution of MHD Radiative Boundary Layer Flow of a Nanofluid past a Stretching Sheet. Proceeding of the International Conference of Mechanical Engineering (ICME-11), FL-011, BUET, Dhaka, Bangladesh; 2011.
24. Khan MS, Alam MM, Ferdows M. MHD Radiative Boundary Layer Flow of a Nanofluid past a Stretching Sheet. Proceeding of the International Conference of Mechanical Engineering and Renewable Energy (ICMERE-11), PI-105, CUET, Chittagong, Bangladesh; 2011.
25. Khan MS, Karim I, Ali LE, Islam A. MHD Free Convection Boundary layer Unsteady Flow of a Nanofluid along a stretching sheet with thermal Radiation and Viscous Dissipation Effects. International Nano Letters. 2012;2:24.
26. Ferdows M, Khan MS, Alam MM, Sun S. MHD Mixed convective boundary layer flow of a nanofluid through a porous medium due to an Exponentially Stretching sheet. Mathematical problems in Engineering. 2012;3:2551-1557.
27. Khan MS, Wahiduzzaman M, Sazad MAK, Uddin MS. Finite difference solution of MHD free convection heat and mass transfer flow of a nanofluid along a Stretching sheet with Heat generation effects. Indian Journal of Theoretical Physics. 2012;60:285-306.
28. Khan MS, Alam MM, Ferdows M. Effects of Magnetic field on Radiative flow of a Nanofluid past a Stretching Sheet. Procedia Engineering, Elsevier. 2013;56:316-322.
29. Ferdows M, Khan MS, Alam MM, Bég OA. Numerical Study of Transient Magnetohydrodynamic Radiative Free Convection Nanofluid Flow from a Stretching Permeable Surface. Journal of Process Mechanical Engineering. 2013:1-16.
30. Beg OA, Khan MS, Karim I, Ferdows M, Alam MM. Explicit Numerical study of Unsteady Hydromagnetic Mixed Convective Nanofluid flow from an Exponential Stretching sheet in Porous media. Applied Nanoscience; 2013.
31. Lotfizadeh Dehkordi B, et al. Investigation of viscosity and thermal conductivity of alumina nanofluids with addition of SDBS. Heat and Mass Transfer. 2013;49(8):1109-1115.
32. Hassan M, et al. "Numerical Study of Entropy Generation in a Flowing Nanofluid Used in Micro-and Minichannels". Entropy. 2013;15(1):144-155.
33. Nachtsheim PR, Swigert P. Satisfaction of the asymptotic boundary conditions in numerical solution of the system of non-linear equations of boundary layer type. NASA, TND-3004; 1965.
34. Lai FC, Kulacki FA. The effect of variable viscosity on convective heat and masstransfer in saturated porous media. Int J Heat and Mass Transfer. 1991;33:1028-1031.

Peer-review history:
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